UNIT I

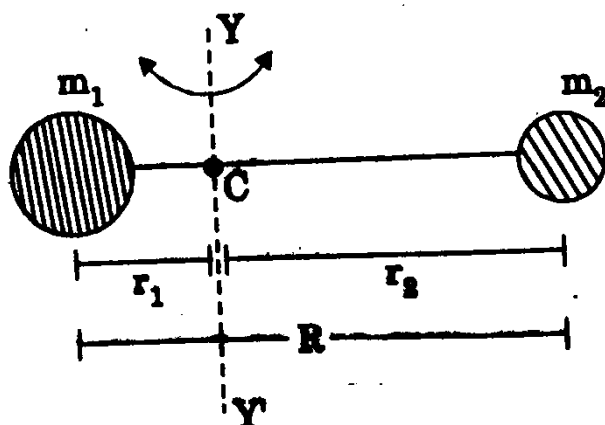## Mechanics

1.5 Rotational energy states of a rigid diatomic molecules

If we consider two atoms of masses m_1 and m_2 which are situated at a distance r_1 and r_2 with respect to the axis of rotation YY' respectively, then this arrangement is called as rigid rotor. Here 'R' is the bond length between the two atoms ($R = r_1 + r_2$).

If the distance diatomic molecule rotates with respect to 'C', then its kinetic energy is given as

$$E = \tfrac{1}{2}\, m_1 r_1 \omega^2 + \tfrac{1}{2}\, m_2 r_2 \omega^2 \qquad \text{------- (1)}$$

(or)

$$E = \tfrac{1}{2}\, \omega^2\, (m_1 r_1 + m_2 r_2)$$

(or)

$$E = \tfrac{1}{2}\, I\, \omega^2$$

(because the moment of inertia $I = I = m_1 r_1^2 + m_2 r_2^2$)

$$E = \tfrac{1}{2}\, I\, \omega^2 \qquad \text{------- (2)}$$

Then eqn. (2) can be rewritten as,

$$E = \frac{1}{2\,I}\, I^2\, \omega^2 \qquad \text{------- (3)}$$

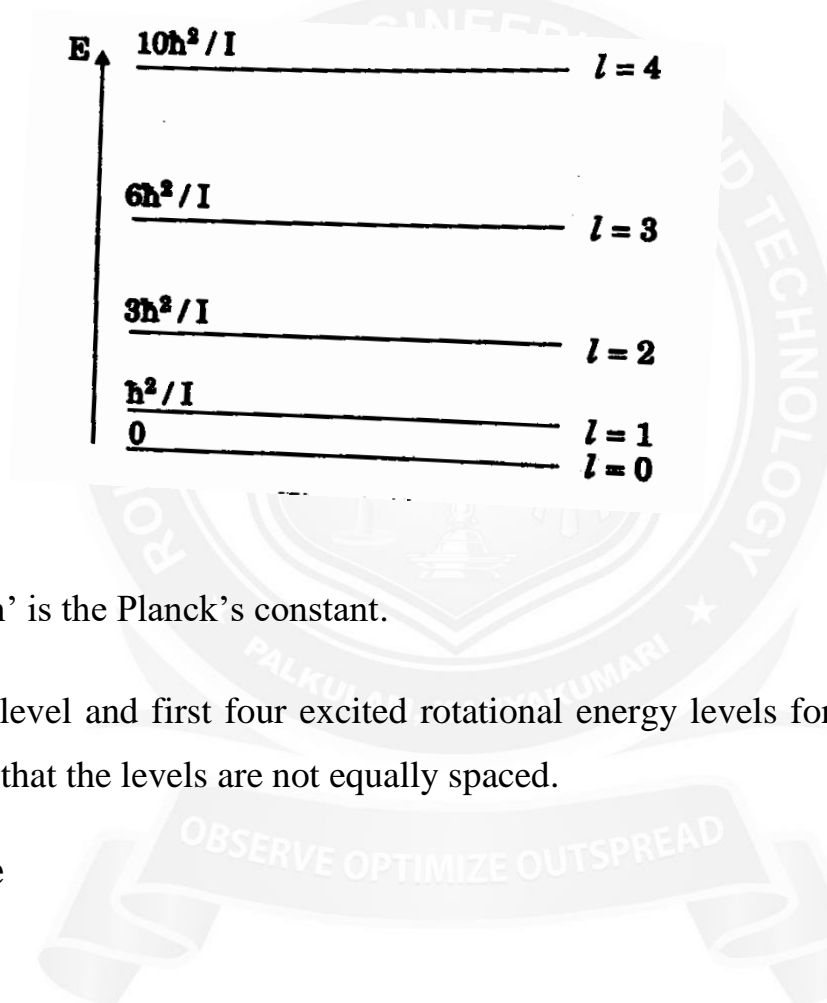As, $I\omega = L$, (the angular momentum of the rigid rotor), the eqn. (3) becomes

$$E = \frac{L^2}{2\,I}$$ ------- (4)

At atomic level, the rotation leads to quantization of the angular momentum with values given by

$$L^2 = l\,(l+1)\,h^2 \qquad\qquad l = 0,1,2,\ \ldots\ldots\ \text{------- (5)}$$

Where 'l' is the rotational quantum number. As 'l' varies interms of integer values, so corresponding the energy levels of a rotating molecules are therefore given by

$$E_l = \frac{l\,(l+1)\,h^2}{2\,I}$$

Here $K = \dfrac{h}{2\pi}$ and 'h' is the Planck's constant.

