DISCRETE MEMORYLESS CHANNEL

- **Transmission rate over a noisy channel**

Repetition code

Transmission rate

- **Capacity of DMC**

Capacity of a noisy channel Examples

➢ All these transition probabilities from xi to yj are gathered in a transition matrix.

➢ The (i ; j) entry of the matrix is P(Y = yj /jX = xi), which is called forward transition probability.

➢ In DMC the output of the channel depends only on the input of the channel at the same instant and not on the input before or after.

➢ The input of a DMC is a RV (random variable) X who selects its value from adiscrete limited set X.

➢ The cardinality of X is the number of the point in the used constellation.

➢ In an ideal channel, the output is equal to the input.

➢ In a non-ideal channel, the output can be different from the input with a given probability.

- **Transmission rate:**

➢ H(X) is the amount of information per symbol at the input of the channel.

➢ H(Y) is the amount of information per symbol at the output of the channel.

➢ H(XjY) is the amount of uncertainty remaining on X knowing Y .

➢ The information transmission is given by:I (X; Y) = H(X) − H(XjY) bits/channel use

➢ For an ideal channel X = Y , there is no uncertainty over X when we observe Y . So all the information is transmitted for each channel use: I (X;Y) = H(X)

➢ If the channel is too noisy, X and Y are independent. So the uncertainty over X remains the same knowing or not Y , i.e. no information passes through the channel: I (X; Y) = 0.

- **Hard and soft decision:**

➢ Normally the size of constellation at the input and at the output are the same, i.e., jXj = jYj

➢ In this case the receiver employs hard-decision decoding.

➢ It means that the decoder makes a decision about the transmitted symbol.

➢ It is possible also that jXj 6= jY j.

➢ In this case the receiver employs a soft-decision.

Channel models and channel capacity:

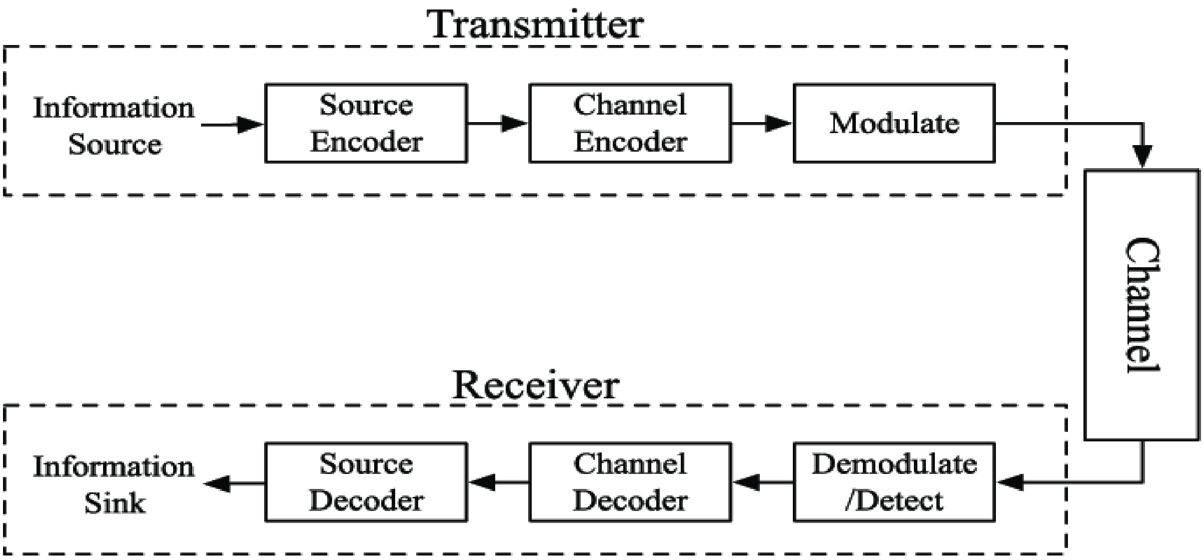

Fig1.2 Block Diagram of Digital Communication System

(Source: https://www.researchgate.net/figure/Block-diagram-of-a-typical-communication-system-doi101371-journalpone0082935g001_fig15_259457178)

1. The encoding process is a process that takes a k information bits at a time and maps each k-bit sequence into a unique n-bit sequence. Such an n-bit sequence is called a code word.

2. The code rate is defined as k/n.

3. If the transmitted symbols are M-ary (for example, M levels), and at the receiver the output of the detector, which follows the demodulator, has an estimate of the transmitted data symbol with

(a). M levels, the same as that of the transmitted symbols, then we say the detector has made a hard decision;

(b). Q levels, Q being greater than M, then we say the detector has made a soft decision.

Channels models:

1. Binary symmetric channel (BSC):

If (a) the channel is an additive noise channel, and (b) the modulator and demodulator/detector are included as parts of the channel. Furthermore, if the modulator employs binary waveforms, and the detector makes hard decision, then the channel has a discrete-time binary input sequence and a discrete-time binary output sequence.

Note that if the channel noise and other interferences cause statistically independent errors in the transmitted binary sequence with average probability p, the channel is called a BSC. Besides, since each output bit from the channel depends only upon the corresponding input bit, the channel is also memoryless.

