Gravity dam

- A gravity dam is a structure so proportioned that its own weight resists the forces exerted upon it. It requires little maintenance and it is most commonly used.
- A Gravity dam has been defined as a “structure which is designed in such a way that its own weight resist the external forces”.
- This type of a structure is most durable and solid and requires very less maintenance.
- Such dams are constructed of masonry or Concrete.
- However, concrete gravity dams are preferred these days and mostly constructed.
- The line of the upstream face or the line of the crown of the dam if the upstream face is sloping, is taken as the reference line for layout purpose etc. and is known as the Base line of the dam or the “Axis of The Dam” When suitable conditions are available such dams can be constructed up to great heights.

The different components of a solid gravity dam are

- Crest.
- Free Board.
- Heel.
- Toe.
- Sluice Way.
- Drainage Gallery.

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- Crest.
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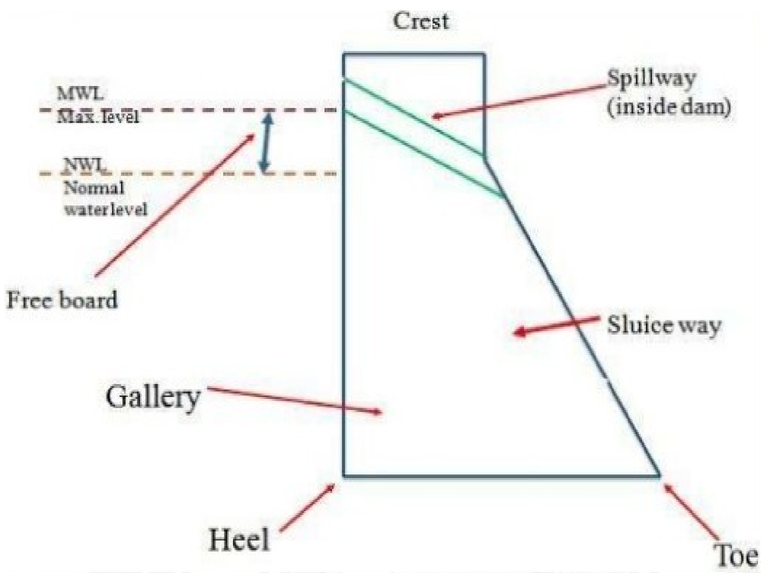
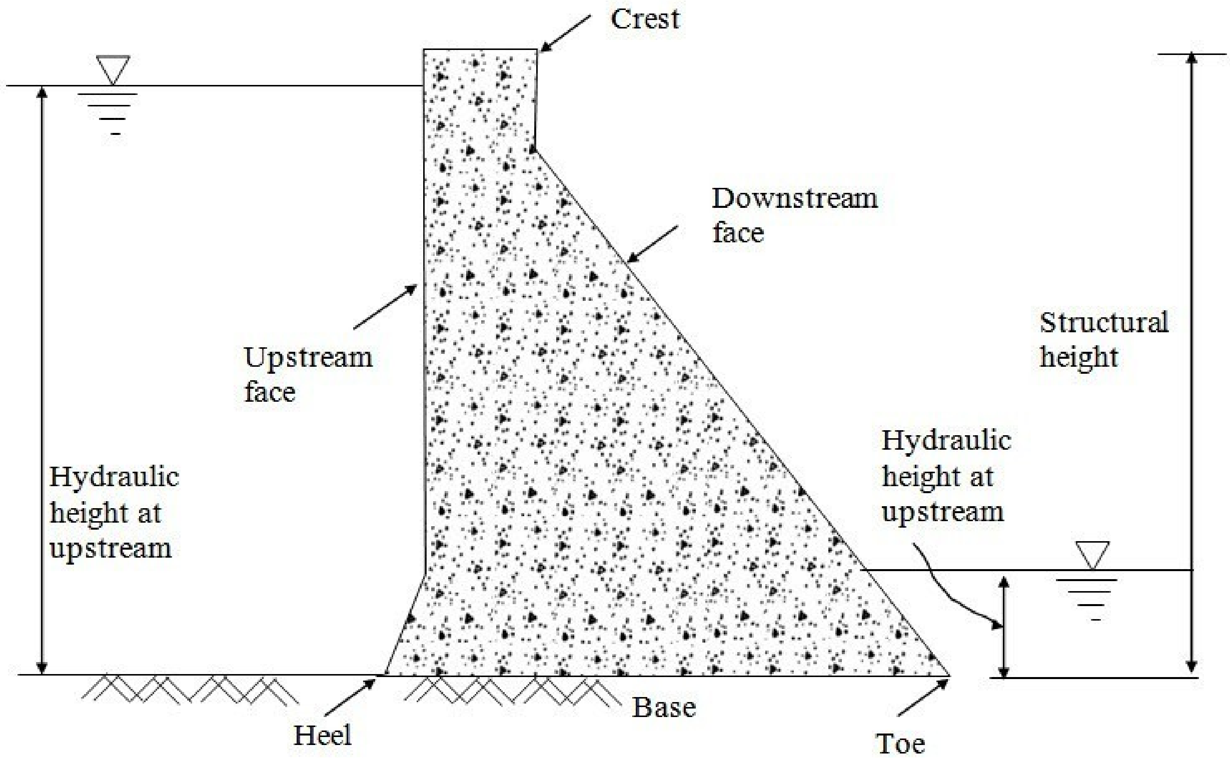


- Drainage Gallery.

Typical cross section of gravity Dam:



Heel: contact with the ground on the upstream side

Toe: contact on the downstream side

Abutment: Sides of the valley on which the structure of the dam rest

Galleries: small rooms like structure left within the dam for checking operations.

Diversion tunnel: Tunnels are constructed for diverting water before the construction of dam. This helps in keeping the river bed dry.

Spillways: It is the arrangement near the top to release the excess water of the reservoir to downstream side

Sluice way: An opening in the dam near the ground level, which is used to clear the silt accumulation in the reservoir side.

