UNIT-II

DESIGN OF EXPERIMENTS

ANOVA – Analysis of Variance

2.1. Working Rule (One – Way Classification)

Set the null hypothesis H_0: There is no significance difference between the treatments.

Set the alternative hypothesis H_1: There is a significance difference between the treatments.

Step: 1 Find N = number of observations

Step: 2 Find T = The total value of observations

Step: 3 Find the Correction Factor C . F $= \dfrac{T^2}{N}$

Step: 4 Calculate the total sum of squares and find the total sum of squares

$$\text{TSS} = \left(\sum X_1{}^2 + \sum X_2{}^2 + \sum X_3{}^2 + \ldots\right) - C.F$$

Step: 5 Column sum of squares SSC $\left(\dfrac{(\sum X_1)^2}{N_1} + \dfrac{(\sum X_2)^2}{N_2} + \dfrac{(\sum X_3)^2}{N_3} + \ldots\right) - C.F$

Where N_i = Total number of observation in each column $(i = 1, 2, 3, \ldots)$

Step: 6 Prepare the ANOVA to calculate F – ratio

Source of variation	Sum of Degrees	Degrees of Freedom	Mean Square	F - Ratio
Between Samples	SSC	K - 1	$MSC = \dfrac{SSC}{K-1}$	$F_c = \dfrac{MSC}{MSE}$ if MSC> MSE
Within Samples	SSE	N - K	$MSE = \dfrac{SSE}{N-K}$	$F_c = \dfrac{MSE}{MSC}$ if MSE> MSC

Step: 7 Find the table value (use chi square table)

Step: 8 Conclusion:

Calculated value $<$ Table value, then we accept null hypothesis.

Calculated value $>$ Table value, then we reject null hypothesis.

PROBLEMS ON ONE WAY ANOVA

1.A completely randomised design experiment with 10 plots and 3 treatments gave the following results.

Plot No	1	2	3	4	5	6	7	8	9	10
Treatment	A	B	C	A	C	C	A	B	A	B
Yield	5	4	3	7	5	1	3	4	1	7

Analyse the result for treatment effects.

Solution:

Set the null hypothesis H_0: There is no significance difference between the treatments.

Set the alternative hypothesis H_1: There is a significance difference between the treatments.

Treatments	Yields from plots			
A	5	7	3	1
B	4	4	7	-
C	3	5	1	-

TABLE:

Treatment A   Treatment B   Treatment C

X_1	$X_1{}^2$	X_2	$X_2{}^2$	X_3	$X_3{}^2$
5	25	4	16	3	9
7	49	4	16	5	25
3	9	7	49	7	7
1	1	-	-	-	-
$\sum X_1 = 16$	$\sum X_1{}^2 = 84$	$\sum X_2 = 5$	$\sum X_2{}^2 = 81$	$\sum X_3 = 9$	$\sum X_3{}^2 = 35$

Step: 1 $N = 10$

Step: 2 Sum of all the items (T) $= \sum X_1 + \sum X_2 + \sum X_3 = 16 + 15 + 9 = 40$

Step: 3 Find the Correction Factor C . F $= \dfrac{T^2}{N} = \dfrac{(40)^2}{10} = 160$

Step: 4 TSS = Total sum of squares

= sum of squares of all the items – C. F

$$\text{TSS} = \left(\sum X_1{}^2 + \sum X_2{}^2 + \sum X_3{}^2 + \ldots \right) - C.F$$

$$= (84 + 81 + 35) - 160 = 40$$

Step: 5 SSC = Sum of squares between samples

$$\text{SSC} = \left(\frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_2} + \frac{(\sum X_3)^2}{N_3} + \ldots \right) - C.F$$

$$\text{SSC} = \left(\frac{(16)^2}{4} + \frac{(15)^2}{3} + \frac{(9)^2}{3} + \ldots \right) - 160$$

