MATCHED FILTERS

- The matched filter is the optimal linear filter for maximizing the signal to noise ratio (SNR) in the presence of additive stochastic noise.

- Matched filters are commonly used in radar, in which a signal is sent out, and we measure the reflected signals, looking for something similar to what was sent out.

- Two-dimensional matched filters are commonly used in image processing, e.g., to improve SNR for X-ray pictures

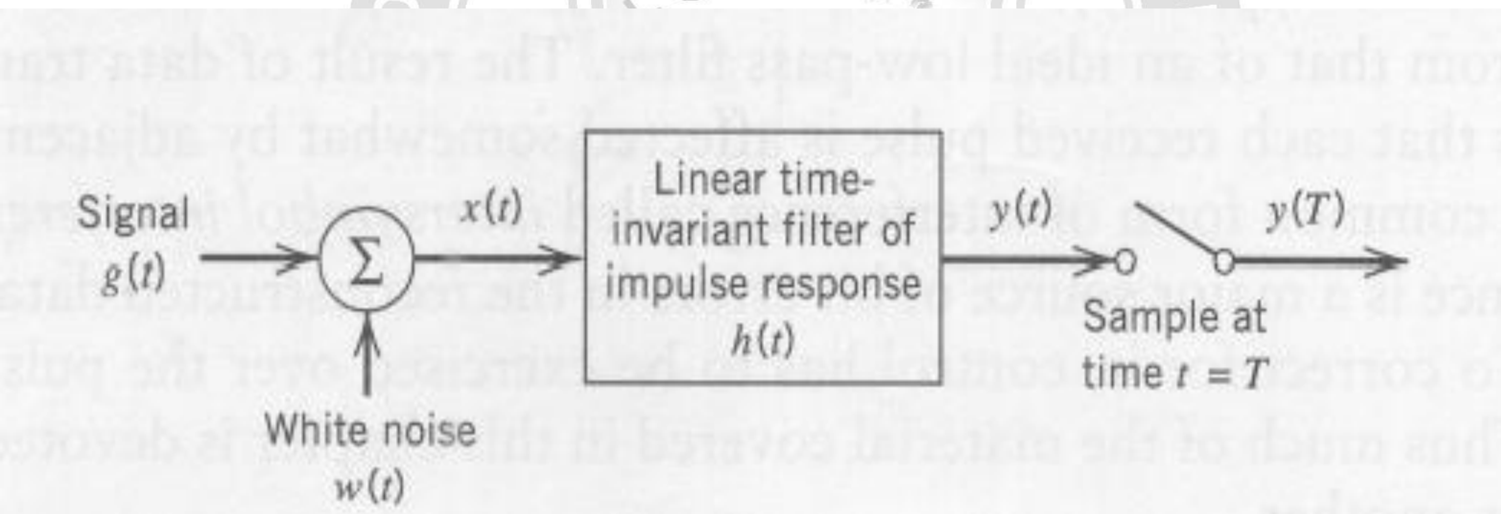

Fig:3.5.1 Matched filter

(Source: Tutorial Point)

- The filter input *x(t)* consists of a pulse signal *g(t)* corrupted by additive channel noise w(t), as shown by

$$x(t) = g(t) + w(t), \qquad 0 \le t \le T$$

where T is an arbitrary observation interval. The pulse signal *g(t)* may represent a binary symbol I or 0 in a digital communication system.

- The *w(t) is* the sample function of a white noise process of zero mean and power spectral density No/2.
- The source of uncertainty lies in the noise w(t).
- The function of the receiver is to detect the pulse signal *g(t)* in an optimum manner, given the received signal x(t).
- To satisfy this requirement, we have to optimize the design of the filter so as to minimize the effects of noise at the filter output in some statistical sense, and thereby enhance the detection of the pulse signal g(t).
- Since the filter is linear, the resulting output *y(t)* may be expressed as
- where $g_o(t)$ and *n(t)* are produced by the signal and noise components of the input *x(t),* respectively.

A simple way of describing the requirement that the output signal component $g_o(t)$ be considerably greater than the output noise component *n(t) is* to have the filter make the instantaneous power in the output signal $g_o(t)$, measured at time ***t* =** *T,* as large as possible compared with the average power of the output noise n(t). This is equivalent to maximizing the *peak pulse signal-to-noise ratio,* defined as

White Noise

For the case of white noise, the description of the matched filter simplified as follows: For white noise,

$$y(t) = g_o(t) + n(t)$$

= No / 2.

Thus equation becomes,

$$\eta = \frac{|g_o(T)|^2}{E[n^2(t)]}$$

It is obtained by correlating a known delayed signal, or *template*, with an unknown signal to detect the presence of the template in the unknown signal. This is equivalent to convolving the unknown signal with a conjugated time-reversed version of the template. The matched filter is the optimal linear filter for maximizing the signal-to-noise ratio (SNR) in the presence of additive stochastic noise.

Matched filters are commonly used in radar, in which a known signal is sent out, and the reflected signal is examined for common elements of the out-going signal. Pulse compression is an example of matched filtering. It is so called because the impulse response is matched to input pulse signals. Two-dimensional matched filters are commonly used in image processing, e.g., to

Improve the SNR of X-ray observations. Matched filtering is a demodulation technique with LTI (linear time invariant) filters to maximize SNR. It was originally also known as a *North filter*.

CORRELATION RECIEVER

A binary communication system uses the following equally-likely signals, Above we said that

$$x_k = \int_{t=-\infty}^{\infty} r(t)\phi_k(t)dt$$

$$x_k = \int_{t=0}^{T} r(t)\phi_k(t)dt$$

$$x_k = \int_{t=0}^{T} r(t)h_k(T-t)dt$$

where hk(t) = φk(T − t). (Plug in (T − t) in this formula and the Ts cancel out and only positive t is left.) This is the output of a convolution, taken at time T,

Notes:

- The xk can be seen as the output of a 'matched' filter at time T.
- This works at time T. The output for other times will be different in the correlation and matched filter.
- These are just two different physical implementations.

We might, for example, have a physical filter with the impulse response φk(T − t) and thus it is easy to do a matched filter implementation. Our signal set can be represented by the orthonormal basis functions,

{φk(t)} K−1 k=0 .

Correlation with the basis functions gives

xk = hr(t), φk(t)i = ai,k + nk

or $k = 0 \ldots K - 1$.

We denote vectors: $x = [x_0, x_1, \ldots, x_{K-1}]^T$

$a_i = [a_{i,0}, a_{i,1}, \ldots, a_{i,K-1}]^T$

$n = [n_0, n_1, \ldots, n_{K-1}]^T$

Hopefully, x and ai should be close since i was actually sent. In an example Matlab simulation, ece5520 lec07.m, we simulated sending ai = [1, 1]T and receiving r(t) (and thus x) in noise. In general, the pdf of x is multivariate Gaussian with each component xk independent, because: • xk = ai,k + nk • ai,k is a deterministic constant • The {nk} are i.i.d. Gaussian. The joint pdf of x is fX(x) = K Y−1 k=0 fXk (xk) = 1 [2π(N0/2)]N/2 e − PN k=1(xk−ai,k) 2 2(N0/2) (16) We showed that there are two ways to implement this: the correlation receiver, and the matched filter receiver. Both result in the same output x.