

## UNIT IV

### NOISE CHARACTERISATION

#### Noise Source Introduction:

Noise is often described as the limiting factor in communication systems: indeed if there was no noise there would be virtually no problem in communications. Noise is a general term which is used to describe an unwanted signal which affects a wanted signal. These unwanted signals arise from a variety of sources which may be considered in one of two main categories:-

- a) Interference, usually from a human source (man made)
- b) Naturally occurring random noise.

Interference arises for example, from other communication systems (cross talk), 50 Hz supplies (hum) and harmonics, switched mode power supplies, thyristor circuits, ignition (car spark plugs) motors ... etc. Interference can in principle be reduced or completely eliminated by careful engineering (i.e. good design, suppression, shielding etc). Interference is essentially deterministic (i.e. random, predictable), however observe.

When the interference is removed, there remains naturally occurring noise which is essentially random (non-deterministic),. Naturally occurring noise is inherently present in electronic communication systems from either external sources or internal sources.

Naturally occurring external noise sources include atmosphere disturbance (e.g. electric storms, lightning, ionospheric effect etc), so called 'Sky Noise' or Cosmic noise which includes noise from galaxy, solar noise and hot spot due to oxygen and water vapor resonance in the earth's atmosphere. These sources can seriously affect all forms of radio transmission and the design of a radio system (i.e. radio, TV, satellite) must take these into account. The figure 4.1.1 below shows noise

temperature (equivalent to noise power, we shall discuss later) as a function of frequency for sky noise.

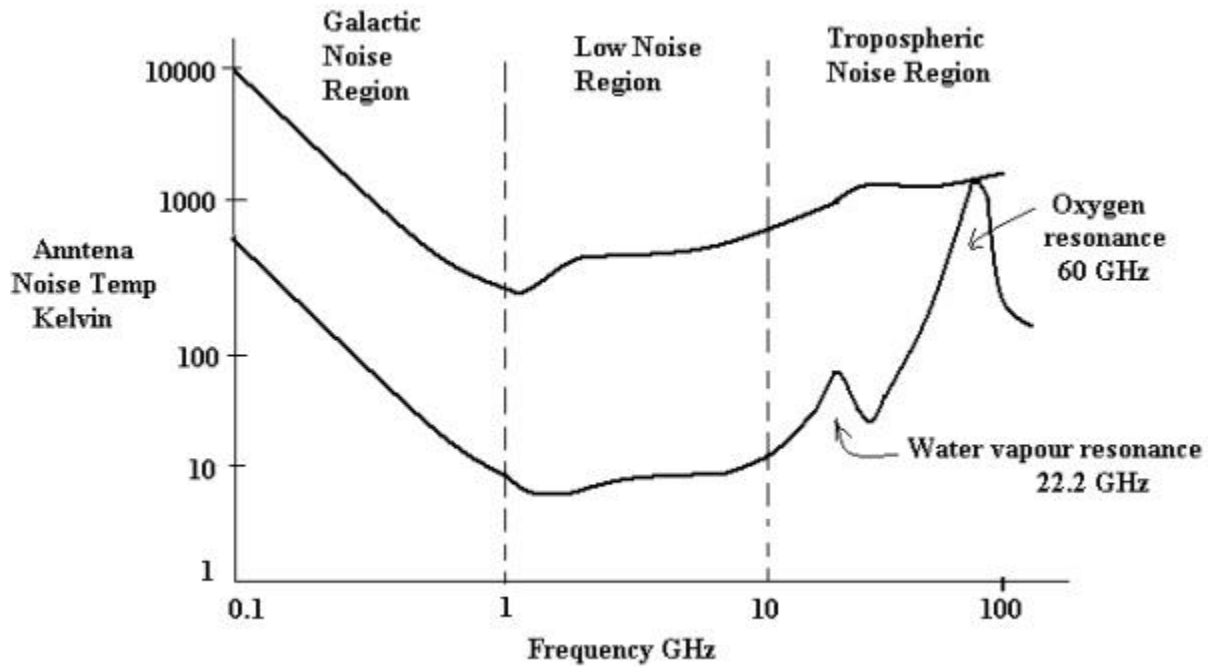


Fig 4.1.1 Noise temperature as a function of frequency for sky noise

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The upper curve represents an antenna at low elevation ( $\sim 5^\circ$  above horizon), the lower curve represents an antenna pointing at the zenith (i.e.  $90^\circ$  elevation).

