

## 5.4 CONVOLUTION SUM

The convolution sum provides a concise, mathematical way to express the output of an LTI system based on an arbitrary discrete-time input signal and the system's response. The convolution sum is expressed as

$$y[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k]$$

1. Convolution is commutative

$$x[n] * h[n] = h[n] * x[n]$$

2. Convolution is Distributive

$$x[n] * (h_1[n] + h_2[n]) = x[n] * h_1[n] + x[n] * h_2[n]$$

3. System connected in cascade

$$y[n] = h_1[n] * [h_2[n] * x[n]] = [h_1[n] * h_2[n]] * x[n]$$

4. System connected in parallel

$$y[n] = h_1[n] * x[n] + h_2[n] * x[n] = [h_1[n] + h_2[n]] * x[n]$$

LTI Systems are said to be stable if ,

$$\sum_{k=-\infty}^{\infty} |h[k]| < \infty$$

LTI system are causal if,

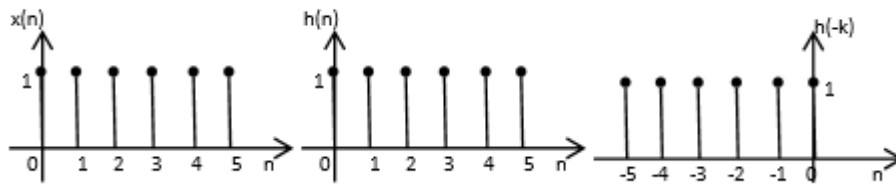
$$h(n) = 0, n < 0.$$

EXAMPLE 1: Convolve the following discrete time signals using graphical convolution  $x(n) = h(n) = u(n)$ .

*Solution:*

$$x(n) = u(n) = 1; n \geq 0$$

$$h(n) = u(n) = 1; n \geq 0$$



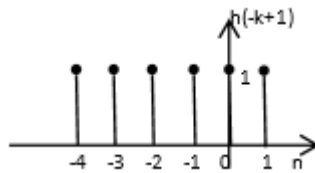
$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

when  $n = 0$

$$y(0) = (1)(1) = 1$$

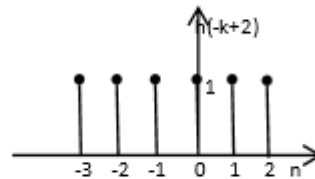
when  $n = 1$

$$y(1) = (1)(1) + (1)(1) = 2$$



when  $n = 2$

$$y(2) = (1)(1) + (1)(1) + (1)(1) = 3$$



$$\therefore y(n) = \{1, 2, 3, 4, 5, \dots\}$$

Example 2: Compute linear convolution  $x(n) = \{2, 2, 0, 1, 1\}$   $h(n) = \{1, 2, 3, 4\}$ .

*Solution:*

	2	2	0	1	1
1	2	2	0	1	1
2	4	4	0	2	2
3	6	6	0	3	3
4	8	8	0	4	4

$$y(n) = x(n) * h(n) = \{2, 6, 10, 15, 11, 5, 7, 4\}$$