Greens ,Gauss Divergence and Stokes theorem

Green's Theorem

Green's theorem relates a line integral to the double integral taken over the region bounded by the closed curve.

Statement

If $M(x,y)$ and $N(x,y)$ are continuous functions with continuous, partial derivatives in a region R of the xy – plane bounded by a simple closed curve C, then

$\oint_{c} Mdx + Ndy = \iint_{R} \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\, dy$,where C is the curve described in the positive direction.

Vector form of Green's theorem

$$\oint_{c} \vec{F} \cdot d\vec{r} = \iint_{R} \left(\nabla \times \vec{F}\right) \cdot \vec{\mathrm{k}}\, \mathrm{dR}$$

STOKE'S THEOREM

Statement of Stoke's theorem

If S is an open surface bounded by a simple closed curve C if $\vec{F}$ is continuous having continuous partial derivatives in S and C, then

$$\int_{c} \vec{F} \cdot d\vec{r} = \iint_{S} curl\, \vec{F} \cdot \hat{n}\, ds$$

(or)

$$\int_{c} \vec{F} \cdot d\vec{r} = \iint_{S} \nabla \times \vec{F} \cdot \hat{n}\, ds$$

$\hat{n}$ is the outward unit normal vector and C is traversed in the anti – clockwise direction.

GAUSS DIVERGENCE THEOREM

This theorem enables us to convert a surface integral of a vector function on a closed surface into volume integral.

Statement of Gauss Divergence theorem

If V is the volume bounded by a closed surface S and if a vector function $\vec{F}$ is continuous and has continuous partial derivatives in V and on S, then

$$\iint_S \vec{F} \cdot \hat{n}\, ds = \iiint_V \nabla \cdot \vec{F}\, dv$$

Where $\hat{n}$ is the unit outward normal to the surface S and $dV = dxdydz$