Forces Acting on Gravity Dam

The Various external forces acting on Gravity dam may be:

- Water Pressure
- Uplift Pressure
- Pressure due to Earthquake forces
- Silt Pressure
- Wave Pressure
- Ice Pressure
- The stabilizing force is the weight of the dam itself

Self weight of the Dam

Self weight of a gravity dam is main stabilizing force which counter balances all the external forces acting on it.

For construction of gravity dams the specific weight of concrete & stone masonry shouldn't be less than 2400 kg/m^3 & 2300 kg/m^3 respectively.

The self weight of the gravity dam acts through the centre of gravity of

the. Its calculated by the following formula – $W = \gamma_m \ X \ Volume$

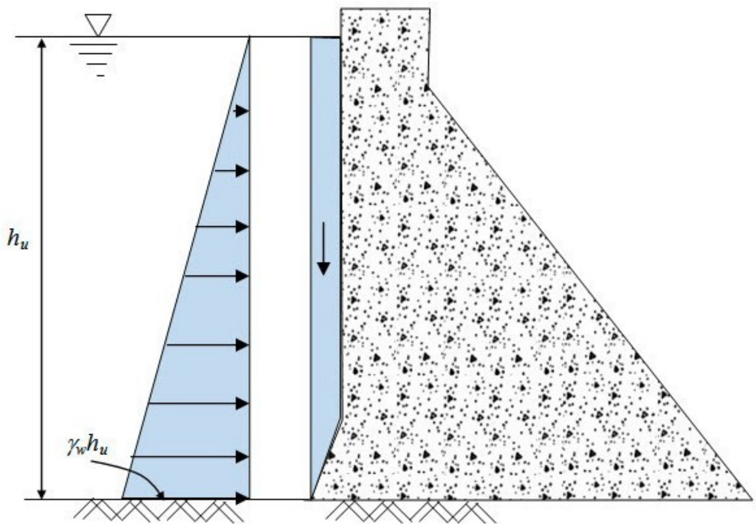
Where γ_m is the specific weight of the dam's material.

Water pressure

- Water pressure on the upstream side is the main destabilizing force in gravity dam.
- Downstream side may also have water pressure.
- Though downstream water pressure produces counter overturning moment, its magnitude is much smaller as compared to the upstream water pressure and therefore generally not considered in stability analysis.

- Water Pressure is the most major external force acting on a gravity dam.
- On upstream face pressure exerted by water is stored upto the full reservoir level. The upstream face may either be vertical or inclined.
- On downstream face the pressure is exerted by tail water. The downstream face is always inclined. It is calculated by the following formula – $P = \frac{1}{2}\gamma_w x\, h^2$

Where γ_w is the unit weight of water & h is the height of water.



Uplift water pressure

- The uplift pressure is the upward pressure of water at the base of the dam as shown in Figure 29.3. It also exists within any cracks in the dam.
- The water stored on the upstream side of the dam has a tendency to seep through the soil below foundation.
- While seeping, the water exerts a uplift force on the base of the dam depending upon the head of water.
- This uplift pressure reduces the self weight of the dam.
- To reduce the uplift pressure, drainage galleries are provided on the base of the dams.
- It is calculated by the following formula – $U = \frac{1}{2}\gamma_w x\, h\, x\, B$

Where 'B' is the width of the base of the dam.

Wave Pressure

When very high wind flows over the water surface of the reservoir, waves are formed which exert pressure on the upstream part of the dam.

The magnitude of waves depend upon –

- The velocity of wind.

- Depth of Reservoir.
- Area of Water Surface.

It is calculated by the following formula - $P_v = 2.4\, \gamma_w\, x h_w$

Where 'h_w ' is the wave height.

WIND PRESSURE :

- The top exposed portion on the dam is small & hence the wind pressure on this portion of dam is negligible.
- But still an allowance should be made for the wind pressure at the rate of about 150 kg/m ^{2} for the exposed surface area of the upstream & downstream faces.

SEISMIC FORCES :

- Dams are subjected to vibration during earthquakes.
- Vibration affects both the body of the dam as well as the water in the reservoir behind the dam.
- The most danger effect occurs when the vibration is perpendicular to the face of the dam.
- Body Forces: Body force acts horizontally at the center of gravity and is calculated as: $P_{em} = a \times W$
- Water Force: Water vibration produces a force on the dam acting horizontally & calculated by: $P_{ew} = \frac{2}{3} C_e\, a\, h^2$

ELEMENTRY PROFILE

- When water is stored against any vertical face, then it exerts pressure perpendicular to the face which is zero at top & maximum at bottom.
- The required top thickness is thus zero & bottom thickness is maximum forming a right angled triangle with the apex at top, one face vertical & some base width.
- Two conditions should be satisfied to achieve stability
  - **When empty -** The external force is zero & its self weight passes through C.G. of the triangle.
  - **When Full -** The resultant force should pass through the extreme right end of the middlethird.

The limiting condition is – $h = \frac{\sigma_c}{\gamma\,(1+S)}$

- where, σ_c=allowable compressive stress

Practical Profile

- Various parameters in fixing the parameters of the dam section are,
- Free Board –IS 6512, 1972 specifies that the free board will be 1.5 times the wave height above normal pool level.
- Top Width – The top width of the dam is generally fixed according to requirements of the roadway to be provided. The most economical top width of the dam is 14 % of its height.

- Base Width – The base width of the dam shall be safe against overturning, sliding & no tension in dam body.

For elementary profile –

- When uplift is considered, $B = \frac{h}{\sqrt{S}}$
- When uplift isn't considered $B = \frac{h}{\sqrt{S-1}}$

Low Gravity Dam

- A low gravity dam is designed on the basis if of elementary profile, where the resultant force passes through the middle-third of its base.
- The principal stress is given by – [illegible] = γ H (S – C + 1) Where, [illegible]=principal stress, γ=unit weight, S=Specific Gravity and C=A constant.
- The principal stress varies with 'H' as all other terms are constant. To avoid failure of the dam the value of '[illegible]' shouldn't exceed allowable working stress(f). F = γ H (S – C + 1)

High Gravity Dam

- The high gravity is a complicated structure, where the resultant force may pass through a point outside the middle-third of the base.
- The section of the dam is modified by providing extra slope on the upstream and downstream side.
- The condition for the high gravity dam are $H > \frac{f}{w(S+1)}$ – Where, f=allowable working stress.