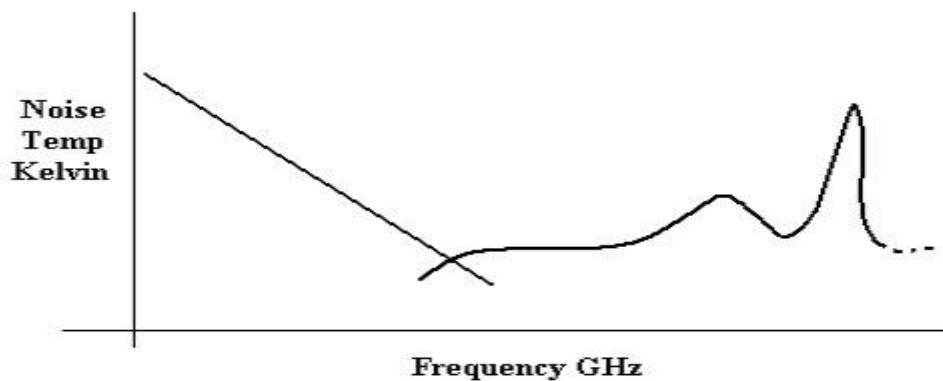


Figure 4.1.2 Relationship curve for noise temperature as a function of frequency

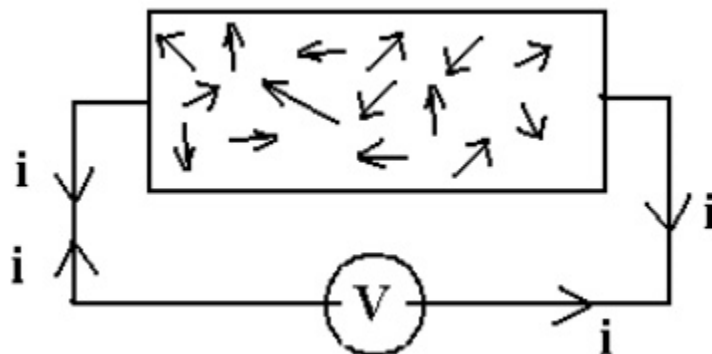
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Contributions to the above diagram are from galactic noise and atmospheric noise as shown below. Note that sky noise is least over the band – 1 GHz to 10 GHz shown in the figure 4.1.2. This is referred to as a low noise ‘window‘ or region and is the main reason why satellite links operate at frequencies in this band (e.g. 4 GHz, 6GHz, 8GHz). Since signals received from satellites are so small it is important to keep the background noise to a minimum.

Naturally occurring internal noise or circuit noise is due to active and passive electronic devices (e.g. resistors, transistors ...etc) found in communication systems. There are various mechanism which produce noise in devices; some of which will be discussed in the following sections.

### **Thermal Noise (Johnson Noise):**

This type of noise is generated by all resistances (e.g. a resistor, semiconductor, the resistance of a resonant circuit shown in figure 4.1.3, i.e. the real part of the impedance, cable etc). Free electrons are in contact random motion for any temperature above absolute zero (0 degree K, ~ -273 degree C). As the temperature increases, the random motion increases, hence thermal noise, and since moving electron constitute a current, although there is no net current flow, the motion can be measured as a mean square noise value across the resistance.



**Figure 4.1.3 Circuit Diagram of Thermal Noise Voltage**

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Experimental results (by Johnson) and theoretical studies (by Nyquist) give the mean square noise

$$\text{voltage as } \bar{V}^2 = 4kTBR \text{ (volt}^2\text{)}$$

Where  $k$  = Boltzmann's constant =  $1.38 \times 10^{-23}$  Joules per K

$T$  = absolute temperature

$B$  = bandwidth noise measured in (Hz)

$R$  = resistance (ohms)

The law relating noise power,  $N$ , to the temperature and bandwidth is

$$N = kTB \text{ watts}$$

These equations will be discussed further in later section.

The equations above held for frequencies up to  $> 10^{13}$  Hz (10,000 GHz) and for at least all practical temperatures, i.e. for all practical communication systems they may be assumed to be valid. Thermal noise is often referred to as 'white noise' because it has a uniform 'spectral density'.

Note – noise power spectral density is the noise power measured in a 1 Hz bandwidth i.e. watts per Hz. A uniform spectral density means that if we measured the thermal noise in any 1 Hz bandwidth from  $\sim 0\text{Hz} \rightarrow 1\text{ MHz} \rightarrow 1\text{GHz} \dots\dots\dots 10,000\text{ GHz}$  etc we would measure the same amount of noise. From the equation  $N=kTB$ , noise power spectral density is  $P_o$  is proportional to  $kT$  watts per Hz. I.e. Graphically figure 4.1.4 is shown as,

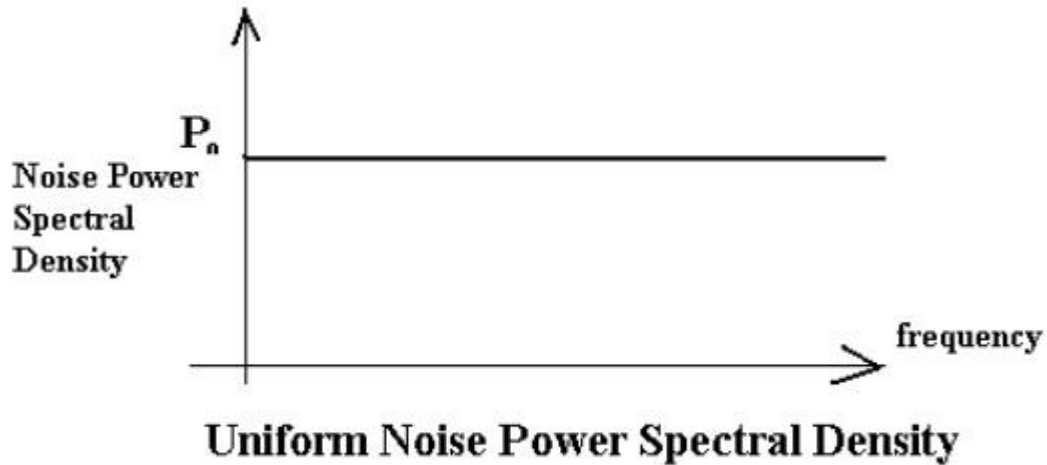


Figure 4.1.4 Uniform Noise Power Spectral Density

*Diagram Source Electronics Post*

### Shot Noise:

Shot noise was originally used to describe noise due to random fluctuations in electron emission from cathodes in vacuum tubes (called shot noise by analogy with lead shot). Shot noise also occurs in semiconductors due to the liberation of charge carriers, which have discrete amount of charge, in to potential barrier region such as occur in pn junctions. The discrete amounts of charge give rise to a current which is effectively a series of current pulses.

