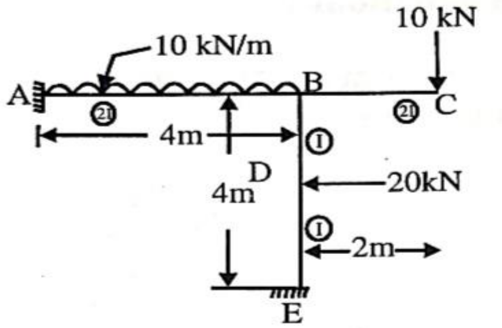
2.3 ANALYSIS OF RIGID FRAMES IN SLOPE DEFLECTION METHOD.

2.3.1 NUMERICAL EXAMPLES ON (RIGID FRAMES):

PROBLEM NO:01

Analysis the rigid frame shown in fig.,Calculate the support moments using slope deflection method.Draw the SF and BM diagrams.



Solution:

- **Fixed End Moments:**

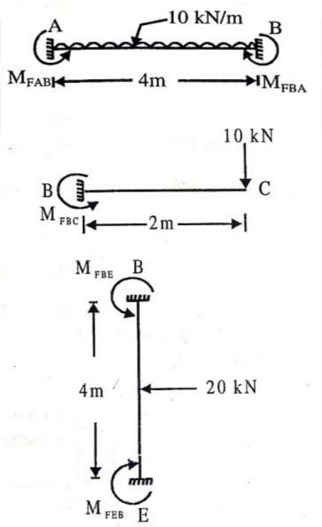
  MFAB = $-Wl^2/12$ = $-10x4^2/12$ = - 13.33 kNm;

  MFBA = $Wl^2/12$ = $10x4^2/12$ = 13.33 kNm;

  MFBC = -10 x 2 = - 20 kNm;

  MFBE = -Wl/8 = - 20x4/8 = - 10 kNm;

  MFEB = Wl/8 = 20x4/8 = 10 kNm;



- **Slope Deflection Equations:**

  MAB = MFAB + 2E(2I)/4(2θA + θB + 3δ/l)

  = - 13.33 + EIθB --- (1)

  MBA = MFBA + 2E(2I)/4(2θB + θA + 3δ/l)

  = 13.33 + EIθB --- (2)

  MBE = MFBE + 2EI/3(2θB + θE + 3δ/l)

  = - 10 + EIθB --- (3)

  MEB = MFEB + 2EI/3(2θE + θB + 3δ/l)

  = 10 + 0.5EIθB --- (4)

- **Joint Equilibrium Equations:**

  Joint B, ΣM = 0;

  MBA + MBC + MBE = 0

  13.33 + 2EIθB – 10 + EIθB – 20 = 0

  3EIθB – 16.67 = 0

  θB = 5.557/EI;

- **Final Moments:**

  MAB = - 7.773 kNm;

  MBA = 24.44 kNm;

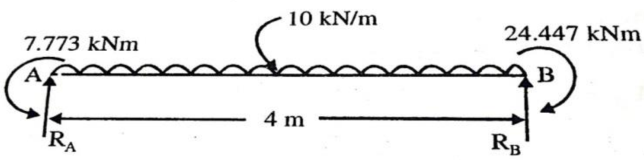
  MBC = - 20 kNm;

  MCD = - 4.33 kNm;

  MDC = 12.78 kNm;

- **To Draw S.F.D:**

  Span AB:

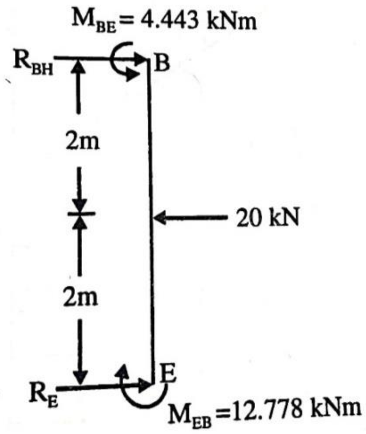


Taking moments about A.

