UNIT-I

STEADY STRESSES AND VARIABLE STRESSES IN MACHINE MEMBERS

CHAPTER - 4

Bending Stress in Straight Beams

In engineering practice, the machine parts of structural members may be subjected to static or dynamic loads which cause bending stress in the sections besides other types of stresses such as tensile, compressive and shearing stresses.

Consider a straight beam subjected to a bending moment M as shown in Fig. 4.1 The following assumptions are usually made while deriving the bending formula.

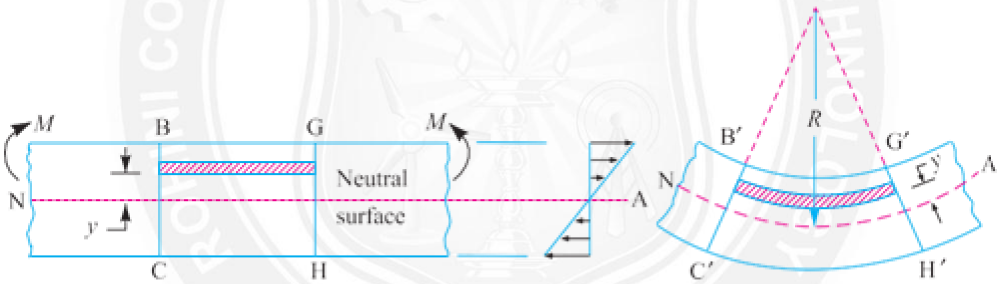


Fig 4.1 Bending stress in straight beams

[Source: "A Textbook of Machine Design by R.S. Khurmi J.K. Gupta, Page: 128]

1. The material of the beam is perfectly homogeneous (i.e. of the same material throughout) and isotropic (i.e. of equal elastic properties in all directions).

2. The material of the beam obeys Hooke's law.

3. The transverse sections (i.e. BC or GH) which were plane before bending, remain plane after bending also.

4. Each layer of the beam is free to expand or contract, independently, of the layer, above or below it.

5. The Young's modulus (E) is the same in tension and compression.

6. The loads are applied in the plane of bending.

A little consideration will show that when a beam is subjected to the bending moment, the fibres on the upper side of the beam will be shortened due to compression and those on the lower side will be elongated due to tension. It may be seen that somewhere between the top and bottom fibres there is a surface at which the fibres are neither shortened nor lengthened. Such a surface is called neutral surface. The intersection of the neutral surface with any normal cross-section of the beam is known as neutral axis. The stress distribution of a beam is shown in Fig. 4.1 The bending equation is given by

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

where M = Bending moment acting at the given section,

σ = Bending stress,

I = Moment of inertia of the cross-section about the neutral axis,

y = Distance from the neutral axis to the extreme fibre,

E = Young's modulus of the material of the beam, and

R = Radius of curvature of the beam.

From the above equation, the bending stress is given by

$$\sigma = y \times \frac{E}{R}$$

Since E and R are constant, therefore within elastic limit, the stress at any point is directly proportional to y, i.e. the distance of the point from the neutral axis.

Also from the above equation, the bending stress,

$$\sigma = \frac{M}{I} \times y = \frac{M}{I/y} = \frac{M}{Z}$$

The ratio I/y is known as section modulus and is denoted by Z.

Bending Stress in Curved Beams

We have seen in the previous article that for the straight beams, the neutral axis of the section coincides with its centroidal axis and the stress distribution in the beam is linear. But in case of curved beams, the neutral axis of the cross-section is shifted towards the centre of curvature of the beam causing a non-linear (hyperbolic) distribution of stress, as shown in Fig. 5.8. It may be noted that the neutral axis lies between the centroidal axis and the centre of curvature and always occurs within the curved beams. The application of curved beam principle is used in crane hooks, chain links and frames of punches, presses, planers etc.

