UNIT II

SHAFTS AND COUPLINGS

CHAPTER 5

Flexible Coupling

We have already discussed that a flexible coupling is used to join the abutting ends of shafts when they are not in exact alignment. In the case of a direct coupled drive from a prime mover to an electric generator, we should have four bearings at a comparatively close distance. In such a case and in many others, as in a direct electric drive from an electric motor to a machine tool, a flexible coupling is used so as to permit an axial misalignment of the shaft without undue absorption of the power which the shaft is transmitting. Following are the different types of flexible couplings:

1. Bushed pin flexible coupling,
2. Oldham's coupling, and
3. Universal coupling.

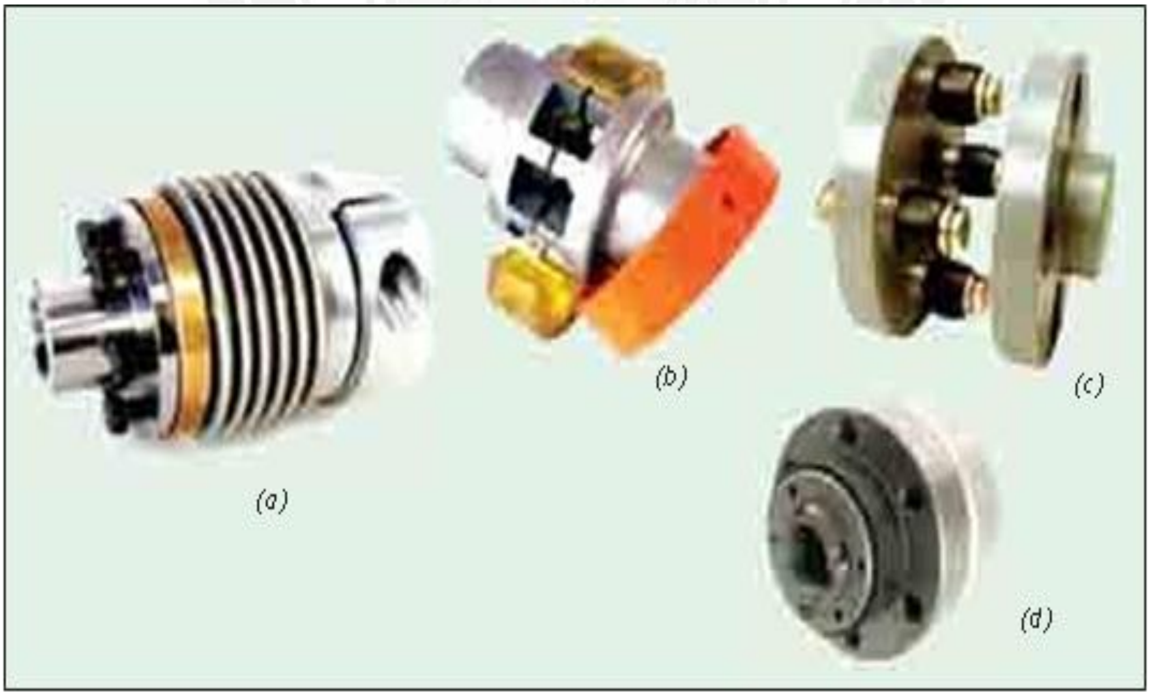

Fig 5.1(a) Bellows coupling, (b) Elastomeric coupling, (c) Flanged coupling, (d) Flexible coupling

[Source: "A Textbook of Machine Design by R.S. Khurmi J.K. Gupta, Page: 498]

Bushed-pin Flexible Coupling

A bushed-pin flexible coupling, as shown in Fig. 5.2, is a modification of the rigid type of flange coupling. The coupling bolts are known as pins. The rubber or leather bushes are used over the pins. The two halves of the coupling are dissimilar in construction. A clearance of 5 mm is left between the face of the two halves of the coupling. There is no rigid connection between them and the drive takes place through the medium of the compressible rubber or leather bushes.