RA x 4 – 10 x 4 x 4/2 – 7.773 + 24.447 = 0; RA = 15.83 KN

RBH = 10 x 4 – RA; RBH = 24.168 KN

Span BE:

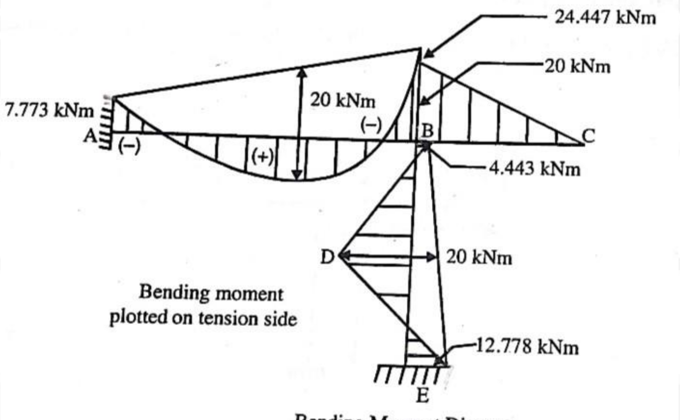


Taking moments about B.

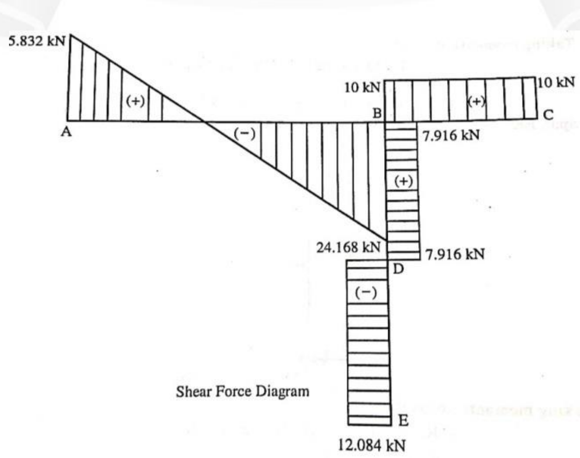
-RE x 4 – 4.443 + 12.778 + 20 x 2 = 0; RE = 12.083 KN

RBH = Total load – RE = 7.916 KN

- **BMD and SFD:**

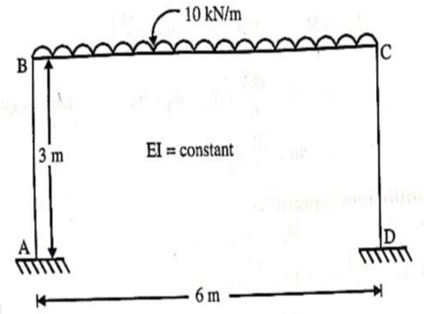


Bending Moment Diagram



PROBLEM NO:02

Analysis the rigid frame shown in fig.,Calculate the support moments using slope deflection method.Draw the SF and BM diagrams.



Solution:

- **Fixed End Moments:**

  MFAB = MFBA = 0

  MFBC = $-Wl^2/12$ = - $10x6^2/12$ = - 30 kNm

  MFCB = $Wl^2/12$ = $10x6^2/12$ = 30 kNm

  MFCD = MFDC = 0

- **Slope Deflection Equations:**

  MAB = MFAB + 2EI/3(2θA + θB + 3δ/l)

  = 2/3EIθB --- (1)

  MBA = MFBA + 2EI/3(2θB + θA + 3δ/l)

  = 4/3EIθB --- (2)

  MBC = MFBC + 2EI/6(2θB + θC + 3δ/l)

  = - 30 + 1/3EIθB --- (3)

  MCB = MFCB + 2EI/3(2θC + θB + 3δ/l)

  = 30 + 1/3EIθB --- (4)

  MCD = MFCD + 2EI/6(2θC + θD + 3δ/l)

  = 4/3EIθB --- (5)

  MDC = MFDC + 2EI/6(2θD + θC + 3δ/l)

  = 2/3EIθB --- (6)

- **Joint Equilibrium Equations:**

  Joint B:

  MBA + MBC = 0

  4/3θB – 30 + 1/3EIθB = 0 --- (7)

  Joint C:

  MCB + MCD = 0

  4/3θC – 30 + 1/3EIθC = 0 --- (8)

  Equvating (7 & 8);we get

  θC = - 18/EI; θB = 18/EI;

- **Final Moments:**

  MAB = 12 kNm;