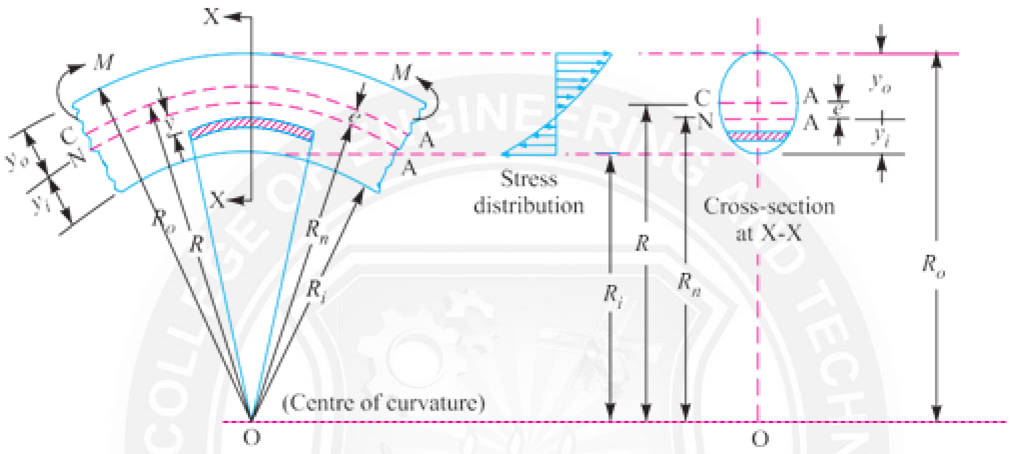


Fig 4.2 Bending stress in a curved beam.

[Source: "A Textbook of Machine Design by R.S. Khurmi J.K. Gupta, Page: 137]

Consider a curved beam subjected to a bending moment M, as shown in Fig. 4.2. In finding the bending stress in curved beams, the same assumptions are used as for straight beams. The general expression for the bending stress (σ_b) in a curved beam at any fibre at a distance y from the neutral axis, is given by

$$\sigma_b = \frac{M}{A.e}\left(\frac{y}{R_n - y}\right)$$

M = Bending moment acting at the given section about the centroidal axis,

A = Area of cross-section,

e = Distance from the centroidal axis to the neutral axis = R – R_n,

R = Radius of curvature of the centroidal axis,

R_n = Radius of curvature of the neutral axis, and

y = Distance from the neutral axis to the fibre under consideration.

It is positive for the distances towards the centre of curvature and negative for the distances away from the centre of curvature.

Notes:

1. The bending stress in the curved beam is zero at a point other than at the centroidal axis.

2. If the section is symmetrical such as a circle, rectangle, I-beam with equal flanges, then the maximum bending stress will always occur at the inside fibre.

3. If the section is unsymmetrical, then the maximum bending stress may occur at either the inside fibre or the outside fibre. The maximum bending stress at the inside fibre is given by

$$\sigma_{bi} = \frac{M.y_i}{A.e.R_i}$$

where y_i = Distance from the neutral axis to the inside fibre = Rn – Ri, and

R_i = Radius of curvature of the inside fibre.

The maximum bending stress at the outside fibre is given by

$$\sigma_{bo} = \frac{M.y_o}{A.e.R_o}$$

where y_o = Distance from the neutral axis to the outside fibre = Ro – Rn, and

R_o = Radius of curvature of the outside fibre.

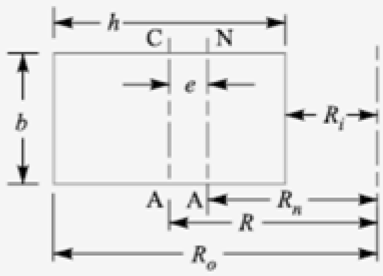
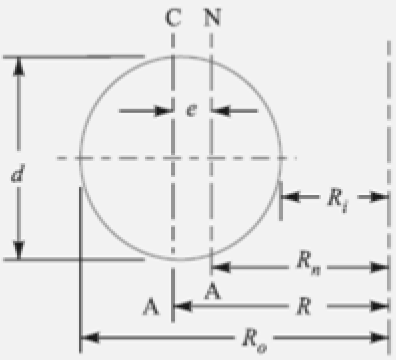
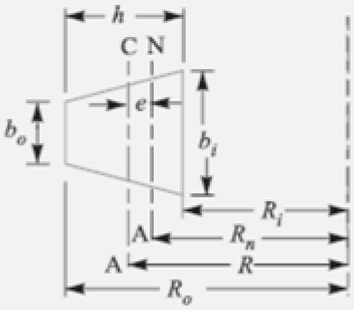
It may be noted that the bending stress at the inside fibre is tensile while the bending stress at the outside fibre is compressive.