2. Discrete memory less channels (DMC):

A channel is the same as above, but with q-ary symbols at the output of the channel encoder, and Q-ary symbols at the output of the detector, where $Q \geq q$. If the channel and the modulator are memory less, then it can be described by a set of qQ conditional probabilities

$P(Y = y_i | X = x_j) \equiv P(y_i | x_j), i = 0,1,...,Q - 1; j = 0,1,..., q - 1$

Such a channel is called discrete memory channel (DSC).

If the input to a DMC is a sequence of n symbols $u_1, u_2, ..., u_n$ selected from the alphabet X and the corresponding output is the sequence $v_1, v_2, ..., v_n$ of symbols from the alphabet Y, the joint conditional probability is

$$P(Y_1 = v_1, Y_2 = v_2, ..., Y_n = v_n | X_1 = u_1, X_2 = u_2, ..., X_n = u_n) = \prod_{k=1}^{n} P(Y_k = v_k | X_k = u_k)$$

The conditional probabilities $P(y_i | x_j)$ can be arranged in the matrix form $\mathbf{P} = [p_{ji}]$, P is called

the probability transition matrix for the channel.

3. Discrete-input, continuous-output channels:

Suppose the output of the channel encoder has q-ary symbols as above, but the output of the detector is un quantized $(Q = \infty)$. The conditional probability density functions

$p(y | X = x_k), k = 0,1, 2,..., q - 1$

AWGN is the most important channel of this type.

$Y = X + G$

where $G \square N(0, \sigma^2)$. Accordingly

$$P(y/X = \mathrm{xk}) = \frac{1}{\sqrt{2\pi}} e^{(-x-y)^2/2\sigma^2} \quad \mathrm{k}=0,1,2,\mathrm{q}-1$$

For any given sequence $X_i, i = 1, 2,..., n$, the corresponding output is $Y_i, i = 1, 2,..., n$

$Y_i = X_i + G_i, i = 1, 2,..., n$

If, further, the channel is memory less, then the joint conditional pdf of the detector‘s output is

$$p(y_1, y_2, ..., y_n | X_1 = u_1, X_2 = u_2, ..., X_n = u_n) = \prod_{i=1}^{n} p(y_i | X_i = u_i)$$

4. Waveform channels:

If such a channel has bandwidth W with ideal frequency response $C(f) = 1$, and if the bandwidth-limited input signal to the channel is $x(t)$, and the output signal, $y(t)$ of the channel is corrupted by AWGN, then

$y(t) = x(t) + n(t)$

The channel can be described by a complete set of orthonormal functions:

$$y(t) = \sum_i y_i f_i(t), \quad x(t) = \sum_i x_i f_i(t), \quad n(t) = \sum_i n_i f_i(t)$$

where

$$y_i = \int_0^T y(t) f_i^*(t) dt = \int_0^T [x(t) + n(t)] f_i^*(t) dt = x_i + n_i$$

The functions $\{f_i(t)\}$ form a complete orthonormal set over (0,T)

The statistical description in such a system is

$$P(y/X = \text{xk}) = \frac{1}{\sqrt{2\pi}} e^{(-x-y)^2/2\sigma^2} \quad \text{k= 0,1,2,q-1}$$

Since $\{n_i\}$ are uncorrelated and are Gaussian, therefore, statistically independent. So

$$p(y_1, y_2, \ldots, y_N \mid x_1, x_2, \ldots, x_n) = \prod_{i=1}^{n} p(y_i \mid x_i)$$

Channel Capacity:

Channel model: DMC

Input alphabet: $X = \{x_0, x_1, x_2, \ldots, x_{q-1}\}$

Output alphabet: $Y = \{y_0, y_1, y_2, \ldots, y_{q-1}\}$

Suppose x_j is transmitted, y_i is received, then

The mutual information (MI) provided about the event $\{X = x_j\}$ by the occurrence of the event

$$\{Y = y_j\} \text{ is } \log\left[P(y_i \mid x_j)/P(y_i)\right] \text{ with } P(y_i) = P(Y = y_i) = \sum_{k=0}^{q-1} P(x_k) P(y_i \mid x_k)$$

Hence, the average mutual information (AMI) provided by the output Y about the input X is

$$I(X,Y) = \sum_{j=0}^{q-1}\sum_{i=0}^{Q-1} P(x_j)P(y_i \mid x_j)\log\left[P(y_i \mid x_j)/P(y_i)\right]$$

To maximize the AMI, we examine the above equation:

(1). $P(y_i)$ represents the jth output of the detector;
(2). $P(y_i \mid x_j)$ represents the channel characteristic, on which we cannot do anything;

(3). $P(x_j)$ represents the probabilities of the input symbols, and we may do something or control them. Therefore, the channel capacity is defined by

$$C = \max_{P(x_j)} \sum_{j=0}^{q-1}\sum_{i=0}^{Q-1} P(x_j)P(y_i \mid x_j)\log\left[P(y_i \mid x_j)/P(y_i)\right]$$

with two constraints: $P(x_j) \geq 0$; $\sum_{j=0}^{q-1} P(x_j) = 1$

- **Unit of C:**
  - bits/channel use when $\log = \log_2$; and
  - nats/input symbol when $\log = \log_e = \ln$
  - If a symbol enters the channel every τ_s seconds (seconds/channel use)
  - Channel capacity: C/τ_s (bits/s or nats/s).