1.1 Magnetic circuits

Introduction

In general, magnetic materials can be classified as magnetically "soft" and "hard" materials. Soft materials are normally used as the magnetic core materials for inductors, transformers, actuators and rotating machines, in which the magnetic fields vary frequently, whereas hard materials, or permanent magnets, are used to replace magnetization coils for generating static magnetic fields in devices such as electric motors and actuators. The B-H relationships and hysteresis loops have been discussed earlier. In this chapter, we are going to examine the power losses in a soft magnetic core under an alternating magnetization, and further develop an electrical circuit model of a magnetic core with a coil. For performance prediction of electromagnetic devices, magnetic field analysis is required.

Analytical magnetic field analysis by the Maxwell's equations, however, has been shown very difficult for engineering problems owing to the fact that most practical devices are of complicated structures. Powerful numerical methods, such as the finite difference and finite element methods, are out of the scope of this subject. In this chapter, we introduce a simple method of magnetic circuit analysis based on an analogy to dc electrical circuits.

Soft Magnetic Materials under Alternating Excitations

Core Losses

Core losses occur in magnetic cores of ferromagnetic materials under alternating magnetic field excitations. The Figure 1.1 on the right hand side plots the alternating core losses of M-36, 0.356 mm steel sheet against the excitation frequency. In this section, we will discuss the mechanisms and prediction of alternating core losses.

As the external magnetic field varies at a very low rate periodically, as mentioned earlier, due to the effects of magnetic domain wall motion the B- H. relationship is a hysteresis loop. The area enclosed by the loop is a power loss known as the hysteresis loss, and can be calculated by,

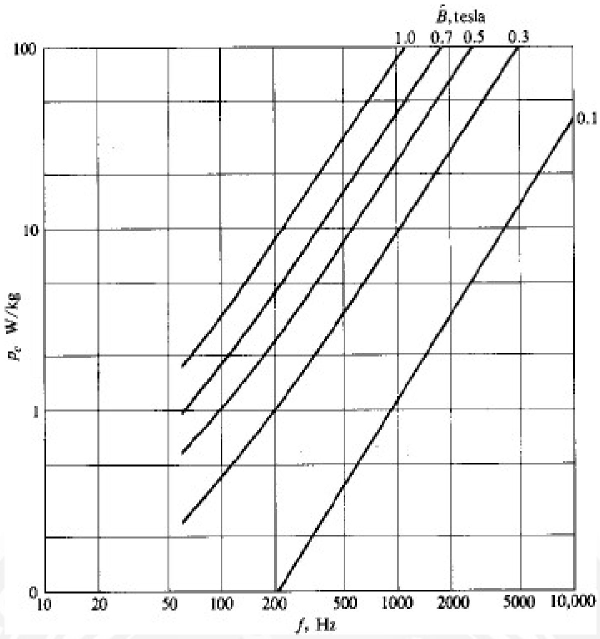


Figure 1.1.1 Alternating core loss of steel sheet at different excitation frequencies

[Source: ""*Electric Machinery Fundamentals" by* Stephen J. Chapman*, Page: 45]*

For magnetic materials commonly used in the construction of electric machines an

$$P_{hyst} = \oint \mathbf{H} \bullet d\mathbf{B} \quad (\text{W/m}^3/\text{cycle}) \text{ or } (\text{J/m}^3)$$

approximate relation is

$$P_{hyst} = C_h f B_p^n \quad (1.5 < n < 2.5) \quad (\text{W/kg})$$

where

*C*h is a constant determined by the nature of the ferromagnetic material,

f is the frequency of excitation, and B_p is the peak value of the flux density.

Example:

A B-H loop for a type of electric steel sheet is shown in the diagram below. Determine approximately the hysteresis loss per cycle in a torus of 300 mm mean diameter and a square cross section of 50*50 mm.

Solution:

The area of each square in the diagram represents

$$(0.1 \text{ T}) \times (25 \text{ A/m}) = 2.5 \text{ (Wb/m}^2) \times \text{(A/m)} = 2.5 \text{ VsA/m}^3 = 2.5 \text{ J/m}^3$$

If a square that is more than half within the loop is regarded as totally enclosed, and one that is more than half outside is disregarded, then the area of the loop is