4. If the section has an axial load in addition to bending, then the axial or direct stress (σ_d) must be added algebraically to the bending stress, in order to obtain the resultant stress on the section. In other words,

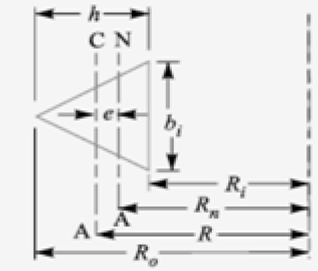
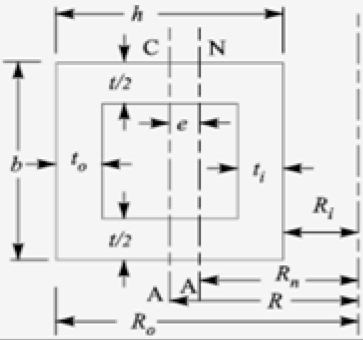
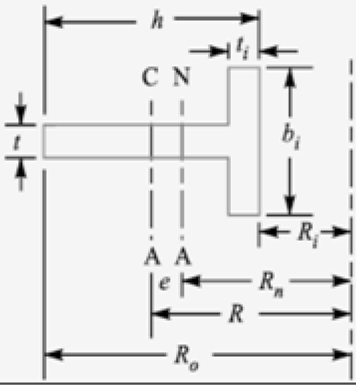
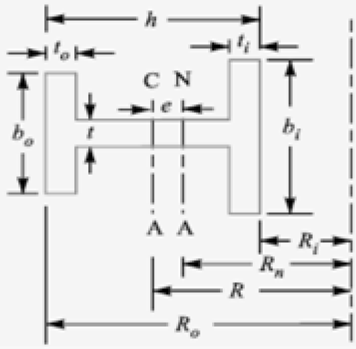
Resultant stress, $\sigma = \sigma_d \pm \sigma_b$

The following table shows the values of Rn and R for various commonly used cross-sections in curved beams.

Table 4.1. Values of Rn and R for various commonly used cross-section in curved beams.

Section	*Values of R_n and R*
	$R_n = \dfrac{h}{\log_e\left(\dfrac{R_o}{R_i}\right)}$ $R = R_i + \dfrac{h}{2}$
	$R_n = \dfrac{\left[\sqrt{R_o} + \sqrt{R_i}\right]^2}{4}$ $R = R_i + \dfrac{d}{2}$
	$R_n = \dfrac{\left(\dfrac{b_i + b_o}{2}\right)h}{\left(\dfrac{b_i R_o - b_o R_i}{h}\right)\log_e\left(\dfrac{R_o}{R_i}\right) - (b_i - b_o)}$ $R = R_i + \dfrac{h\,(b_i + 2b_o)}{3\,(b_i + b_o)}$

[Source: "A Textbook of Machine Design by R.S. Khurmi J.K. Gupta, Page: 138,139]

Section	Values of R_n and R
	$R_n = \dfrac{\frac{1}{2} b_i \times h}{\frac{b_i R_o}{h} \log_e\left(\frac{R_o}{R_i}\right) - b_i}$ $R = R_i + \dfrac{h}{3}$
	$R_n = \dfrac{(b-t)(t_i+t_o)+t.h}{b\left[\log_e\left(\frac{R_i+t_i}{R_i}\right)+\log_e\left(\frac{R_o}{R_o-t_o}\right)\right]+t.\log_e\left(\frac{R_o-t_o}{R_i+t_i}\right)}$ $R = R_i + \dfrac{\frac{1}{2}h^2.t+\frac{1}{2}t_i^2(b-t)+(b-t)\,t_o\,(h-\frac{1}{2}t_o)}{ht+(b-t)(t_i+t_o)}$
	$R_n = \dfrac{t_i(b_i-t)+t.h}{(b_i-t)\log_e\left(\frac{R_i+t_i}{R_i}\right)+t.\log_e\left(\frac{R_o}{R_i}\right)}$ $R = R_i + \dfrac{\frac{1}{2}h^2 t+\frac{1}{2}t_i^2(b_i-t)}{ht+t_i(b_i-t)}$
	$R_n = \dfrac{t_i(b_i-t)+t_o(b_o-t)+t.h}{b_i\log_e\left(\frac{R_i+t_i}{R_i}\right)+t\log_e\left(\frac{R_o-t_o}{R_i+t_i}\right)+b_o\log_e\left(\frac{R_o}{R_o-t_o}\right)}$ $R = R_i + \dfrac{\frac{1}{2}h^2 t+\frac{1}{2}t_i^2(b_i-t)+(b_o-t)\,t_o\,(h-\frac{1}{2}t_o)}{t_i(b_i-t)+t_o(b_o-t)+t.h}$