  MBA = 24 kNm;

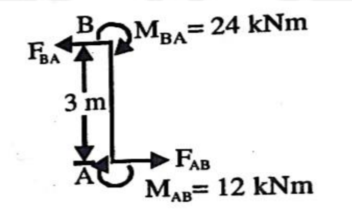
  MBC = - 24 kNm;

  MCB = 24 kNm;

  MCD = - 24 kNm;

  MDC = - 12 kNm;

- **To Draw S.F.D:**



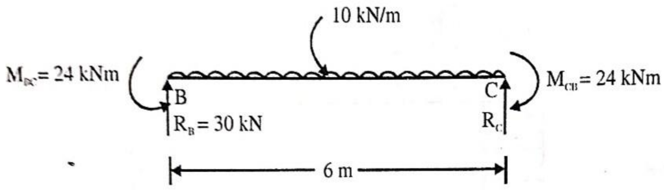
Span AB:

Taking moments about A.

-FBA x 3 + MBA + MAB = 0;

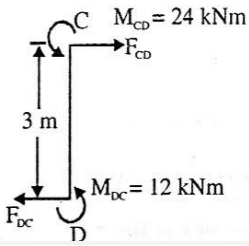
24 + 12 = FBA x 3; FBA = FAB = 12 KN

Span BC:



RB = RC = Total load/2 = 10 x 6/2 = 30 KN

Span CD:

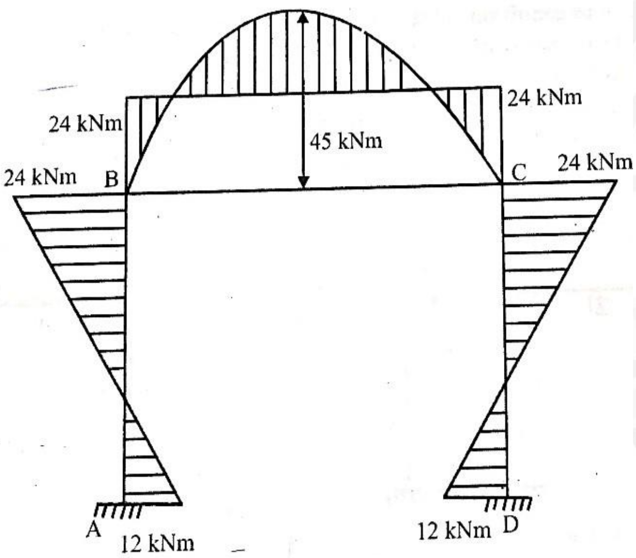


FCD = FDC = 12KN (by symmetry)