For pn junctions the mean square shot noise current is

$$I_n^2 = 2(I_{DC} + 2I_o)q_e B \quad (\text{amps})^2$$

Where

$I_{DC}$  is the direct current as the pn junction (amps)

$I_o$  is the reverse saturation current (amps)

$q_e$  is the electron charge =  $1.6 \times 10^{-19}$  coulombs

B is the effective noise bandwidth (Hz)

Shot noise is found to have a uniform spectral density as for thermal noise.

**Low Frequency Or Flicker Noise:**

Active devices, integrated circuit, diodes, transistors etc also exhibits a low frequency noise, which is frequency dependent (i.e. non uniform) known as flicker noise or ‘one – over –  $f^n$ ’ noise. The mean square value is found to be proportional to  $(1/f)$  where  $f$  is the frequency and  $n= 1$ . Thus the noise at higher frequencies is less than at lower frequencies. Flicker noise is due to impurities in the material which in turn cause charge carrier fluctuations.

**Excess Resistor Noise:**

Thermal noise in resistors does not vary with frequency, as previously noted, by many resistors also generates as additional frequency dependent noise referred to as excess noise. This noise also exhibits a  $(1/f)$  characteristic, similar to flicker noise. Carbon resistor generally generates most excess noise whereas wire wound resistors usually generates negligible amount of excess noise. However the inductance of wire wound resistor limits their frequency and metal film resistor are usually the best choices for high frequency communication circuit where low noise and constant resistance are required.

**Burst Noise Or Popcorn Noise:**

Some semiconductors also produce burst or popcorn noise with a spectral density which is proportional to  $(1/f)^2$

**General Comments:**

The figure 4.1.5 below illustrates the variation of noise with frequency.

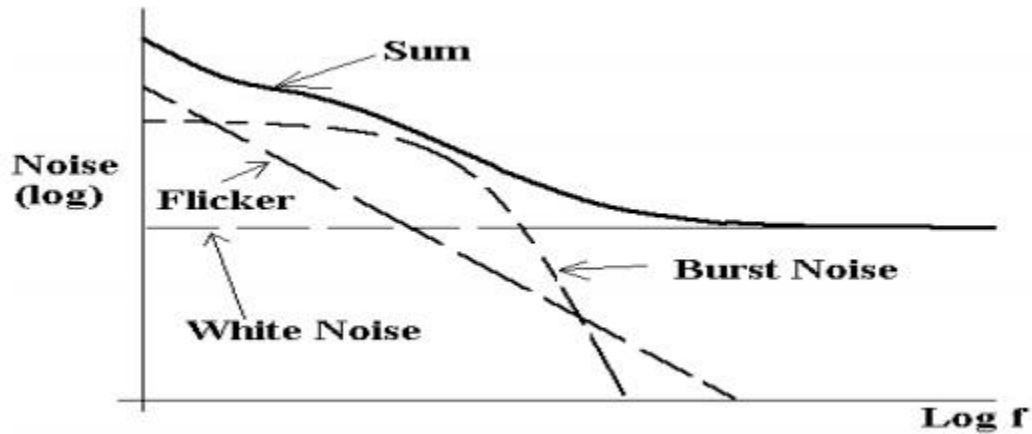


Figure 4.1.5 Variation of noise with frequency

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For frequencies below a few KHz (low frequency systems), flicker and popcorn noise are the most significant, but these may be ignored at higher frequencies where 'white' noise predominates.

Thermal noise is always present in electronic systems. Shot noise is more or less significant depending upon the specific devices used for example as FET with an insulated gate avoids junction shot noise. As noted in the preceding discussion, all transistors generate other types of non-white noise which may or may not be significant depending on the specific device and application. Of all these types of noise source, white noise is generally assumed to be the most significant and system analysis is based on the assumption of thermal noise. This assumption is reasonably valid for radio systems which operate at frequencies where non-white noise is greatly reduced and which have low noise 'front ends' which, as shall be discussed, contribute most of the internal (circuit) noise in a receiver system. At radio frequencies the sky noise contribution is significant and is also (usually) taken into account.

Obviously, analysis and calculations only gives an indication of system performance. Measurements of the noise or signal-to-noise ratio in a system include all the noise, from whatever source, present at the time of measurement and within the constraints of the measurements or system bandwidth.

Before discussing some of these aspects further an overview of noise evaluation as applicable to communication systems will first be presented.

### Noise Evaluation:

#### Overview:

It has been stated that noise is an unwanted signal that accompanies a wanted signal, and, as discussed, the most common form is random (non-deterministic) thermal noise. The essence of calculations and measurements is to determine the signal power to Noise power ratio, i.e. the (S/N) ratio or (S/N) expression in dB.

i.e. Let S= signal power (mW)

N = noise power (mW)

$$\left(\frac{S}{N}\right)_{ratio} = \frac{S}{N}$$

$$\left(\frac{S}{N}\right)_{dB} = 10 \log_{10} \left(\frac{S}{N}\right)$$

*Also recall that*

$$S_{dBm} = 10 \log_{10} \left(\frac{S(mW)}{1mW}\right)$$

$$\text{and } N_{dBm} = 10 \log_{10} \left(\frac{N(mW)}{1mW}\right)$$

$$\text{i.e. } \left(\frac{S}{N}\right)_{dB} = 10 \log_{10} S - 10 \log_{10} N$$

$$\left(\frac{S}{N}\right)_{dB} = S_{dBm} - N_{dBm}$$



Noise, which accompanies the signal is usually considered to be additive (in terms of powers) and its often described as Additive White Gaussian Noise, AWGN, noise. Noise and signals may also Powers are usually measured in dBm (or dBw) in communications systems. The equation  $(S/N)_{dB} = S_{dBm} - N_{dBm}$  is often the most useul. The  $(S/N)$  at various stages in a communication system gives an indication of system quality and performance in terms of error rate in digital data communication systems and ‘fidelity’ in case of analogue communication systems. (Obviously, the larger the  $(S/N)$  , the better the system will be).