[Source: "A Textbook of Machine Design by R.S. Khurmi J.K. Gupta, Page: 139,140]

Problem 4.1

The crane hook carries a load of 20 kN as shown in Fig. 5.3. The section at X-X is rectangular whose horizontal side is 100 mm. Find the stresses in the inner and outer fibres at the given section.

Fig 4.3 Crane Hook

[Source: "A Textbook of Machine Design by R.S. Khurmi J.K. Gupta, Page: 142]

Given Data:

$W = 20\ kN = 20 \times 10^3\ N$

$R_i = 50\ mm$

$R_o = 150\ mm$

$h = 100\ mm$

$b = 20\ mm$

We know that area of section at X-X,

$$A = b.h = 20 \times 100 = 2000\ mm^2$$

The various distances are shown in Fig. 4.3.

We know that radius of curvature of the neutral axis,

$$R_n = \frac{h}{\log_e(\frac{R_o}{R_i})}$$

$$= \frac{100}{\log_e(\frac{150}{50})}$$

$$= \frac{100}{1.098} = 91.07\ mm$$

and radius of curvature of the centroidal axis,

$$R = R_i + \frac{h}{2}$$

$$= 50 + \frac{100}{2}$$

$$R = 100 \text{ mm}$$

∴ Distance between the centroidal axis and neutral axis,

$$e = R - R_n = 100 - 91.07 = 8.93 \text{ mm}$$

and distance between the load and the centroidal axis,

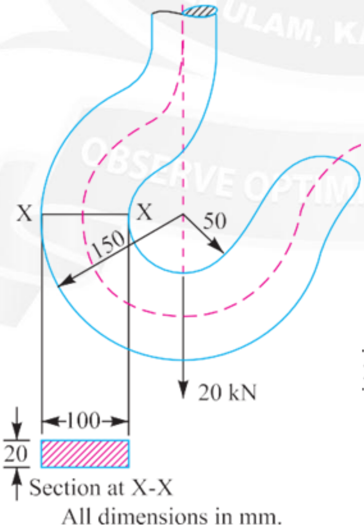
$$x = R = 100 \text{ mm}$$

∴ Bending moment about the centroidal axis,

$$M = W \times x = 20 \times 10^3 \times 100 = 2 \times 10^6 \text{ N-mm}$$

The section at X-X is subjected to a direct tensile load of $W = 20 \times 10^3$ N and a bending moment of $M = 2 \times 10^6$ N-mm. We know that direct tensile stress at section X-X,

$$\alpha_i = \frac{W}{A} = \frac{20\times10^3}{2000} = 10 \text{ N/mm}^2 = 10 \text{ MPa}$$



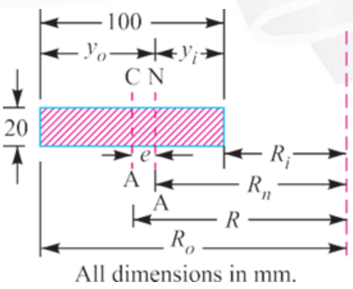


Fig 4.4 **Fig 4.5**

[Source: "A Textbook of Machine Design by R.S. Khurmi J.K. Gupta, Page: 143]

We know that the distance from the neutral axis to the inside fibre,

$$y_i = R_n - R_i = 91.07 - 50 = 41.07 \text{ mm}$$

and distance from the neutral axis to outside fibre,

$$y_o = R_o - R_n = 150 - 91.07 = 58.93 \text{ mm}$$

∴ Maximum bending stress at the inside fibre,

$$\sigma_{bi} = \frac{M.y_i}{A.e.R_i} = \frac{20\times10^3\times41.07}{2000\times8.93\times50}$$

$$= 92 \text{ N/mm}^2 = 92 \text{ MPa}$$

and maximum bending stress at the outside fibre,

$$\sigma_{bo} = \frac{M.y_o}{A.e.R_o} = \frac{20\times10^3\times58.93}{2000\times8.93\times150}$$

$$= 44 \text{ N/mm}^2$$

= 44 MPa (compressive)