Failure of Gravity Dam

Failure of gravity dams are caused due to,

- Sliding – It may take place on a horizontal joint above formation, on the foundation. Sliding takes place when total horizontal forces are grater than the combined shearing resistance of the joint and the static friction induced by total vertical forces.
- Overturning – A dam fails in overturning when total horizontal forces acting on the dam section are quite great in comparison with total vertical forces. In such cases the resultant of two passes outside the limits of the dam.
- Dam may fail when tension is produced in the concrete.
- Dam may fail in crushing.

Precautions against Failure

- To prevent overturning, the resultant of all forces acting on the dam should remain within the middle-third of the base width of the dam.

- In the dam, the sliding should be fully resisted when the condition for no sliding exists in the dam section.

- In the dam section, the compressive stresses of concrete or masonry should not exceed the permissible working stress to avoid failure due to crushing.
- There should be no tension in the dam section to avoid the formation of cracks.
- The factor of safety should be maintained between 4 to 5.

Temperature Control

During setting of concrete heat of hydration is evolved producing internal temperature stresses resulting in development of internal cracks can get formed.

To control the temperature the following steps may be taken

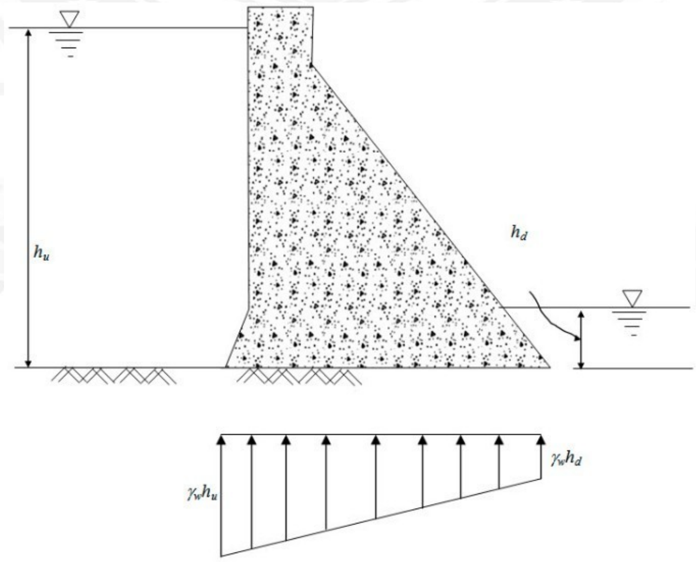
1. Low heat cement may be used in concrete.
2. The water & coarse aggregates should be cooled down to 5°C by suitable means before mixing.
3. During laying the height of concrete blocks should not be more than 1.5 m. It helps radiate heat to the atmosphere more quickly.
4. The water is cooled by crushed ice before using it for the curing purpose.

Advantages

1. Gravity dams are more suitable in narrow valleys.
2. Maintenance cost is lower
3. Failure of these dams is not very sudden.
4. Gravity dams may be built to any height.
5. Loss of water by seepage in gravity dams is less

Disadvantages

1. Initial cost for construction of gravity dams is very higher.
2. Gravity dams of greater height can only be constructed on sound rock foundations.
3. Require skill labour for construction.
4. Design of gravity dams is very complicated.



General Requirement for Stability

A gravity dam may fail in the following modes,

- Overturning
- Sliding
- Compression
- Tension

Therefore, the requirements for stability are,

- The dam should be safe against overturning.
- The dam should be safe against sliding.
- The induced stresses (either tension or compression) in the dam or in the foundation should not exceed the permissible value.

DESIGN OF GRAVITY DAM

Example 19.2. *Fig. 19.20 (a) shows the section of a gravity dam built of concrete. Examine the stability of this section at the base.*

The earthquake forces may be taken as equivalent to 0.1 g for horizontal forces and 0.05 g for vertical forces. The uplift may be taken as equal to the hydrostatic pressure at the either ends and is considered to act over 60% of the area of the section.

A tail water depth of 6 m is assumed to be present when the reservoir is full and there is no tail water when the reservoir is empty.

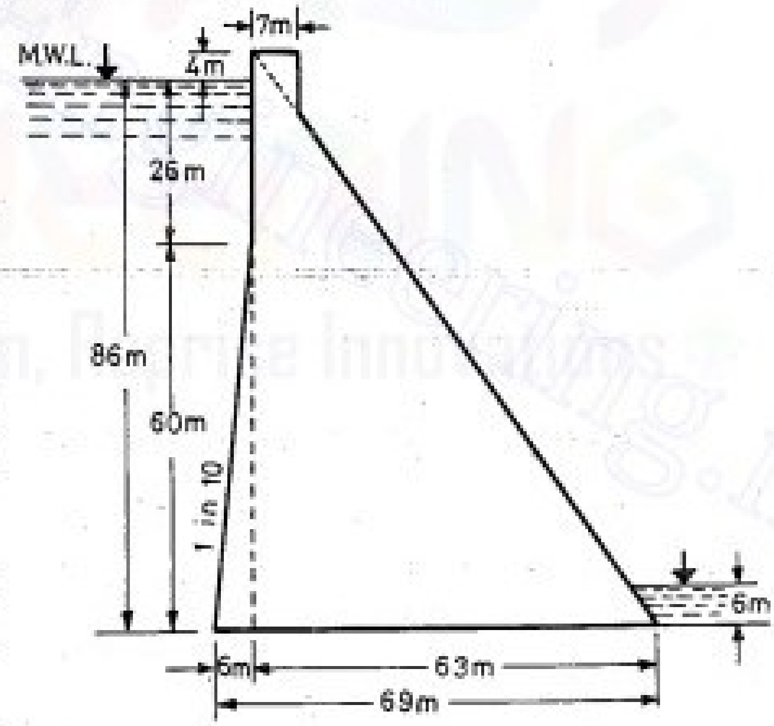


Fig. 19.20 (*a*)

Also indicate the values of various kinds of stresses that are developed at heel and toe. Assume the unit wt. of concrete as 24 kN/m³ ; and unit wt. of water = 10 kN/m³.