AWGN.In order to evaluate noise various mathematical models and techniques have to be used, particularly concepts from statistics and probability theory, the major starting point being that random noise is assumed to have a Gaussian or Normal distribution.

We may relate the concept of white noise with a Gaussian distribution as follows:

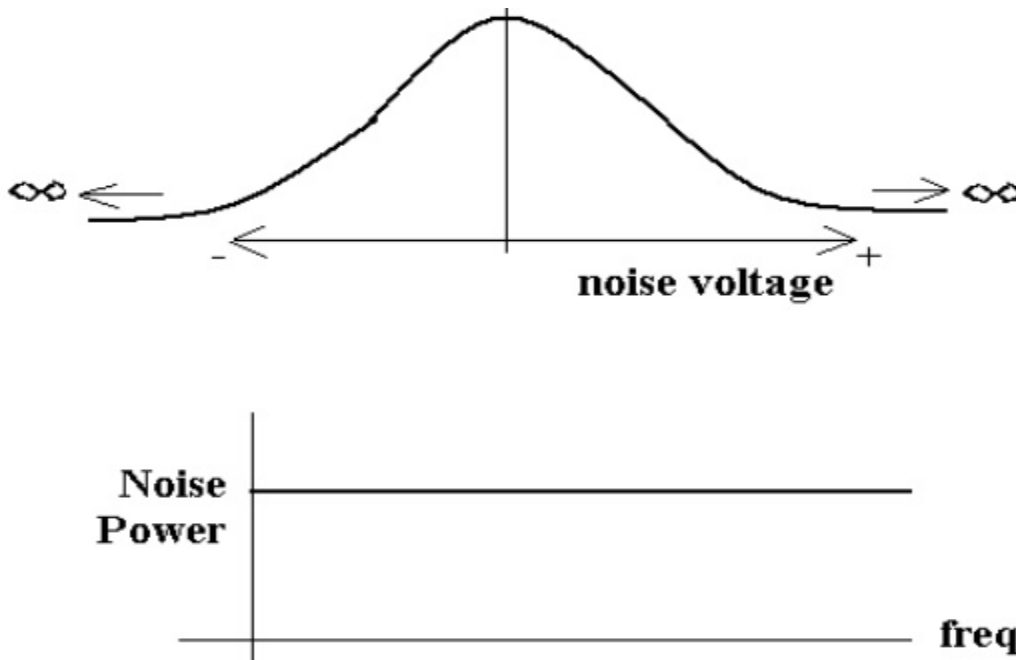
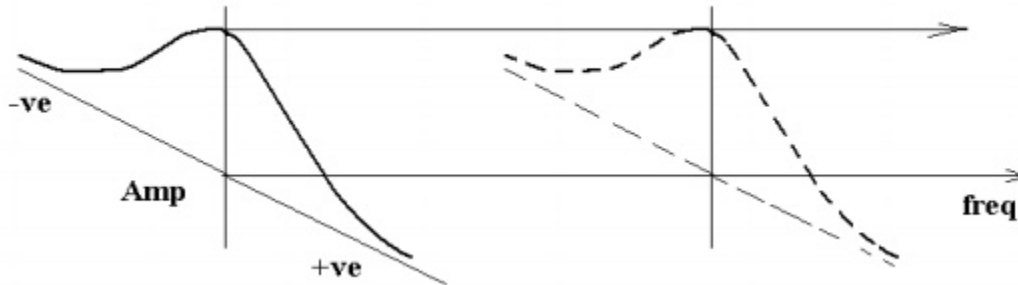


Figure 4.1.6 Probability of Noise Vs Voltage and Frequency Vs Noise Power

*Diagram Source Brain Kart*

Gaussian distribution – ‘graph’ shows Probability of noise voltage vs voltage – i.e. most probable noise voltage is 0 volts (zero mean) shown in figure 4.1.6. There is a small probability of very large +ve or –ve noise voltages. White noise – uniform noise power from ‘DC’ to very high frequencies. Although not strictly consistent, we may relate these two characteristics of thermal noise as follows:



**Figure 4.1.7 Characteristics of Thermal Noise**

*Diagram Source Brain Kart*

The probability of amplitude of noise at any frequency or in any band of frequencies (e.g. 1 Hz, 10Hz... 100 KHz .etc) is a Gaussian distribution. Noise may be quantified in terms of noise power spectral density,  $p_0$  watts per Hz, from which Noise power shown in the figure 4.1.7 may be expressed as

$$N = p_0 B_n \text{ watts}$$

Where  $B_n$  is the equivalent noise bandwidth, the equation assumes  $p_0$  is constant across the band (i.e. White Noise).

Note -  $B_n$  is not the 3dB bandwidth, it is the bandwidth which when multiplied by  $p_0$

Gives the actual output noise power  $N$ . This is illustrated further below the figure 4.1.8.