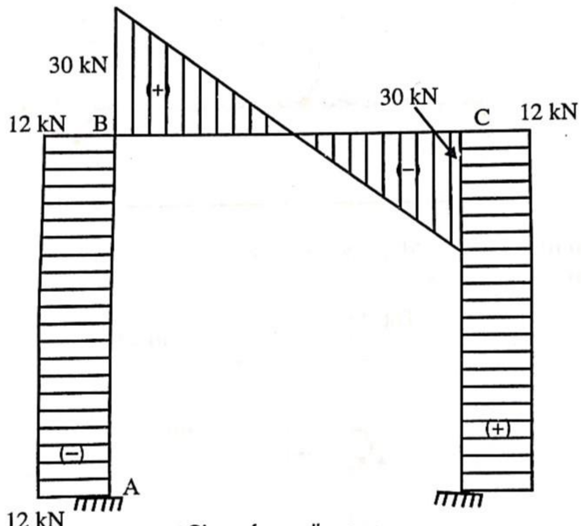
- **Free BMD:**

  MBC = $Wl^2/8 = 10 \times 6^2/8 = 45$ kNm

- **BMD and SFD:**



Bending Moment Diagram

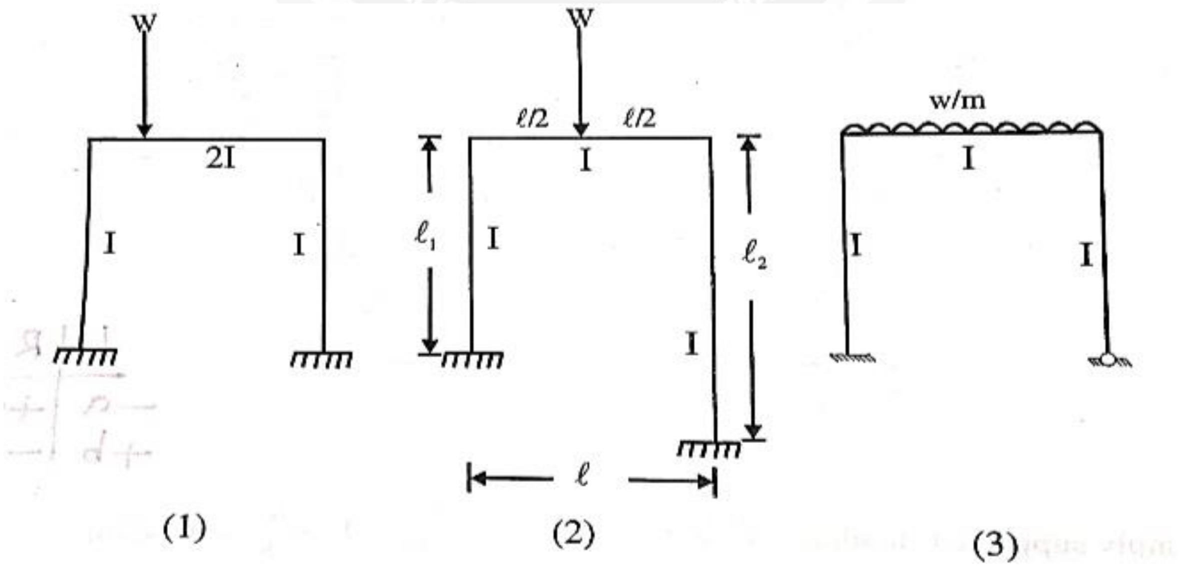


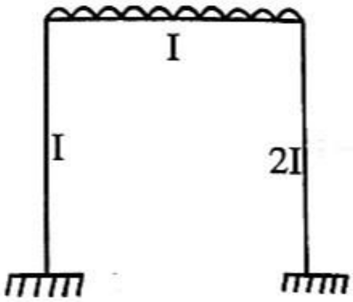
Shear force diagram

2.3.2.RIGID FRAMES WITH SWAY IN SLOPE DEFLECTION METHOD.

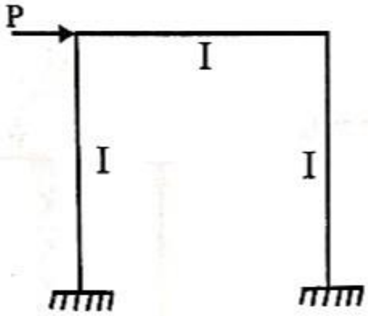
Portal frames may sway due to one of the following reasons:

- Eccentric or unsymmetrical loading on the portal frames.
- Unsymmetrical shape of the frames.
- Different end conditions of the columns of the portal frames.
- Non uniform section of the members of the frame.
- Horizontal loading on the columns of the frame.
- Settlement of the supports of the frame.
- A combination of the above.

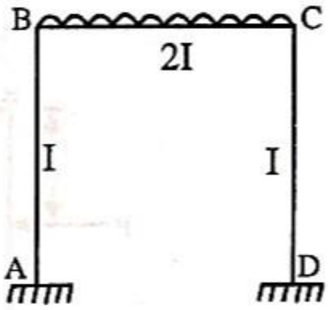




(4)

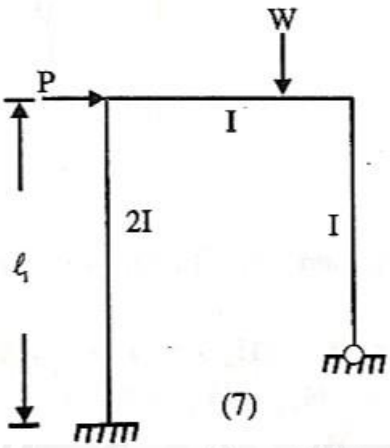


(5)



(6)

D Settles down / Sinks by δ



(7)

2.3.3. NUMERICAL EXAMPLES ON (RIGID FRAMES WITH SWAY):

PROBLEM NO:03

Analysis the rigid frame shown in fig.,Calculate the support moments using slope deflection method.Draw the SF and BM diagrams.

