

## 2.2 FOURIER TRANSFORM

The Fourier representation of periodic signals has been extended to non-periodic signals by letting the fundamental period  $T$  tend to infinity and this Fourier method of representing non-periodic signals as a function of frequency is called Fourier transform.

Definition of Continuous time Fourier Transform

The Fourier transform (FT) of Continuous time signals is called Continuous Time Fourier Transform

Let  $x(t) = \text{continuous time signal}$

$$X(j\omega) = F\{x(t)\}$$

The Fourier transform of continuous time signal,  $x(t)$  is defined as

$$X(j\omega) = F\{x(t)\} = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

Conditions for existence of Fourier transform

The Fourier transform  $x(t)$  exist if it satisfies the following Dirichlet's condition

1.  $x(t)$  should be absolutely integrable

$$\text{ie, } \int_{-\infty}^{\infty} x(t) dt < \infty$$

2.  $x(t)$  should have a finite number of maxima and minima with in any finite interval.
3.  $x(t)$  should have a finite number of discontinuities with in any interval.

Definition of Inverse Fourier Transform

The inverse Fourier Transform of  $X(j\omega)$  is defined as,

$$x(t) = F^{-1}\{X(j\omega)\} = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega$$

**EXAMPLE 1:** Find Fourier transform of impulse signal.

Solution:

$$F\{x(t)\} = X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t} dt$$

$$F[\delta(t)] = \int_{-\infty}^{\infty} \delta(t)e^{-j\omega t} dt$$

$$[\because \text{Impulse signal } \delta(t) = \begin{cases} 1 & \text{for } t = 0 \\ 0 & \text{for } t \neq 0 \end{cases}]$$

$$F[\delta(t)] = \delta(0)e^{-j\omega(0)} = 1$$

**EXAMPLE 2:** Find Fourier transform of double sided exponential signal.

Solution:

$$F[e^{-a|t|}] = \begin{cases} e^{-at} & : t \geq 0 \\ e^{at} & : t \leq 0 \end{cases}$$

$$F[e^{-a|t|}] = \int_{-\infty}^0 e^{at} e^{-j\omega t} dt + \int_0^{\infty} e^{-at} e^{-j\omega t} dt$$

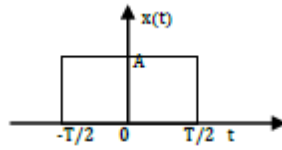
$$= \int_{-\infty}^0 e^{(a-j\omega)t} dt + \int_0^{\infty} e^{-(a+j\omega)t} dt$$

$$= \left[ \frac{e^{(a-j\omega)t}}{(a-j\omega)} \right]_{-\infty}^0 + \left[ \frac{e^{-(a+j\omega)t}}{-(a+j\omega)} \right]_0^{\infty}$$

$$= \frac{1}{(a-j\omega)} + \frac{1}{(a+j\omega)} = \frac{a+j\omega+a-j\omega}{a^2+\omega^2}$$

$$= \frac{2a}{a^2+\omega^2}$$

**EXAMPLE 3:** Find Fourier transform of rectangular pulse function shown in figure



Solution:

$$x(t) = \pi(t) = A; \frac{-T}{2} \leq t \leq \frac{T}{2}$$

$$\begin{aligned} F[\pi(t)] &= \int_{-\frac{T}{2}}^{\frac{T}{2}} A e^{-j\omega t} dt = A \left[ \frac{e^{-j\omega t}}{-j\omega} \right]_{-\frac{T}{2}}^{\frac{T}{2}} \\ &= \frac{A}{-j\omega} \left[ e^{-j\omega \frac{T}{2}} - e^{-j\omega \left(-\frac{T}{2}\right)} \right] = \frac{2A}{j\omega} \left[ \frac{e^{j\omega \frac{T}{2}} - e^{-j\omega \frac{T}{2}}}{2} \right] = \frac{2A}{\omega} \sin \omega \frac{T}{2} \\ &= \frac{2A}{\omega T} T \sin \omega \frac{T}{2} = AT \frac{\sin \omega \frac{T}{2}}{\omega \frac{T}{2}} = AT \operatorname{sinc} \omega \frac{T}{2} \end{aligned}$$

**EXAMPLE 4:** Find inverse Fourier transform  $X(j\omega) = \delta(\omega)$ .

Solution:

$$F^{-1}[X(j\omega)] = F^{-1}[\delta(\omega)]$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} \delta(\omega) e^{j\omega t} d\omega = \frac{1}{2\pi} [1]$$

$$\delta(\omega) = \begin{cases} 1 & \text{for } \omega = 0 \\ 0 & \text{for } \omega \neq 0 \end{cases}$$

$$F^{-1}[\delta(\omega)] = \frac{1}{2\pi}$$