Ideal low pass filter

$$\text{Bandwidth } B \text{ Hz} = B_n$$

$$N = p_0 B_n \text{ watts}$$

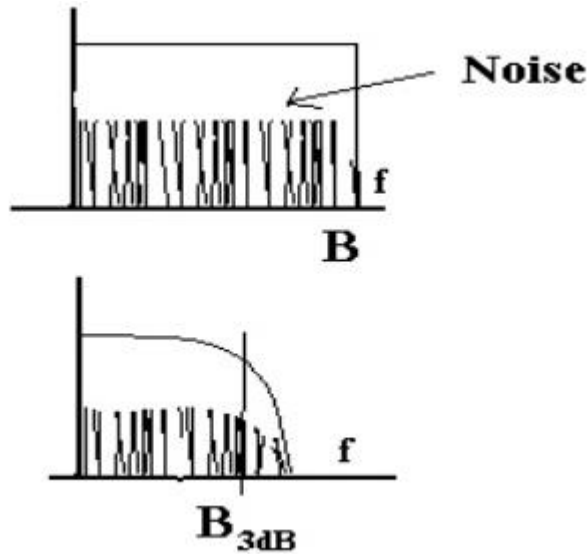


Figure 4.1.8 Basic ideal Low Pass Filter

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### Practical LPF

3 dB bandwidth shown, but noise does not suddenly cease at B<sub>3dB</sub>

Therefore, B<sub>n</sub> > B<sub>3dB</sub>, B<sub>n</sub> depends on actual filter.

$$N = p_0 B_n$$

In general the equivalent noise bandwidth is > B<sub>3dB</sub>.

Alternatively, noise may be quantified in terms of ‘mean square noise’ i.e. bar (V<sup>2</sup>), which is effectively a power. From this a ‘Root mean square (RMS)’ value for the noise voltage may be determined.

$$\text{i.e. RMS} = \sqrt{\bar{V^2}}$$

In order to ease analysis, models based on the above quantities are used. For example, if we imagine noise in a very narrow bandwidth, δf, as δf → df, the noise approaches a sine wave (with frequency ‘centred’ in df). Since an RMS noise voltage can be determined, a ‘peak’ value of the noise may be invented since for a sine wave

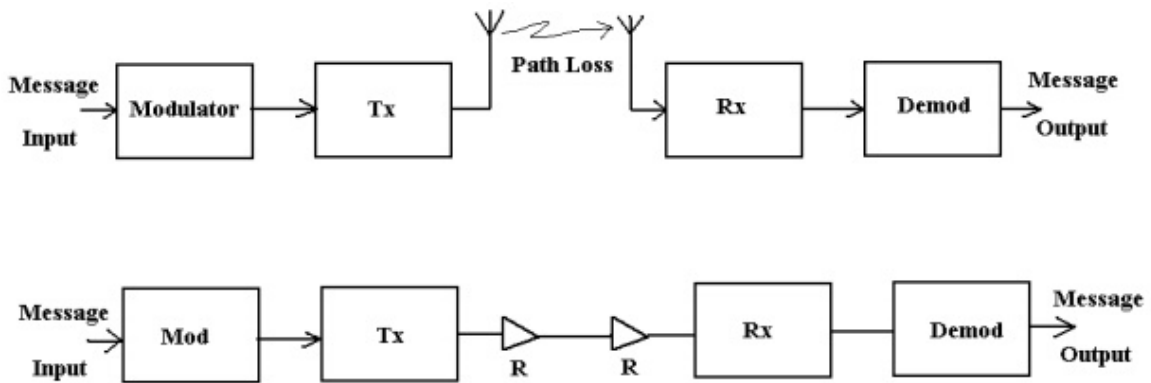
$$\text{RMS} = \frac{\text{Peak}}{\sqrt{2}}$$

Note – the peak value is entirely fictitious since in theory the noise with a Gaussian distribution could have a peak value of + ∞ or - ∞ volts.

Hence we may relate

Mean square → RMS →  $\sqrt{2}$  (RMS) → Peak noise voltage (invented for convenience)

Problems arising from noise are manifested at the receiving end of a system and hence most of the analysis relates to the receiver / demodulator with transmission path loss and external noise sources (e.g. sky noise) if appropriate, taken into account.



**Figure 4.1.9 Block Diagram of Communication Systems**

*Diagram Source Electronic Tutorials*

R = repeater (Analogue) or Regenerators (digital)

Figure 4.1.9 shows the block Diagram of Communication Systems. These systems may facilitate analogue or digital data transfer

**Thermal Noise (Johnson noise)**

The thermal noise in a resistance R has a mean square value given by

$$\overline{V^2} = 4kTBR \text{ (volt}^2\text{)}$$

Where k = Boltzmann's constant =  $1.38 \times 10^{-23}$  Joules per K

T = absolute temperature

B = bandwidth noise measured in (Hz)

R = resistance (ohms)

This is found to hold for large bandwidth ( $>1013$  Hz) and large range in temperature.

This thermal noise may be represented by an equivalent circuit as shown below the figure 4.1.10.