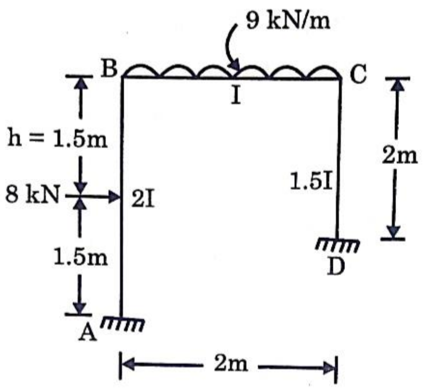


Fig. 2.16

Solution:

- **Fixed End Moments:**

  MFAB = -Wl/8 = - 8x3/8 = - 3 kNm;

  MFBA = Wl/8 = 8x3/8 = 3 kNm;

  MFAB = -Wl^2/12 = - 20x4^2/12 = - 26.67 kNm;

  MFBA = Wl^2/12 = 20x4^2/12 = 26.67 kNm;

  MFCD = 0;

  MFDC = 0;

- **Slope Deflection Equations:**

  MAB = MFAB + 2E(2I)/3(2θA + θB + 3δ/l)

  = - 3 + 4/3EI(θB – δ) --- (1)

  MBA = MFBA + 2E(2I)/3(2θB + θA + 3δ/l)

  = 3 + 4/3EI(2θB – δ) --- (2)

  MBC = MFBC + 2EI/2(2θB + θC + 3δ/l)

  = - 3 + EI(2θB + θC) --- (3)

  MCB = MFCB + 2EI/2(2θC + θB + 3δ/l)

  = 3 + EI(2θC + θB) --- (4)

  MCD = MFCD + 2E(1.5I)/2(2θC + θD + 3δ/l)

  = 1.5EI(2θC - 3δ/2) --- (5)

  MDC = MFDC + 2EI/6(2θD + θC + 3δ/l)

  = 1.5EI(2θC - 3δ/2) --- (6)

- **Equilibrium and Shear Equations:**

  MBA + MBC = 0

  14 θB - 4δ + 3θC = 0 ---(7)

  MCB + MCD = 0

  θB - 2.25δ + 5θC = 0 ---(8)

  Using Shear Equations,we get;

  MAB + MBA – Ph/l +MCD + MDC/l + P = 0

  (θC = - 0.044/EI; θB = 0.414/EI).

- **Final Moments:**

  MAB = - 4.34 kNm;

  MBA = 2.24 kNm;

  MBC = - 2.24 kNm;

  MCB = 3.33 kNm;

  MCD = - 3.33 kNm;

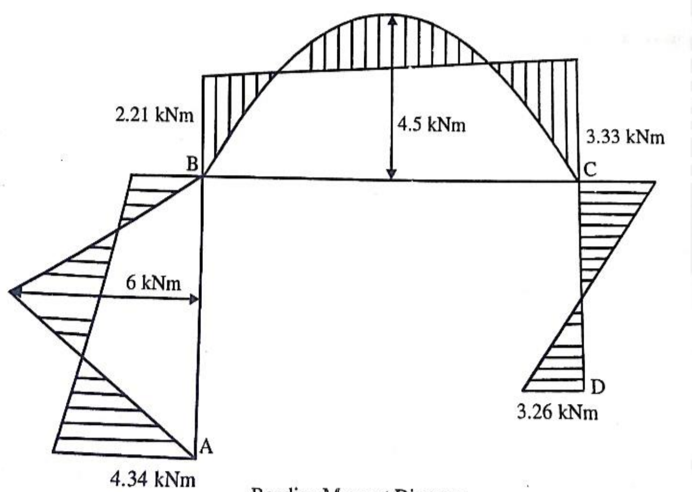
  MDC = - 3.26 kNm;

- **Free Bending Moments:**

  AB = Wl/4 = 8 x 3/4 = 6 kNm

  BC = $Wl^2/8 = 9 \times 2^2/8 = 4.5$ kNm

- **BMD:**



Bending Moment Diagram