Solution. The stability analysis shall be carried out for both the cases, *i.e.* (1) Reservoir Empty, and (2) Reservoir Full.

Case (I) Reservoir Empty. Consider 1 m length of the dam.

When the reservoir is empty, the various forces are worked out in Table 19.2 (*a*) with reference to Fig. 19.20 (*b*). Horizontal earthquake forces acting towards upstream are considered. Stability is examined for two sub-cases, *i.e.* (*a*) When vertical earthquake forces are additive to the weight of the dam ; (*b*). When vertical earthquake forces are subtractive to the dam weight.

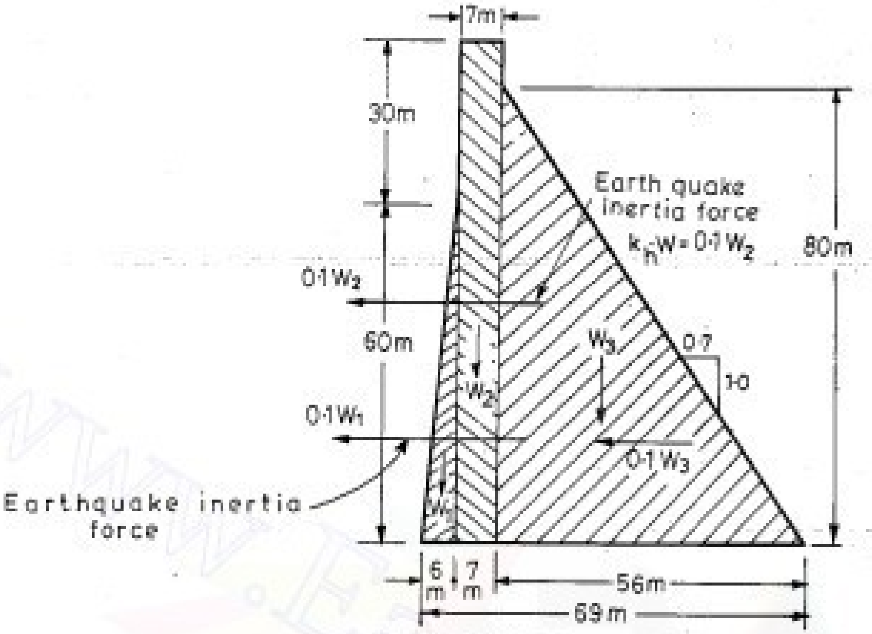


Fig. 19.20 (*b*). Reservoir empty case.

Table 19.2 (*a*)

Name of the force	*Designation if any*	*Magnitude of force in kN.* Vertical	Horizontal	*Lever arm m*	*Moments about the toe anti-clockwise (+ve) in kN.m.*
Downward wt. of dam	W_1	(+) $\frac{1}{2} \times 6 \times 60 \times 24 = 4{,}320$		65.0	(+) 2,80,400
	W_2	(+) $7 \times 90 \times 24 = 15{,}110$		59.5	(+) 8,99,000
	W_3	(+) $\frac{1}{2} \times 56 \times 80 \times 24 = 53700$		37.33	(+) 20,00,000
		$\Sigma V_1 = 73{,}130$			$\Sigma M_1 = (+)\ 31{,}79{,}400$
Horizontal earthquake forces	P_{w1}		$0.1\ W_1 = 0.1 \times 4320 = 432$	20.0	(+) 8640
	P_{w2}		$0.1\ W_2 = 0.1 \times 1{,}511 = 1511$	45.0	(+) 68000
	P_{w3}		$0.1\ W_3 = 0.1 \times 5{,}370 = 5370$	26.67	(+) 1,43,200
			$\Sigma H = 7313$		$\Sigma M_2 = 2{,}19{,}840$
Vertical earthquake forces		$\Sigma V_2 = 0.05 \times \Sigma V_1 = 0.05 \times 73130 = 3{,}657$			$\Sigma M_2 = 0.05 \times \Sigma M_1 = 0.05 \times 31{,}79{,}400 = 1{,}58{,}970$

Case (I). (*a*) *Reservoir empty and vertical earthquake forces are acting downward.*
From table 19.2 (*a*), we have $\Sigma M = \Sigma M_1 + \Sigma M_2 + \Sigma M_3$

$$= 31,79,400 + 2,19,840 + 1,58,970 = \mathbf{35,58,210\ kN \cdot m}$$

Also, $\Sigma V = \Sigma V_1 + \Sigma V_2 = 73,130 + 3,657 = 76,787\ \text{kN}$

$$\bar{x} = \frac{\Sigma M}{\Sigma V} = \frac{35,58,210}{76,787} = 47.3\ \text{m}$$

$$e = \frac{B}{2} - \bar{x} = \frac{69}{2} - 46.3 = 34.5 - 46.3 = -\mathbf{11.8\ m} > \frac{B}{6},\ \textit{i.e.}\ 11.5\ \text{m}.$$

Resultant acts near the heel and slight tension will develop at toe.

$$p_{max/min} = \frac{\Sigma V}{B}\left[1 \pm \frac{6e}{B}\right]$$

$$\therefore \quad p_{max/min} = \frac{76,787}{69}\left[1 \pm \frac{6 \times 11.8}{69}\right] = 1114\ [1 \pm 1.026]$$

p_v at heel $= 1114 \times 2.026 = 2260\ \text{kN/m}^2$; which is ≤ 3000 (safe)

p_v at toe $= 1114 \times (-0.026) = -29\ \text{kN/m}^2$; which is < 420 (*safe*)

Average vertical stress

$$= \frac{\Sigma V}{B} = \frac{76787}{69} = 1114\ \text{kN/m}^2 \text{ ; which is} < 3000 \text{ (safe)}$$

Principal stress at toe,

$$\sigma = p_v \sec^2 \alpha \text{ ; } (\tan \alpha = 0.7)$$

$$= -29\ (1 + 0.49) = -29 \times 1.49 = -43\ \text{kN/m}^2 \text{ ; which is} < 420 \text{ (safe)}$$

Principle stress at heel

$$\sigma_1 = p_{v \cdot (heel)} \sec^2 \phi \qquad \text{where } \tan \phi = 0.1$$

$$\text{or} \qquad \sec^2 \phi = 1 + \tan^2 \phi = 1 + 0.01 = 1.01.$$

or $\sigma_1 = 2260 \times 1.01 = \mathbf{2280\ kN/m^2}$; which is < 3000 (safe).