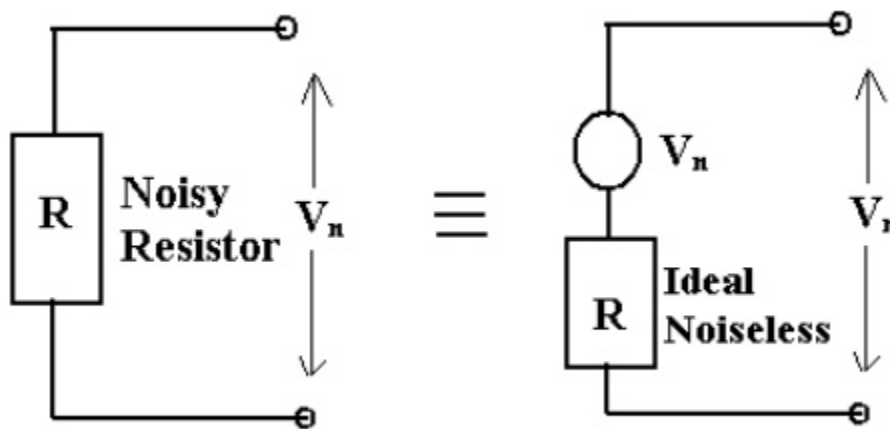


Figure 4.1.10 Equivalent Circuit of Thermal Noise

*Diagram Source Brain Kart*

i.e. equivalent to the ideal noise free resistor (with same resistance R) in series with a voltage source with voltage  $V_n$ . Since

$$\overline{V^2} = 4kTBR \text{ (mean square value , power)}$$

then  $V_{\text{RMS}} = \sqrt{\overline{V^2}} = 2\sqrt{kTBR} = V_n$  in above

i.e.  $V_n$  is the RMS noise voltage. The above equation indicates that the noise power is proportional to bandwidth. For a given resistance  $R$ , at a fixed temperature  $T$  (Kelvin)

We have  $\overline{V^2} = (4kTR) B$ , where  $(4kTR)$  is a constant – units watts per Hz.

For a given system, with  $(4kTR)$  constant, then if we double the bandwidth from  $B$  Hz to  $2B$  Hz, the noise power will double (i.e. increased by 3 dB). If the bandwidth were increased by a factor of 10, the noise power is increased by a factor of 10. For this reason it is important that the system bandwidth is only just 'wide' enough to allow the signal to pass to limit the noise bandwidth to a minimum.

Signal Spectrum Signal Power =  $S$  and the System Band width is  $W = B$  Hz

### Noise Voltage Spectral Density

Since data sheets specify the noise voltage spectral density with unit's volts per  $\sqrt{Hz}$  (volts per root Hz).

This is from  $V_n = (2\sqrt{kTR})\sqrt{B}$  i.e.  $V_n$  is proportional to  $\sqrt{B}$ . The quantity in bracket, i.e.  $(2\sqrt{kTR})$  has units of volts per  $\sqrt{Hz}$ . If the bandwidth  $B$  is doubled the noise voltage will increase by  $\sqrt{2}$ . If bandwidth is increased by 10, the noise voltage will increase by  $\sqrt{10}$ .

### Resistance in Series

Assume that  $R_1$  at temperature  $T_1$  and  $R_2$  at temperature  $T_2$  are connected in series shown in Figure 4.1.11, then

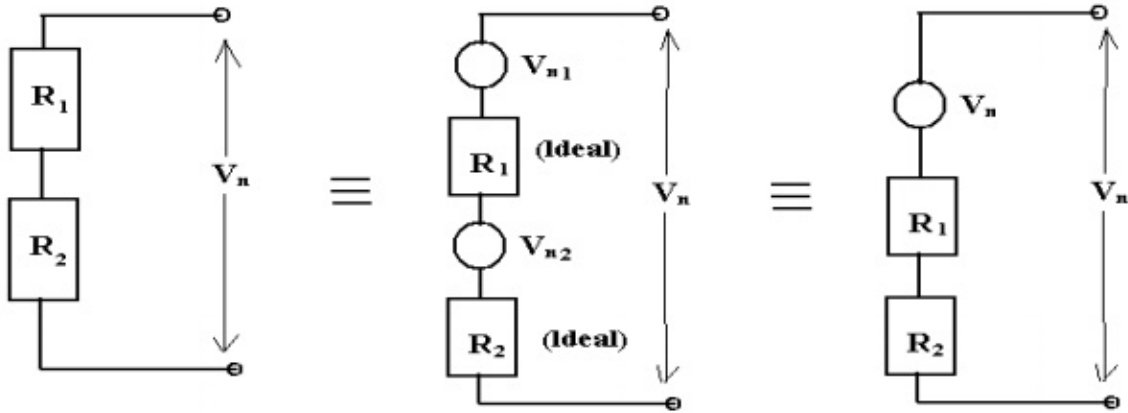


Figure 4.1.11 Circuit diagram for Resistors connected in series

*Diagram Source Brain Kart*

Assume that  $R_1$  at temperature  $T_1$  and  $R_2$  at temperature  $T_2$ , then

$$\overline{V_n^2} = \overline{V_{n1}^2} + \overline{V_{n2}^2} \quad (\text{we add noise power not noise voltage})$$

$$\overline{V_{n1}^2} = 4kT_1BR_1 \quad \overline{V_{n2}^2} = 4kT_2BR_2$$

$$\therefore \overline{V_n^2} = 4kB(T_1R_1 + T_2R_2) \quad \text{Mean square noise}$$

$$\text{If } T_1 = T_2 = T \text{ then } \overline{V_n^2} = 4kTB(R_1 + R_2)$$

i.e. The resistor in series at same temperature behave as a single resistor  $(R_1 + R_2)$

ü Resistance in Parallel shown in Figure 4.1.12.