∴ Resultant stress at the inside fibre

$= \sigma_t + \sigma_{bi} = 10 + 92$

= 102 MPa (tensile)

and resultant stress at the outside fibre

$= \sigma_t - \sigma_{bo} = 10 - 44$

= – 34 MPa

= 34 MPa (compressive)

Principal Stresses and Principal Planes

In the previous chapter, we have discussed about the direct tensile and compressive stress as well as simple shear. Also we have always referred the stress in a plane which is at right angles to the line of action of the force. But it has been observed that at any point in a strained material, there are three planes, mutual perpendicular to each other which carry direct stresses only and no shear stress. It may be noted that out of these three direct stresses, one will be maximum and the other will be minimum. These perpendicular planes which have no shear stress are known as principal planes and the direct stresses along these planes are known as principal stresses. The planes on which the maximum shear stress act are known as planes of maximum shear.

Determination of Principal Stresses for a Member Subjected to Bi-axial Stress

When a member is subjected to bi-axial stress (i.e. direct stress in two mutually perpendicular planes accompanied by a simple shear stress), then the normal and shear stresses are obtained as discussed below:

Consider a rectangular body ABCD of uniform cross-sectional area and unit thickness subjected to normal stresses σ_1 and σ_2 as shown in Fig. 4.6 (a). In addition to these normal stresses, a shear stress τ also acts. It has been shown in books on 'Strength of Materials' that the normal stress across any oblique section such as EF inclined at an angle θ with the direction of σ_2, as shown in Fig. 4.6 (a), is given by

$$\alpha_t = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 + \sigma_2}{2} \cos 2\theta + \tau \sin 2\theta$$

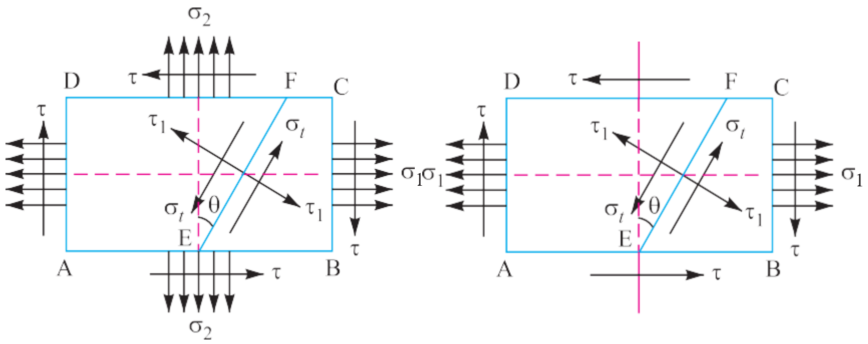
and tangential stress (i.e. shear stress) across the section EF,

$$\tau_1 = \frac{1}{2} (\sigma_1 - \sigma_2) \sin 2\theta - \tau \cos 2\theta$$

Since the planes of maximum and minimum normal stress (i.e. principal planes) have no shear stress, therefore the inclination of principal planes is obtained by equating τ1 = 0 in the above equation (ii), i.e.

$$\frac{1}{2} (\sigma_1 - \sigma_2) \sin 2\theta - \tau \cos 2\theta = 0$$

$$\tan 2\theta = \frac{2\tau}{\sigma_1 - \sigma_2}$$



(a) Direct stress in two mutually perpendicular planes accompanied by a simple shear stress

(b) Direct stress in one plane accompanied by a simple shear stress.

Fig 4.6 Principal stresses for a member subjected to bi-axial stress.

We know that there are two principal planes at right angles to each other. Let θ_1 and θ_2 be the inclinations of these planes with the normal cross-section.