Shear stress at toe

$$\tau_{0(toe)} = p_{v(toe)} \tan \alpha$$

$$= -29 \times 0.7 = -20.3\ \text{kN/m}^2 \text{ ; which is} < 420 \text{ (safe)}$$

Shear stress at heel

$$\tau_{0(heel)} = p_{v \cdot (heel)} \tan \phi$$

$$= 2260 \times 0.1 = 226\ \text{kN/m}^2 \text{ ; which is} < 3000 \text{ (safe).}$$

Case I. (*b*) *Reservoir empty and vertical earthquake forces are acting upward.*

Then $\Sigma V = \Sigma V_1 - \Sigma V_3$

$$= 73,130 - 3657 = 69473\ \text{kN}$$

$$\Sigma M = \Sigma M_1 + \Sigma M_2 - \Sigma M_3$$

$$= 31,79,400 + 2,19,840 - 1,58,970 = 32,40,270\ \text{kN} \cdot \text{m}.$$

$$\bar{x} = \frac{\Sigma M}{\Sigma V} = \frac{32,40,270}{69473} = \mathbf{46.7\ m.}$$

$$e = \frac{B}{2} - \bar{x} = 34.5 - 46.7 = (-)\ 12.2\ \text{m} < \frac{B}{6}$$

[$-$ ve sign shows that resultant lies near the heel and, therefore, tension will develop at toe.]

Average vertical stress

$$= \frac{\Sigma V}{B} = \frac{69{,}473}{69} = 1004\ \text{kN/m}^2$$

$$p_{max/min} = \frac{\Sigma V}{B}\left[1 \pm \frac{6e}{B}\right]$$

$$= \frac{69473}{69}\left[1 \pm \frac{6 \times 12.2}{69}\right] = 1004\,[1 \pm 1.06]$$

$$p_v \text{ at heel} = 1004 \times 2.06 = 2070\ \text{kN/m}^2 < 3000 \text{ (safe)}$$

$$p_v \text{ at toe} = (-)\ 1004 \times 0.06 = -\,60.3\ \text{kN/m}^2 < 420 \text{ (safe)}$$

Principal stress at toe

$$= \sigma = p_{v(toe)} \sec^2 \alpha$$

$$= -\,60.3\,(1 + 0.49) = -\,60.3 \times 1.49 = 90\ \text{kN/m}^2$$

Shear stress at toe

$$= \tau_0 = p_{v(toe)} \tan \alpha = -\,60.3 \times 0.7$$

$$= \mathbf{-\,42.21\ kN/m^2}\ ;\ \text{which is} < 420 \text{ (safe)}$$

stresses at heel remain critical in this 1st case.

Case II. When the reservoir is full

Horizontal earthquake moving towards the reservoir causing upstream acceleration, and thus producing horizontal forces towards downstream is considered, as it is the worst case for this condition. Similarly, a vertical earthquake moving downward and thus, producing forces upward, *i.e.* subtractive to the weight of the dam is considered.

The uplift coefficient C is taken as equal to 0.6, as given in the equation, and thus uplift pressure diagram as shown in Fig. 19.20 (c), is developed.

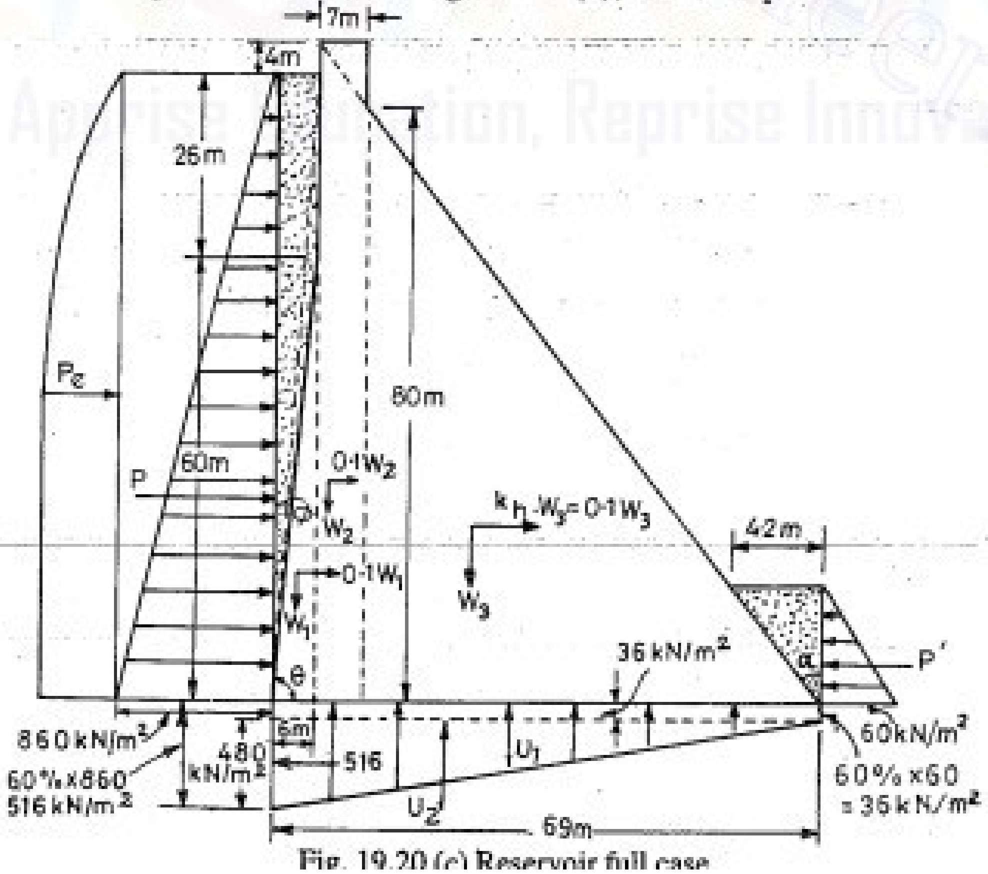


Fig. 19.20 (c) Reservoir full case

The various forces acting in this case are :

(*i*) Hydrostatic pressures P and P'.