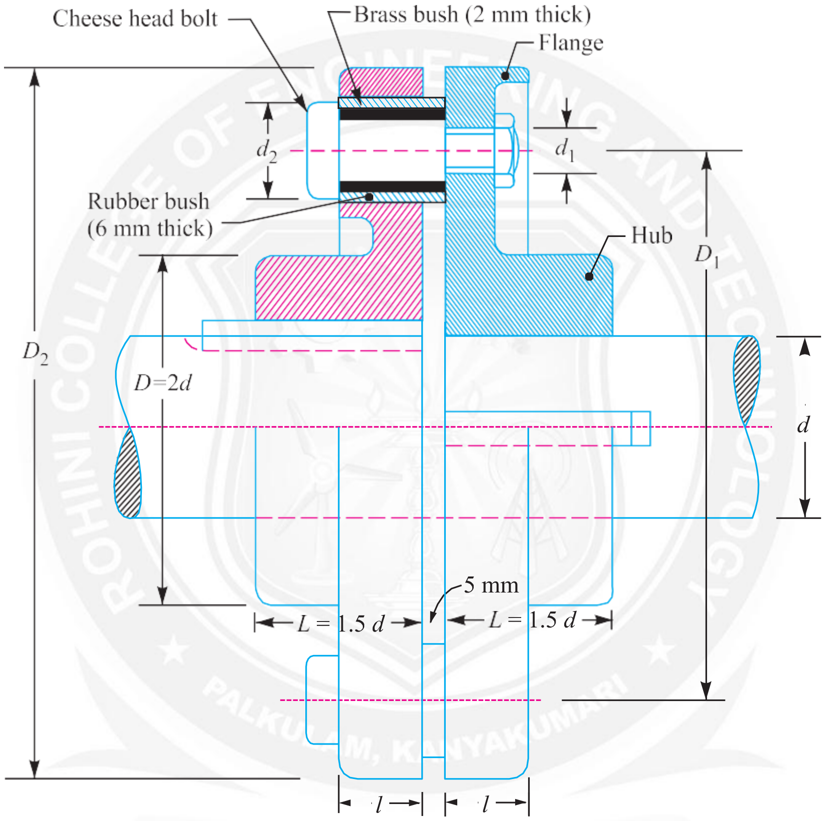

Fig 5.2 Bushed-pin flexible coupling.

[Source: "A Textbook of Machine Design by R.S. Khurmi J.K. Gupta, Page: 499]

In designing the bushed-pin flexible coupling, the proportions of the rigid type flange coupling are modified. The main modification is to reduce the bearing pressure on the rubber or leather bushes and it should not exceed 0.5 N/mm^2. In order to keep the low bearing pressure, the pitch circle diameter and the pin size is increased.

Let l = Length of bush in the flange,

d_2 = Diameter of bush,

p_b = Bearing pressure on the bush or pin,

n = Number of pins, and

D_1 = Diameter of pitch circle of the pins.

We know that bearing load acting on each pin,

$$W = p_b \times d_2 \times l$$

∴ Total bearing load on the bush or pins

$$= W \times n$$

$$= p_b \times d2 \times l \times n$$

and the torque transmitted by the coupling,

$$T = W \times n \left(\frac{D_1}{2}\right)$$

$$T = p_b \times d2 \times l \times n \left(\frac{D_1}{2}\right)$$

The threaded portion of the pin in the right hand flange should be a tapping fit in the coupling hole to avoid bending stresses. The threaded length of the pin should be as small as possible so that the direct shear stress can be taken by the unthreaded neck.

Direct shear stress due to pure torsion in the coupling halves,

$$\tau = \frac{W}{\frac{\pi}{4}(d_1)^2}$$

Since the pin and the rubber or leather bush is not rigidly held in the left hand flange, therefore the tangential load (W) at the enlarged portion will exert a bending action on the pin as shown in Fig. 5.3. The bush portion of the pin acts as a cantilever beam of length l. Assuming a uniform distribution of the load W along the bush, the maximum bending moment on the pin,

$$M = W\left(\frac{l}{2} + 5mm\right)$$

We know that bending stress,

$$\sigma = \frac{M}{Z} = \frac{W\left(\frac{l}{2} + 5mm\right)}{\frac{\pi}{32}(d_1)^2}$$

Since the pin is subjected to bending and shear stresses, therefore the design must be checked either for the maximum principal stress or maximum shear stress by the following relations:

Maximum principal stress

$$= \frac{1}{2}\left[\sigma + \sqrt{\sigma^2 + 4\tau^2}\right]$$

and the maximum shear stress on the pin

$$= \frac{1}{2} [\sqrt{\sigma^2 + 4\tau^2}]$$

The value of maximum principal stress varies from 28 to 42 MPa.

Fig 5.3 (a) Taper bush (b) Locking-assembly (shaft or bush connectors) (c) Friction joint bushing (d) Safety overload coupling.

[Source: "A Textbook of Machine Design by R.S. Khurmi J.K. Gupta, Page: 500]

Problem 5.1

Design a bushed-pin type of flexible coupling to connect a pump shaft to a motor shaft transmitting 32 kW at 960 r.p.m. The overall torque is 20 percent more than mean torque.

The material properties are as follows:

(a) The allowable shear and crushing stress for shaft and key material is 40 MPa and 80 MPa respectively.

(b) The allowable shear stress for cast iron is 15 MPa.