The ground level and first four excited rotational energy levels for a diatomic molecule is shown in fig. Note that the levels are not equally spaced.

1.5.1 Gyroscope

Definition

A gyroscope is a device used for measuring or maintaining orientation and angular velocity. It is a spinning wheel or disc in which the axis of rotation (spin axis) is free to assume any orientation by itself.

When rotating, the orientation of this axis is unaffected by tilting or rotation of the mounting, according to the conservation of angular momentum.

Gyroscopic principle

All spinning objects have gyroscopic properties. The main properties that an object can experience in any gyroscopic motion are rigidity in space and precession.

Description and working

A gyroscope is essentially a heavy wheel rotating at a high speed about an axle passing through its centre of mass and so mounted as to be free to turn about any of three mutually perpendicular axes 1, 2, 3 (Fig. (a) and (b)).

If the wheel rotates with high angular speed about the axis 1, the base may be turned in any manner without exerting any torque on the wheel.
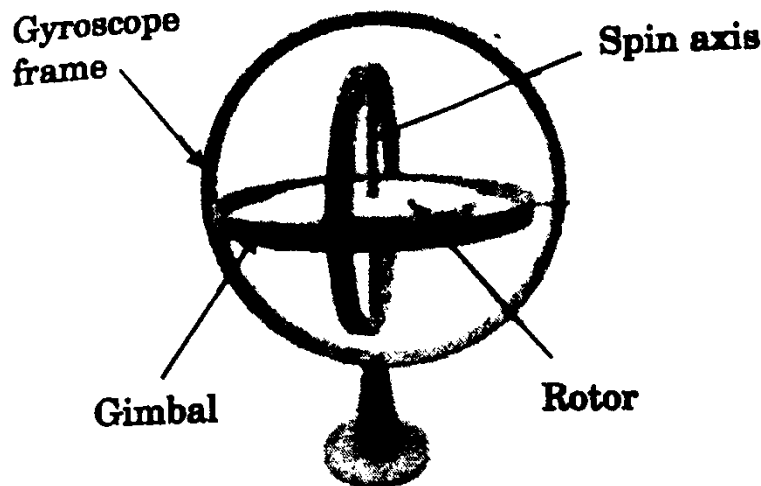
In other words, so long the wheel rotates rapidly, it maintain its axis of rotation unchanged in space as the support is tilted in any manner
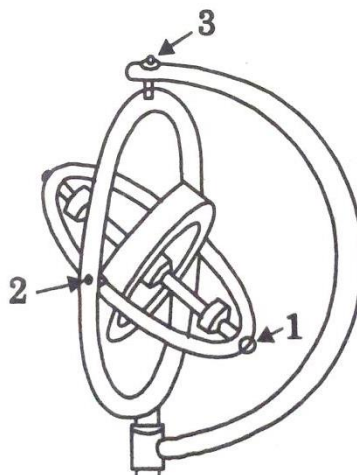
If, any torque is applied perpendicular to the axis of rotation, there will be a precession inversely proportional to the angular momentum (Iω) of the wheel. However, a heavy wheel rotating at high speed, having a large moment of inertia, would suffer very small precession.

Thus gyroscope is a device characterized by the greater stability of its axis of rotation.

Application

1. In view of the property of stability, the gyroscope are used as stabilizers in ships, boats and aeroplanes.

2. Due to the inherent stability of the gyroscope, it used as a compass, and a gyro-compass is preferable to the magnetic compass in many respects.

3. Another important application of the directional stability of a rapidly spinning (rotating) body is the rifling of the barrels of the rifles.

   This spin motion prevents the deflection of the bullet from its path due to air and gravity effects, and causes only very little precession. Thus the uniformity of flight of the bullet is increased.

4. The rolling of hoops and the riding of bicycles (which are statically unstable since both of them cannot remain in equilibrium when at rest) are possible because of the gyroscopic effect. This effect produces a movement of the plane of rotation, tending to counterbalance the disturbing action of gravity.

Many modern aircraft instruments such as automatic pilot, bomb sights, artificial horizon, turn and back indicators, etc. have been developed on gyroscope controlled.