(*ii*) Hydrodynamic pressure P_e. (P_e' is neglected as it is very small and neglection is on conservative side.)

(*iii*) Uplift forces U_1 and U_2

(*iv*) Weight of the dam, W_1, W_2 and W_3.

(*v*) Horizontal inertial earthquake forces acting towards downstream, equal to 0.1 W_1, 0.1 W_2 and 0.1 W_3 at *c.gs.* of these weights W_1, W_2 and W_3 respectively.

(*vi*) A vertical force equal to 0.05 W or (0.05 ΣV_1) acting upward.

Calculation of P_e

P_e and the moment due to this hydrodynamic force is calculated, and then all the forces and their moments are tabulated in Table 19.2 (*b*).

Calculation of P_e from Zanger's formulas

$$P_e = 0.726\, p_e\, H \qquad ...(19.3)$$

$$\text{where } p_e = C_m \cdot K_h \cdot \gamma_w \cdot H \qquad ...(19.4)$$

$$\text{and} \quad C_m \doteq 0.735 \frac{\theta}{90°}$$

Since the u/s inclined face is extended for more than half the depth, the overall slope up to the whole height may be taken.

Fig. 19.20 (*d*)

$$\therefore \qquad \tan\theta = \frac{86}{6} = 14.33$$

$$\theta = 81.9°$$

$$\therefore \qquad C_m = 0.735 \times \frac{81.9°}{90°} = 0.668.$$

$$p_e = 0.668 \times 0.1 \times 10 \times 86 = 57.5$$

$$P_e = 0.726 \times 57.5 \times 86 = \mathbf{3580\ kN.}$$

$$M_e = 0.412 \cdot P_e \cdot H = 0.412 \times 3580 \times 86 = \mathbf{1{,}26{,}500\ kN.m.}$$

Case 2 (*a*) **Reservoir full with all forces including uplift**

$$\Sigma M = [31{,}79{,}400 + 2{,}23{,}380 - 8{,}47{,}500 - 1{,}58{,}970 - 10{,}59{,}730 - 1{,}26{,}500 - 2{,}19{,}840]$$

$$= \mathbf{34{,}02{,}780 - 24{,}12{,}540 = 9{,}90{,}240\ kN/m.}$$

$$\Sigma V = 73130 + 3486 - 19030 - 3657 = 53929 \text{ kN}$$

$$\bar{x} = \frac{\Sigma M}{\Sigma V} = \frac{9{,}90{,}240}{53{,}929} = 18.36 \text{ m}$$

$$e = \frac{B}{2} - \bar{x} = 34.5 - 18.36 = 16.14 > \frac{B}{6}$$

The resultant is nearer the toe and tension is developed at the heel.

Average vertical stress

$$= \frac{\Sigma V}{B} = \frac{53929}{69} = 782 \text{ kN/m}^2.$$

$$p_{max/min} = \frac{\Sigma V}{B}\left[1 \pm \frac{6e}{B}\right]$$

Table 19.2 (*b*)

Name of force	*Designation if any*	*Magnitude of force in kN*: *Vertical forces Downward = +ve Upward = −ve*	*Magnitude of force in kN*: *Horizontal forces Towards Upstream = +ve Towards Downstream = −ve*	*Lever arm in m*	*Moments about toe in kN. Anticlock wise (+ve) and clockwise (− ve) in kN.m*
(1)	(2)	(3)	(4)	(5)	(6)
Weight of Dam	W_1	(+) $\frac{1}{2} \times 6 \times 60 \times 1 \times 24 = 4320$		65.0	(+) 2,80,400
	W_2	(+) $7 \times 90 \times 1 \times 24 = 15{,}110$		59.5	(+) 8,99,000
	W_3	(+) $\frac{1}{2} \times 56 \times 80 \times 1 \times 24 = 53{,}700$		37.33	(+) 20,00,000
		$\Sigma V_1 = (+)\ 73{,}130$			$\Sigma M_1 = 31{,}79{,}400$
Weight of water supported on u/s slope water on d/s slope.	—	(+) $26 \times 6 \times 1 \times 10 = 1560$		66.0	(+) 1,02,800
	—	(+) $\frac{1}{2} \times 60 \times 6 \times 1 \times 10 = 1800$		67.0	(+) 1,10,400
	—	(+) $\frac{1}{2} \times 6 \times 4.2 \times 1 \times 10 = 126$		1.4	(+) 180
		$\Sigma V_2 = (+)\ 3486$			$\Sigma M_2 = (+)\ 2{,}23{,}380$
Uplift forces	U_1	(−) $69 \times 3.6 \times 10 = 2{,}480$		34.5	(−) 85,500
	U_2	(−) $\frac{1}{2} \times 69 \times 48 \times 10 = 16{,}550$		46.0	(−) 7,62,000
		$\Sigma V_3 = (-)\ 19{,}030$			$\Sigma M_3 = (-)\ 8{,}47{,}500$
Upward vertical earthquake forces 0.05 W		$\Sigma V_4 = (-)\ 0.05 \cdot \Sigma V_1$ $= (-)\ 0.05 \times 73{,}130$ $= (-)\ 3{,}657$			$= (-)\ 0.05 \cdot \Sigma M_1$ $= (-)\ 0.05 \times 31{,}79{,}400$ $\Sigma M_4 = (-)\ 1{,}58{,}970$
Horizontal hydrostatic pressure	P		(−) $\frac{1}{2} \times 10 \times 86 \times 86 \times 1$ $= (-)\ 36{,}980$	28.67	(−) 10,60,090
	P'		(+) $\frac{1}{2} \times 10 \times 6 \times 6 \times 1 = (+)\ 180$	2.0	(−) 360
			$\Sigma H_1 = (-)\ 36{,}800$		$\Sigma M_5 = (-)\ 10{,}59{,}730$
Horizontal hydro-dynamic pressure	P_e		Calculated separately earlier : $= (-)\ 3{,}580$		$\Sigma M_e = (-)\ 1{,}26{,}500$ (calculated separately earlier)
			$\Sigma H_2 = (-)\ 3{,}580$		
Horizontal inertia forces due to earthquake	P_{w_1}		(−) $0.1\ W_1 = (-)\ 432$	20.0	(−) 8,640
	P_{W_2}		(−) $0.1\ W_2 = (-)\ 1{,}511$	45.0	(−) 68,000
	P_{W_3}		(−) $0.1\ W_3 = (-)\ 5{,}370$	26.67	(−) 1,43,200
			$\Sigma H_3 = (-)\ 7{,}313$		$\Sigma M_7 = (-)\ 2{,}19{,}840$