$$2 \times 43 \times 2.5 = 215 \ \text{J/m}^3$$

The volume of the torus is

$$0.05^2 \times 0.3\pi = 2.36 \times 10^{-3} \ \text{m}^3$$

Energy loss in the torus per cycle is thus

$$2.36 \times 10^{-3} \times 215 = 0.507 \ \text{J}$$

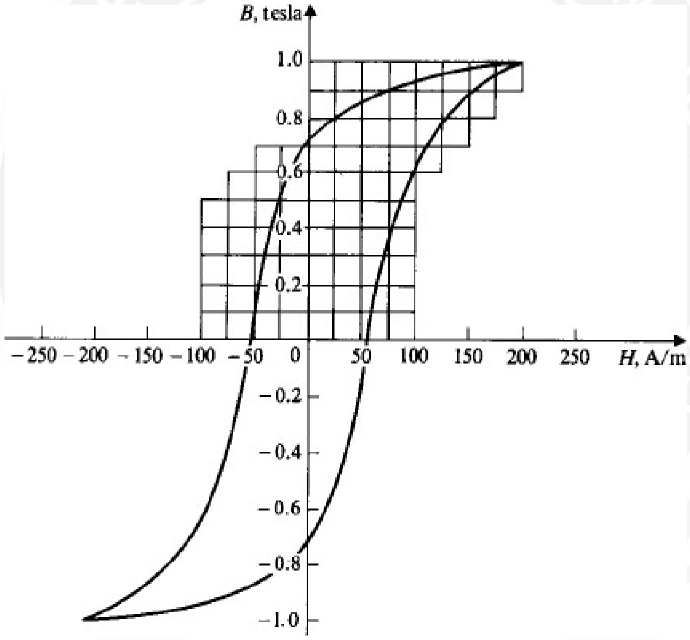


Figure 1.1.2 Hysteresis loop of M-36 steel sheet

[Source: "Electric Machinery Fundamentals" by Stephen J. Chapman*, Page: 46]*

A Simple Magnetic Circuit

Consider a simple structure consisting of a current carrying coil of *N* turns and a magnetic core of mean length *l*c and a cross sectional area *A*c as shown in the diagram below. The permeability of the core material is μ_c. Assume that the size of the device and the operation frequency are such that the displacement current in Maxwell's equations are negligible, and that the permeability of the core material is very high so that all magnetic flux will be confined within the core. By Ampere's law,

$$\oint_C \mathbf{H} \bullet d\mathbf{l} = \oint_S \mathbf{J} \bullet d\mathbf{a}$$

we can write

$$H_c l_c = Ni$$

where Hc is the magnetic field strength in the core, and Ni the magneto motive force. The magnetic flux through the cross section of the core can expressed as

$$\phi_c = B_c A_c$$

where ϕ_c is the flux in the core and *B*c the flux density in the core. The constitutive equation of the core material is

$$B_c = \mu H_c$$

Therefore, we obtain

$$\phi_c = \frac{Ni}{l_c/(\mu_c A_c)} = \frac{F}{R_c}$$

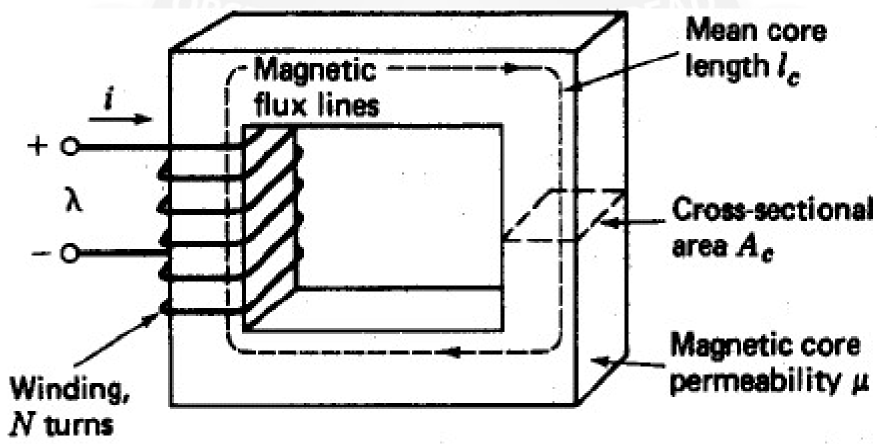
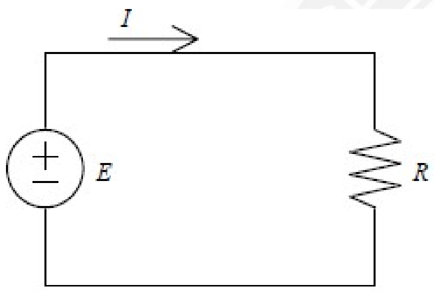


Figure 1.1.3 A Simple Magnetic Circuit

[Source: "Electric Machinery Fundamentals" by Stephen J. Chapman*, Page: 49]*

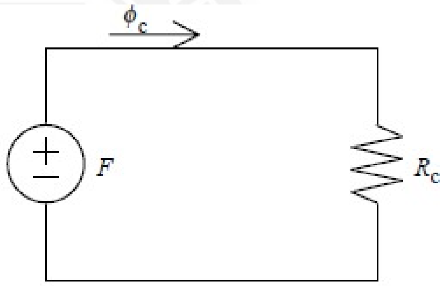
If we take the magnetic flux ϕ_c as the "current", the magneto motive force F=Ni as the "emf of a voltage source", and $Rc=l_c/(\phi_c A_c)$ (known as the magnetic reluctance) as the "resistance" in the magnetic circuit, we have an analog of Ohm's law in electrical circuit theory.

Electric Circuit



$$I = \frac{E}{R}$$

Magnetic Circuit



$$\phi_c = \frac{F}{R_c}$$