From Fig. 4.6, we find that

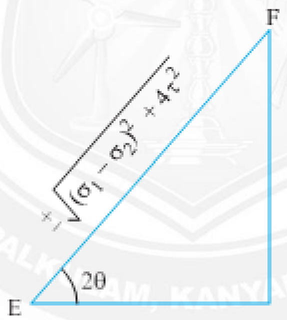


Fig 4.7

[Source: "A Textbook of Machine Design by R.S. Khurmi J.K. Gupta, Page: 146]

$$\sin 2\theta = \pm\frac{2\tau}{\sqrt{(\sigma_1-\sigma_2)^2+4\tau^2}}$$

$$\sin 2\theta_1 = +\frac{2\tau}{\sqrt{(\sigma_1-\sigma_2)^2+4\tau^2}}$$

$$\sin 2\theta_2 = -\frac{2\tau}{\sqrt{(\sigma_1-\sigma_2)^2+4\tau^2}}$$

$$\cos 2\theta = \pm\frac{(\sigma_1-\sigma_2)}{\sqrt{(\sigma_1-\sigma_2)^2+4\tau^2}}$$

$$\cos 2\theta_1 = +\frac{(\sigma_1-\sigma_2)}{\sqrt{(\sigma_1-\sigma_2)^2+4\tau^2}}$$

$$\cos 2\theta_2 = -\frac{(\sigma_1-\sigma_2)}{\sqrt{(\sigma_1-\sigma_2)^2+4\tau^2}}$$

The maximum and minimum principal stresses may now be obtained by substituting the values of sin 2θ and cos 2θ in equation (i).

∴ Maximum principal (or normal) stress,

$$\sigma_{t1} = \frac{\sigma_1+\sigma_2}{2} + \frac{1}{2}\sqrt{(\sigma_1-\sigma_2)^2+4\tau^2}$$

and minimum principal (or normal) stress,

$$\sigma_{t1} = \frac{\sigma_1+\sigma_2}{2} - \frac{1}{2}\sqrt{(\sigma_1-\sigma_2)^2+4\tau^2}$$

The planes of maximum shear stress are at right angles to each other and are inclined at 45° to the principal planes. The maximum shear stress is given by one-half the algebraic difference between the principal stresses, i.e.

$$\tau_{max} = \frac{\sigma_{t1}-\sigma_{t2}}{2} = \frac{1}{2}\sqrt{(\sigma_1-\sigma_2)^2+4\tau^2}$$

Problem 4.2

An overhang crank with pin and shaft is shown in Fig. 5.18. A tangential load of 15 kN acts on the crank pin. Determine the maximum principal stress and the maximum shear stress at the centre of the crankshaft bearing.

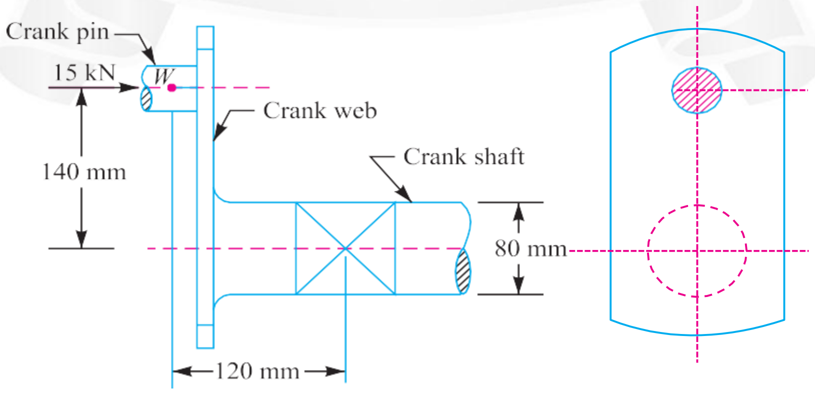


Fig 4.8

[Source: "A Textbook of Machine Design by R.S. Khurmi J.K. Gupta, Page: 151]

Given Data:

$W = 15 \text{ kN} = 15 \times 10^3 \text{ N}$

$d = 80 \text{ mm}$

$y = 140 \text{ mm}$

$x = 120 \text{ mm}$

Bending moment at the centre of the crankshaft bearing,

$M = W \times x = 15 \times 10^3 \times 120 = 1.8 \times 10^6 \text{ N-mm}$

and torque transmitted at the axis of the shaft,

$T = W \times y = 15 \times 10^3 \times 140 = 2.1 \times 10^6 \text{ N-mm}$

We know that bending stress due to the bending moment,

$$\sigma_b = \frac{M}{Z} = \frac{M \times 32}{\pi \times d^3}$$

$$= \frac{1.8 \times 10^3 \times 32}{\pi \times 80^3}$$

$$= 35.8 \text{ N/mm}^2$$

$$= 35.8 \text{ MPa}$$

and shear stress due to the torque transmitted,

$$\tau = \frac{16T}{\pi d^3}$$

$$= \frac{16 \times 2.1 \times 10^6}{\pi \times 80^3}$$

$$= 20.9 \text{ N/mm}^2$$

$$= 20.9 \text{ MPa}$$

Maximum principal stress

We know that maximum principal stress,

$$\sigma_{t(max)} = \frac{\sigma_t}{2} + \sqrt{(\sigma_t)^2 + 4\tau^2}$$

$= \frac{35.8}{2} + \sqrt{35.8)^2 + 4(20.9)^2}$

= 17.9 + 27.5

= 45.4 MPa

Maximum shear stress

We know that maximum shear stress,

$\tau_{max} = \frac{1}{2} [\sqrt{(\sigma_t)^2 + 4\tau^2}]$

$= \frac{1}{2} \sqrt{35.8)^2 + 4(20.9)^2}$

= 27.5 MPa

Factor of Safety

It is defined, in general, as the ratio of the maximum stress to the working stress.

Mathematically,

Factor of safety = Maximum stress/ Working or design stress

In case of ductile materials e.g. mild steel, where the yield point is clearly defined, the factor of safety is based upon the yield point stress. In such cases,

Factor of safety = Yield point stress/ Working or design stress

In case of brittle materials e.g. cast iron, the yield point is not well defined as for ductile materials. Therefore, the factor of safety for brittle materials is based on ultimate stress.

Factor of safety = Ultimate stress/ Working or design stress

This relation may also be used for ductile materials.

The above relations for factor of safety are for static loading.

Design for strength and rigidity:

Design for strength:

All the concepts discussed so far and the problems done are strength based, i.e., there will be some permissible stress or strength and our task is to limit the stresses below the given permissible value and accordingly sizing the machine element.

Design for rigidity or stiffness:

It the ability to resist deformations under the action of external load. Along with strength, rigidity is also very important operating property of many machine components. Ex: helical and leaf springs, elastic elements in various instruments, shafts, bearings, toothed and worm gears and so on.

In many cases, this parameter of operating capacity proves to be most important and to ensure it the dimensions of the part have to be increased to such an extent that the actual induced stresses become much lower that the allowable ones. Rigidity also necessary to ensure that the mated parts and the machine as a whole operate effectively. Forces subject the parts to elastic deformations: shafts are bent and twisted, bolts are stretched ect.,

1.When a shaft is deflected, its journals are misaligned in the bearings there by causing the uneven wear of the shells, heating and seizure in the sliding bearings.

2.Deflections and angles of turn of shafts at the places where gears are fitted cause non- uniform load distribution over the length of the teeth.

3.With the deflection of an insufficiently rigid shaft, the operating conditions or antifriction bearings sharply deteriorate if the bearings cannot self aligning.

4.Rigidity is particularly important for ensuring the adequate accuracy of items produced on machine tools.

Rigidity of machine elements is found with the help of formulae from the theory of strength of materials. The actual displacements like deflections, angles of turn, angles of twist should not be more that the allowable values. The most important design methods for increasing the rigidity of machine elements are as follows.

a) The decrease in the arms of bending and twisting forces.

b) The incorporation of additional supports.

c) The application of cross sections which effectively resist torsion (closed tubular) and bending (in which the cross section is removed as far as possible from the neutral axis).

d) The decrease of the length of the parts in tension and the increase of their cross section area.

From the above it's clear that the stiffness of a member depends not only on the shape and size of its cross section but also on elastic modulus of the material used.

Preferred Numbers

When a machine is to be made in several sizes with different powers or capacities, it is necessary to decide what capacities will cover a certain range efficiently with minimum number of sizes. It has been shown by experience that a certain range can be covered efficiently when it follows a geometrical progression with a constant ratio. The preferred numbers are the conventionally rounded off values derived from geometric series including the integral powers of 10 and having as common ratio of the following factors:

$$\sqrt[5]{10}, \sqrt[10]{10}, \sqrt[20]{10}, \sqrt[40]{10}$$

These ratios are approximately equal to 1.58, 1.26, 1.12 and 1.06. The series of preferred numbers are designated as *R5, R10, R20 and R40 respectively. These four series are called basic series. The other series called derived series may be obtained by simply multiplying or dividing the basic sizes by 10, 100, etc. The preferred numbers in the series R5 are 1, 1.6, 2.5, 4.0 and 6.3.