$$\Sigma H = \Sigma H_1 + \Sigma H_2 + \Sigma H_3 = (-)36{,}800 - 3580 - 7313 = (-)\ 47{,}693$$

$$= \frac{53929}{69}\left[1 \pm \frac{6 \times 18.32}{69}\right] = 782\,[1 \pm 1.595]$$

p_v (at toe) $= 782 \times 2.595 = \mathbf{2030\ kN/m^2}$; which is $< 3000\ \mathbf{kN/m^2}$ (∴ Safe)

p_v (at heel) $= -782 \times 0.405$

$= \mathbf{-316.7\ kN/m^2}$; which is $< 420\ kN/m^2$ (∴ Safe)

Since the tensile stress developed is less than the safe allowable value, the dam is safe even when examined with seismic forces, under reservoir full condition.

Principal stress at toe

$= \sigma = p_v \cdot \sec^2 \alpha - p' \tan^2 \alpha$ *i.e.* Eq. (19.17)

where $\tan \alpha = 0.7$, $p' = 60\ kN/m^2$; $p_v = 2030\ kN/m^2$

$$\sigma = 2030\,(1 + \tan^2 \alpha) - p' \tan^2 \alpha$$
$$= 2030\,(1 + 0.49) - 60 \times 0.49 = 2030 \times 1.49 - 29$$
$$= 3025 - 29 = \mathbf{2996\ kN/m^2} : \textbf{which is} < \mathbf{3000}$$ (just Safe)

Principal stress at heel is

$\sigma_1 = p_{v(heel)} \sec^2 \phi - (p + p_e) \tan^2 \phi$ *i.e.* Eq. (19.19)

where ϕ is the angle which the upstream face makes with the vertical

$\tan \phi = 0.1$

∴ $$\sigma_1 = -316.7\,[1 + (0.1)^2] - (860 + 57.5)\,(0.1)^2$$
$$= -316.7 \times 1.01 - 917.5 \times 0.01 = -319.9 - 9.2$$
$$= \mathbf{-329.1\ kN/m^2}\ ;\ \text{which is} < 420\ kN/m^2$$ (Hence, safe)

Shear stress at toe

$$\tau_{0(toe)} = (p_{v(toe)} - p') \tan \alpha = (2030 - 60)\,0.7$$
$$= 1970 \times 0.7 = \mathbf{1379\ kN/m^2}.$$

Shear Stress at heel

$$\tau_{0(heel)} = -[p_{v(heel)} - (p + p_e)] \tan \phi$$
$$= -[-329.1 - (860 + 57.5)]\,0.1$$
$$= -[-329.1 - 917.5]\,0.1 = +1246.6 \times 0.1 = \mathbf{124.7\ kN/m^2}$$

Factor of safety against overturning

$$= \frac{\Sigma M\,(+)}{\Sigma M\,(-)} = \frac{34{,}02{,}780}{24{,}12{,}540} = 1.41\ ;\ \text{which is} < 1.5$$ **(Hence, Unsafe)**

Factor of safety against sliding

$$= \frac{\mu \cdot \Sigma V}{\Sigma H}$$

where $\mu = 0.7$

$\Sigma V = 53{,}929$

$\Sigma H = \Sigma H_1 + \Sigma H_2 + \Sigma H_3$

$= -36800 - 3580 - 7313 = -47{,}693$ kN

Sliding factor $= \dfrac{0.7 \times 53929}{47693} = 0.79$, which is < 1 (Hence, Unsafe)

Shear friction factor

$$\text{S.F.F.} = \frac{\mu \cdot \Sigma V + B \cdot q}{\Sigma H}$$

$$= \frac{0.7 \times 53929 + 69 \times 1400}{47693}$$

$= 2.81$; which is less than 3 (Hence, slightly unsafe)

Case 2 (*b*). Reservoir full, without uplift

Sometimes, values of stresses at toe and heel are worked out when there is no uplift, s the vertical downward forces are maximum in this case. For this case, we shall alculate ΣM and ΣV by ignoring the corresponding values of ΣV_3 and ΣM_3 caused by plift.

$$\therefore \quad \Sigma M = \Sigma M_1 + \Sigma M_2 + \Sigma M_4 + \Sigma M_5 + \Sigma M_6 + \Sigma M_7$$

$$= 31{,}79{,}400 + 2{,}23{,}380 - 1{,}58{,}970 - 10{,}59{,}730 - 1{,}26{,}500 - 2{,}19{,}840$$

$$= 34{,}02{,}780 - 15{,}65{,}040 = 18{,}37{,}740$$

$$\Sigma V = \Sigma V_1 + \Sigma V_2 + \Sigma V_4 = 73130 + 3486 - 3657 = 72{,}959 \text{ kN}$$

$$\bar{x} = \frac{\Sigma M}{\Sigma V} = \frac{18{,}37{,}740}{72{,}959} = 25.19 \text{ m}$$

$$e = \frac{B}{2} - \bar{x} = 34.5 - 25.9 = 9.31 \text{ m} > \frac{B}{6} \quad i.e. \quad \frac{69}{6} = 11.5 \text{ m}$$