(c) The allowable bearing pressure for rubber bush is 0.8 N/mm^2.

(d) The material of the pin is same as that of shaft and key.

Draw neat sketch of the coupling.

Given Data:

$P = 32 \text{ kW} = 32 \times 10^3 \text{ W}$

$N = 960$ r.p.m.

$T_{max} = 1.2\ T_{mean}$

$\tau_s = \tau_k = 40$ MPa = 40 N/mm^2

$\sigma_{cs} = \sigma_{ck} = 80$ MPa = 80 N/mm^2

$\tau_c = 15$ MPa = 15 N/mm^2

$p_b = 0.8$ N/mm^2

The bushed-pin flexible coupling is designed as discussed below:

1. Design for pins and rubber bush

First of all, let us find the diameter of the shaft (d). We know that the mean torque transmitted by the shaft,

$$T_{mean} = \frac{P \times 60}{2\pi N} = \frac{32 \times 10^3 \times 60}{2\pi \times 960}$$

$$T_{mean} = 318.3 \text{ N-m}$$

and the maximum or overall torque transmitted,

$$T_{max} = 1.2\ T_{mean}$$

$$= 1.2 \times 318.3$$

$$T_{max} = 382 \text{ N-m}$$

$$T_{max} = 382 \times 10^3 \text{ N-mm}$$

We also know that the maximum torque transmitted by the shaft (T_{max}),

$$382 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3$$

$$382 \times 10^3 = \frac{\pi}{16} \times 40 \times d^3$$

$$382 \times 10^3 = 7.86\ d^3$$

$$d^3 = 382 \times 10^3 / 7.86$$

$$d^3 = 48.6 \times 103 \text{ or}$$

$$\therefore d = 36.5 \text{ say } 40 \text{ mm}$$

We have discussed in rigid type of flange coupling that the number of bolts for 40 mm diameter shaft are 3. In the flexible coupling, we shall use the number of pins

(n) as 6.

$\therefore$ Diameter of pins, $d_1 = \frac{0.5d}{\sqrt{n}} = \frac{0.5 \times 40}{\sqrt{6}}$

$d_1 = 8.2$mm.

In order to allow for the bending stress induced due to the compressibility of the rubber bush, the diameter of the pin (d_1) may be taken as 20 mm. The length of the pin of least diameter i.e. $d_1 = 20$ mm is threaded and secured in the right hand coupling half by a standard nut and washer. The enlarged portion of the pin which is in the left hand coupling half is made of 24 mm diameter. On the enlarged portion, a brass bush of thickness 2 mm is pressed. A brass bush carries a rubber bush. Assume the thickness of rubber bush as 6 mm.

$\therefore$ Overall diameter of rubber bush,

$d_2 = 24 + 2 \times 2 + 2 \times 6 = 40$ mm.

and diameter of the pitch circle of the pins,

$D_1 = 2\,d + d_2 + 2 \times 6$

$D_1 = 2 \times 40 + 40 + 12$

$D_1 = 132$ mm.

Let l = Length of the bush in the flange.

We know that the bearing load acting on each pin,

$W = p_b \times d_2 \times l$

$W = 0.8 \times 40 \times l$

$W = 32\,l$ N

and the maximum torque transmitted by the coupling (T_{max}),

$382 \times 10^3 = W \times n \times \frac{D_1}{2}$

$382 \times 10^3 = 32\,l \times 6 \times \frac{132}{2}$

$382 \times 10^3 = 12672\,l$

$l = 382 \times 10^3/12672$

$l = 30.1$ say

$l = 32$ mm

and $W = 32\,l = 32 \times 32 = 1024$ N

∴ Direct stress due to pure torsion in the coupling halves,

$$\tau = \frac{1024}{\frac{\pi}{4}(d_1)^2}$$

$$\tau = \frac{W}{\frac{\pi}{4}(20)^2}$$

$$\tau = 3.26 \text{ N/mm}^2$$

Since the pin and the rubber bush are not rigidly held in the left hand flange, therefore the tangential load (W) at the enlarged portion will exert a bending action on the pin. Assuming a uniform distribution of load (W) along the bush, the maximum bending moment on the pin,