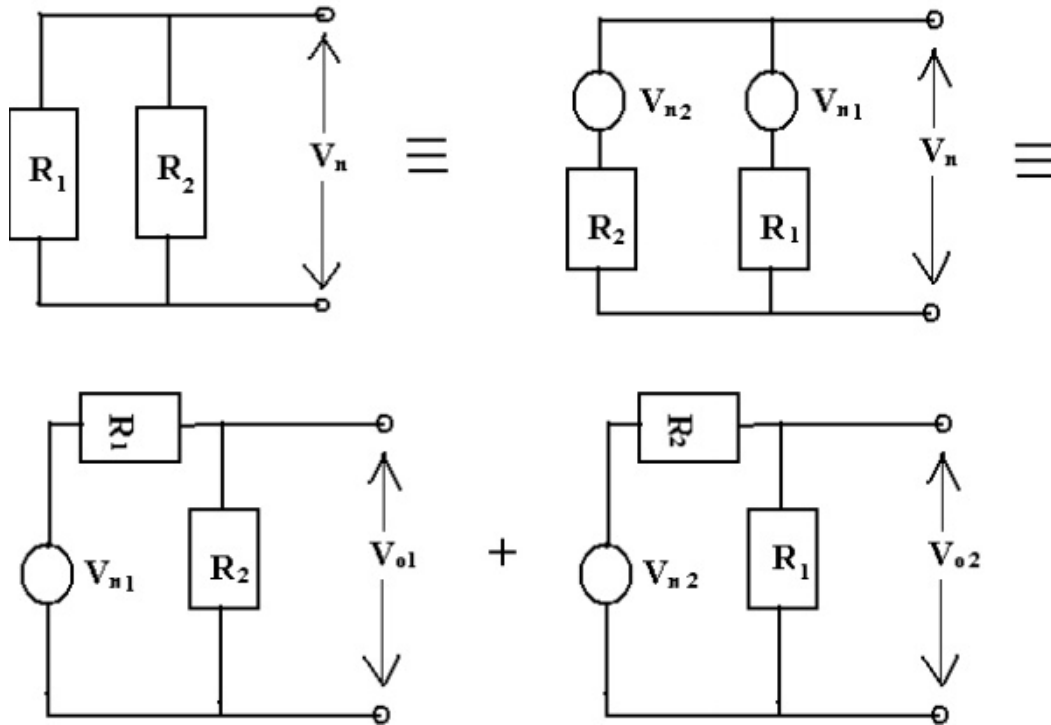


Figure 4.1.12 Circuit diagram for Resistors connected in Parallel

*Diagram Source Brain Kart*

Since an ideal voltage source has zero impedance, we can find noise as an output  $V_{o1}$ , due to  $V_{n1}$ , an output voltage  $V_{o2}$  due to  $V_{n2}$ .





$$V_{o1} = V_{n1} \frac{R_2}{R_1 + R_2} \qquad V_{o2} = V_{n2} \frac{R_1}{R_1 + R_2}$$

$$\overline{V_n^2} = \overline{V_{o1}^2} + \overline{V_{o2}^2}$$

Assuming  $R_1$  at temperature  $T_1$  and  $R_2$  at temperature  $T_2$

$$\overline{V_{n1}^2} = 4kT_1 B R_1 \qquad \text{and} \qquad \overline{V_{n2}^2} = 4kT_2 B R_2$$

Hence,

$$\overline{V_{o1}^2} = 4kT_1 B R_1 \left( \frac{R_2}{R_1 + R_2} \right)^2$$

and

$$\overline{V_{o2}^2} = 4kT_2 B R_2 \left( \frac{R_1}{R_1 + R_2} \right)^2$$

$$\left\{ \begin{array}{l} \text{e.g. } V_{o1} = V_{n1} \frac{R_2}{R_1 + R_2}, \quad \overline{V_{o1}^2} = (V_{n1})^2 \left( \frac{R_2}{R_1 + R_2} \right)^2 \\ V_{n1} = \text{RMS}, \therefore (V_{n1})^2 = \text{Meansqaure} = \overline{V_{n1}^2} \\ \overline{V_{o1}^2} = \overline{V_{n1}^2} \left( \frac{R_2}{R_1 + R_2} \right)^2 \end{array} \right\}$$

$$\text{Thus } \overline{V_n^2} = \overline{V_{o1}^2} + \overline{V_{o2}^2} = \frac{4kB}{(R_1 + R_2)^2} [R_2^2 T_1 R_1 + R_1^2 T_2 R_2] \times \left( \frac{R_1 R_2}{R_1 R_2} \right)$$

$$\overline{V_n^2} = \frac{4kB R_1 R_2 (T_1 R_1 + T_2 R_2)}{(R_1 + R_2)^2}$$

or

$$\overline{V_n^2} = 4kTB \left( \frac{R_1 R_2}{R_1 + R_2} \right)$$

i.e. the two noisy resistors in parallel behave as a resistance  $\left( \frac{R_1 R_2}{R_1 + R_2} \right)$  which is the equivalent resistance of the parallel combination.

### Noise Equivalent Voltage

An equivalent circuit, when the line is connected to the receiver is shown below the figure 4.1.13 in fi. (Note we omit the noise due to  $R_{in}$  – this is considered in the analysis of the receiver section).