$$= 64 + 75 + 27 - 160 = 6$$

Step: 6 MSC = Mean squares between samples

$$= \frac{Sum\ of\ squares\ between\ samples}{d.f}$$

$$= \frac{6}{2} = 3$$

SSE = Sum of squares within samples

= Total sum of squares – Sum of squares between samples

$$= 40 - 6 = 34$$

Step:7 MSE = Mean squares within samples

$$= \frac{Sum\ of\ squares\ within\ samples}{d.f}$$

$$= \frac{34}{7} = 4.86$$

ANOVA TABLE

Source of variation	Sum of Degrees	Degrees of Freedom	Mean Square	F - Ratio
Between Samples	SSC = 6	K – 1 = 3 – 1 =2	$\text{MSC} = \frac{SSC}{K-1} = 3$	
Within Samples	SSE = 34	N – K =10 – 3 = 7	$\text{MSE} = \frac{SSE}{N-K} = 4.86$	$F_c = \frac{MSE}{MSC} = 1.62$

d.f for (7, 2) at 5% level of significance is 19.35

Step: 8 Conclusion:

Calculated value $<$ Table value, then we accept null hypothesis.

2. Three different machines are used for a production. On the basis of the outputs, set up one – way ANOVA table and test whether the machines are equally effective.

Outputs		
Machine I	**Machine II**	**Machine III**
10	9	20
15	7	16
11	5	10
10	6	14

Given that the value of F at 5% level of significance for (2, 9) d. f is 4.26

Solution:

Set the null hypothesis H_0: The machines are equally effective.

TABLE:

Treatment A		Treatment B		Treatment C	
X_1	X_1^2	X_2	X_2^2	X_3	X_3^2
10	100	9	81	20	400
15	225	7	49	16	256
11	121	5	25	10	100
20	400	6	36	14	196
$\sum X_1 = 56$	$\sum X_1^2 = 846$	$\sum X_2 = 27$	$\sum X_2^2 = 191$	$\sum X_3 = 60$	$\sum X_3^2 = 952$

Step: 1 N $=12$

Step: 2 Sum of all the items (T) $= \sum X_1 + \sum X_2 + \sum X_3 = 56 + 27 + 60 = 143$

Step: 3 Find the Correction Factor C . F $= \dfrac{T^2}{N} = \dfrac{(143)^2}{12} = 1704.08$

Step: 4 TSS = Total sum of squares

$$= \text{sum of squares of all the items} - \text{C. F}$$

$$\text{TSS} = \left(\sum X_1{}^2 + \sum X_2{}^2 + \sum X_3{}^2 + \ldots \right) - C.F$$

$$= (846 + 191 + 952) - 1704.08 = 284.92$$

Step: 5 SSC = Sum of squares between samples

$$\text{SSC} = \left(\frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_2} + \frac{(\sum X_3)^2}{N_3} + \ldots \right) - C.F$$

$$\text{SSC} = \left(\frac{(56)^2}{4} + \frac{(27)^2}{4} + \frac{(60)^2}{4} + \ldots \right) - 1704.08$$

$$= 784 + 182.25 + 900 - 1704.08 = 162.17$$

Step: 6 MSC = Mean squares between samples

$$= \frac{Sum\ of\ squares\ between\ samples}{d.f}$$

$$= \frac{162.17}{2} = 81.085$$

SSE = Sum of squares within samples

$$= \text{Total sum of squares} - \text{Sum of squares between samples}$$

$$= 284.92 - 162.17 = 122.75$$

Step: 7 MSE = Mean squares within samples

$$= \frac{Sum\ of\ squares\ within\ samples}{d.f}$$

$$= \frac{122.75}{9} = 13.63$$

ANOVA TABLE

Source of variation	Sum of Degrees	Degrees of Freedom	Mean Square	F - Ratio
Between Samples	SSC = 162.17	K – 1 = 3 – 1 = 2	$MSC = \frac{SSC}{K-1} =$ 81.085	
Within Samples	SSE = 122.75	N – K = 12 – 3 = 9	$MSE = \frac{SSE}{N-K} =$ 13.63	$F_c = \frac{MSE}{MSC} = 5.95$

d.f for (2, 9) at 5% level of significance is 4.26.

Step: 8 Conclusion:

Calculated value > Table value, then we reject the null hypothesis.

i.e., the three machines are not equally effective.