$$M = W(\frac{l}{2} + 5)$$

$$M = 1024(\frac{l}{2} + 5)$$

$$M = 21504 \text{ N-mm}$$

and section modulus, $Z = \frac{\pi}{32}(d_1)^3 = \frac{\pi}{32}(20)^3$

$$Z = 785.5 \text{ mm}^3$$

We know that bending stress,

$$\sigma = \frac{M}{Z} = \frac{21504}{785.5}$$

$$\sigma = 27.4 \text{ N/mm}^2$$

∴ Maximum principal stress

$$= \frac{1}{2}[\sigma + \sqrt{\sigma^2 + 4\tau^2}]$$

$$= \frac{1}{2}[27.4 + \sqrt{27.2^2 + 4(3.26)^2}]$$

$$= 13.7 + 14.1 = 27.8 \text{ N/mm}^2$$

and the maximum shear stress on the pin

$$= \frac{1}{2}[\sqrt{\sigma^2 + 4\tau^2}]$$

$$= \frac{1}{2}[\sqrt{27.8^2 + 4(3.26)^2}]$$

$= 14.1 \text{ N/mm}^2$

Since the maximum principal stress and maximum shear stress are within limits, therefore the design is safe.

2. Design for hub

We know that the outer diameter of the hub,

$$D = 2\, d = 2 \times 40$$

$$D = 80 \text{ mm}$$

and length of hub, $L = 1.5\, d = 1.5 \times 40$

$$L = 60 \text{ mm}$$

Let us now check the induced shear stress for the hub material which is cast iron. Considering the hub as a hollow shaft. We know that the maximum torque transmitted (T_{max}),

$$382 \times 10^3 = \frac{\pi}{16} \times \tau_c \left[\frac{D^4 - d^4}{D}\right]$$

$$382 \times 10^3 = \frac{\pi}{16} \times \tau_c \left[\frac{80^4 - d^4}{80}\right]$$

$$382 \times 10^3 = 94.26 \times 10^3\, \tau_c$$

$$\therefore \tau_c = 382 \times 10^3 / 94.26 \times 10^3$$

$$\tau_c = 4.05 \text{ N/mm}^2$$

$$\tau_c = 4.05 \text{ MPa}$$

Since the induced shear stress for the hub material (i.e. cast iron) is less than the permissible value of 15 MPa, therefore the design of hub is safe.

3. Design for key

Since the crushing stress for the key material is twice its shear stress (i.e. $\sigma_{ck} = 2\, \tau_k$), therefore a square key may be used. From Table, we find that for a shaft of 40 mm diameter,

Width of key, $w = 14$ mm.

and thickness of key, $t = w = 14$ mm.

The length of key (L) is taken equal to the length of hub, i.e.

$$L = 1.5\, d = 1.5 \times 40$$

$$L = 60 \text{ mm}$$

Let us now check the induced stresses in the key by considering it in shearing and crushing. Considering the key in shearing. We know that the maximum torque transmitted (T_{max}),

$$382 \times 10^3 = L \times w \times \tau_k \times \frac{d}{2}$$

$$382 \times 10^3 = 60 \times 14 \times \tau_k \times \frac{40}{2}$$

$$382 \times 10^3 = L \times w \times \tau_k \times \frac{d}{2}$$

$$382 \times 10^3 = 16\,800\ \tau_k$$

$$\tau_k = 382 \times 10^3/16\,800$$

$$\tau_k = 22.74\ N/mm^2$$

$$\therefore \tau_k = 22.74\ MPa$$

Considering the key in crushing. We know that the maximum torque transmitted (T_{max}),

$$382 \times 10^3 = L \times \frac{t}{2} \times \sigma_{ck} \times \frac{d}{2}$$

$$382 \times 10^3 = 60 \times \frac{14}{2} \times \sigma_{ck} \times \frac{40}{2}$$

$$382 \times 10^3 = 8400\ \sigma_{ck}$$

$$\sigma_{ck} = 382 \times 10^3/8400$$

$$\sigma_{ck} = 45.48\ N/mm^2$$

$$\therefore \sigma_{ck} = 45.48\ MPa$$

Since the induced shear and crushing stress in the key are less than the permissible stresses of 40 MPa and 80 MPa respectively, therefore the design for key is safe.

4. Design for flange

The thickness of flange (t_f) is taken as 0.5 d.

$$\therefore t_f = 0.5\ d = 0.5 \times 40$$

$$\therefore t_f = 20\ mm$$

Let us now check the induced shear stress in the flange by considering the flange at the junction of the hub in shear.

We know that the maximum torque transmitted (T_{max}),

$$382 \times 10^3 = \frac{\pi D^2}{2} \times t_f \times \tau_c$$

$382 \times 10^3 = \frac{\pi 80^2}{2} \times 20 \times \tau_c$

$382 \times 10^3 = 201 \times 10^3\ \tau_c$

$\tau_c = 382 \times 10^3 / 201 \times 10^3$

$\tau_c = 1.9$ N/mm2

$\therefore \tau_c = 1.9$ MPa

Since the induced shear stress in the flange of cast iron is less than 15 MPa, therefore the design of flange is safe.