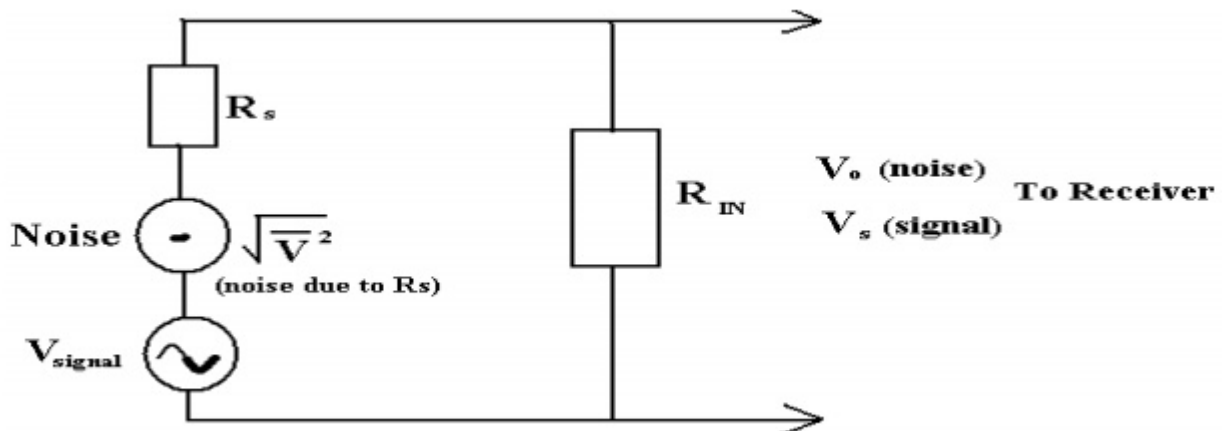


Figure 4.1.13 Equivalent Circuit of Noise Voltage

*Diagram Source Brain Kart*

The RMS voltage output,  $V_o$  (noise) is

$$V_o(\text{noise}) = \sqrt{\overline{v^2}} \left( \frac{R_{IN}}{R_{IN} + R_S} \right)$$

Similarly, the signal voltage output due to  $V_{\text{signal}}$  at input is

$$V_{S(\text{signal})} = (V_{\text{signal}}) \left( \frac{R_{IN}}{R_{IN} + R_S} \right)$$

For maximum power transfer, the input  $R_{IN}$  is matched to the source  $R_S$ , i.e.  $R_{IN} = R_S = R$  (say)

Then 
$$V_o(\text{noise}) = \sqrt{\overline{v^2}} \left( \frac{R}{2R} \right) = \frac{\sqrt{\overline{v^2}}}{2} \text{ (RMS Value)}$$

And signal, 
$$V_{S(\text{signal})} = \frac{V_{\text{signal}}}{2}$$

Continuing the analysis for noise only, the mean square noise is  $(\text{RMS})^2$ .

$$(V_o(\text{noise}))^2 = \left( \frac{\sqrt{\overline{v^2}}}{2} \right)^2 = \frac{\overline{v^2}}{4}$$

But  $\overline{v^2}$  is noise due to  $R_S = R$ , i.e.  $\overline{V^2} = 4kTBR$  (volt<sup>2</sup>).

Hence 
$$(V_o(\text{noise}))^2 = \frac{4kTBR}{4} = kTBR$$

Since average power = 
$$\frac{(V_{rms})^2}{2}$$

Then 
$$N = \frac{\overline{V^2}}{R} = kTB_n$$

i.e. Noise Power =  $kTB_n$  watts

For a matched system,  $N$  represents the average noise power transferred from the source to the load. This may be written as

$$p_0 = \frac{N}{B_n} = kT \text{ watts per Hz}$$

where  $p_0$  is the noise power spectral density (watts per Hz)

$B_n$  is the noise equivalent bandwidth (Hz)  $k$  is the Boltzmann's constant  
 $T$  is the absolute temperature K.

Note: that  $p_0$  is independent of frequency, i.e. white noise.

These equations indicate the need to keep the system bandwidth to a minimum, i.e. to that required to pass only the band of wanted signals, in order to minimize noise power,  $N$ .

For example, a resistance at a temperature of 290 K (17 deg C),  $p_0 = kT$  is  $4 \times 10^{-21}$  watts per Hz. For a noise bandwidth  $B_n = 1$  KHz,  $N$  is  $4 \times 10^{-18}$  watts (-174 dBW). If the system bandwidth is increased to 2 KHz,  $N$  will decrease by a factor of 2 (i.e.  $8 \times 10^{-18}$  watts or -171 dBW) which will degrade the (S/N) by 3 dB. Care must also be exercised when noise or (S/N) measurements are made, for example with a power meter or spectrum analyser, to be clear which bandwidth the noise is measured in, i.e. system or test equipment. For example, assume a system bandwidth is 1 MHz and the measurement instrument bandwidth is 250 KHz.

In the above example, the noise measured is band limited by the test equipment rather than the system, making the system appear less noisy than it actually is. Clearly if the relative bandwidths are known (they should be) the measured noise power may be corrected to give the actual noise power in the system bandwidth. If the system bandwidth was 250 KHz and the test equipment was 1 MHz then the measured result now would be -150 dBW (i.e. the same as the actual noise)

because the noise power monitored by the test equipment has already been band limited to 250 KHz.

### (ii) Signal to Noise Ratio

The signal to noise ratio is given by

$$\frac{S}{N} = \frac{\text{Signal Power}}{\text{Noise Power}}$$

The signal to noise in dB is expressed by

$$\left(\frac{S}{N}\right)_{dB} = 10 \log_{10} \left(\frac{S}{N}\right)$$

$$\text{or} \quad \left(\frac{S}{N}\right)_{dB} = 10 \log_{10} S - 10 \log_{10} N$$

since  $10 \log_{10} S = S \text{ dBm}$  if S in mW

and  $10 \log_{10} N = N \text{ dBm}$

$$\text{then} \quad \left(\frac{S}{N}\right)_{dB} = S_{dBm} - N_{dBm} \quad \text{for S and N measured in mW}$$