Resultant is nearer the toe and no tension is developed any where.

$$p_{max/min} = \frac{\Sigma V}{B}\left[1 \pm \frac{6e}{B}\right]$$

$$= \frac{72{,}959}{69}\left[1 \pm \frac{6 \times 9.31}{69}\right] = 1057\,[1 \pm 0.81]$$

p_v at toe $= 1057 \times 1.81 = \mathbf{1913\ kN/m^2} < 3000$ ($\therefore$ Safe)

p_v at heel $= 1057 \times 0.19 = \mathbf{201\ kN/m^2} < 3000$ ($\therefore$ Safe)

Principal stress at toe $= \sigma = p_v \cdot \sec^2 \alpha - p' \tan^2 \alpha$ ...(19.17)

$$p' = 60, \tan \alpha = 0.7$$

$$\therefore \quad \sigma = 1913\,(1 + 0.49) - 60 \times 0.49 = 1913 \times 1.49 - 29 = 2821 \text{ kN/m}^2 < 3000$$

(Hence, Unsafe)

Principal stress at heel

$$\sigma_1 = p_{v(heel)} \sec^2 \phi - (p + p_e) \tan^2 \phi \quad ...(19.19)$$

where $\tan \phi = 0.1$

$$\therefore \quad \sigma_1 = 201(1 + 0.01) - (860 + 57.5) \times 0.01$$

$$= 203 - 9 = 194 \text{ kN/m}^2 < 420 \quad \text{(Safe)}$$

Shear stress at toe

$$\tau_0 = (p_v - p') \tan \alpha \qquad i.e. \text{ Eq. (19.20)}$$

$$= (1913 - 60)\,0.7$$

$$= 1853 \times 0.7 = 1297 \text{ kN/m}^2 < 1400$$

Note. Shear friction factor, etc. are not worked out here as they were more critical in the 1st case, *i.e.* in 'Reservoir full with uplift' case.

Conclusion. The dam is unsafe only in sliding and S.F.F., for which shear key etc. can be provided.

Example 19.3. *Examine the stability of the dam section given in the previous example, if there are no seismic forces acting on the dam. Also state the magnitude of maximum compressive stress and maximum shear stress that may develop under any conditions of loading in the dam and also state whether tension is developed anywhere or not.*

Solution. The figures calculated earlier in Table 19.2 (*a*) and (*b*) shall be used here.

Case I. When the reservoir is empty

$$\Sigma V = \Sigma V_1 \text{ from Table 10.2 } (a) = 73130$$

$$\Sigma M = \Sigma M_1 \text{ from Table 19.2 } (b) = 3179400$$

$$\therefore \quad \bar{x} = \frac{\Sigma M}{\Sigma V} = \frac{3179400}{73130} = 43.4 \text{ m}$$

$$e = \frac{B}{2} - \bar{x} = 34.5 - 43.4 = -8.9 \text{ m}$$

–ve sign means that the resultant is towards left side, *i.e.* nearer to the heel, and since $e < \frac{B}{6}$, no tension is develoned

$$p_{max/min} = \frac{\Sigma V}{B}\left[1 \pm \frac{6e}{B}\right]$$

$$= \frac{73130}{69}\left[1 \pm \frac{6 \times 8.9}{69}\right]$$

$$= 1060\,[1 \pm 0.774]$$

$$p_v \text{ at heel} = 1060(1 + 0.774) = 1060 \times 1.774 = \mathbf{1880\ kN/m^2}$$

$$p_v \text{ at toe} = 1060(1 - 0.774) = 1060 \times 0.226 = \mathbf{239\ kN/m^2}$$

Average vertical stress

$$= \frac{\Sigma V}{B} = \frac{73130}{69} = \mathbf{1060\ kN/m^2}$$

Principal stress at toe

$$\sigma = p_{v(toe)} \sec^2 \alpha$$

$$= 239(1 + 0.49) = 239 \times 1.49 = \mathbf{357\ kN/m^2}$$

Principal stress at heel,

$$\sigma = p_{v(heel)} \sec^2 \phi$$

where $\tan \phi = 0.1$

$$= 1880(1 + 0.01) = 1880 \times 1.01 = \mathbf{1896\ kN/m^2}$$

Shear stress at toe

$$\tau_0 = p_{v(toe)} \tan \alpha$$

$$= 239 \times 0.7 = 167.3 \text{ kN/m}^2$$

$$= 734\,(1 + 0.01) - 860 \times 0.01$$
$$= 742 - 9 = \mathbf{733\ kN/m^2}$$

Shear stress at toe

$$\tau_0 = [p_{v(toe)} - p'] \tan \alpha$$
$$= (1490 - 60)\ 0.7 = 1430 \times 0.7 = 1001\ \text{kN/m}^2$$

Shear stress at heel

$$= -\,[p_{v(heel)} - p] \tan \phi$$
$$= -\,[734 - 860] \times 0.7 = 126 \times 0.7 = \mathbf{88.2\ kN/m^2}$$

Conclusions. We find that the dam is safe throughout except that the S.F.F. is equal to 3.72, while generally it should be between 4 to 5. The dam thus remains slightly unsafe in S.F.F. even when the seismic forces are not considered.

The results of stability analysis are given below :

The maximum shear stress developed in dam	$= 1001\ \text{kN/m}^2$.	
Maximum compressive stress developed in dam	$= 2191\ \text{kN/m}^2$	
No tension is developed anywhere.		**Ans.**
Factor of safety against sliding	$= 1.10$	
S.F.F.	$= 3.72$	
Factor of safety against overturning	$= 1.78$	