4.3. ANALOG FILTER DESIGN – BUTTERWORTH FILTER

The popular methods of designing IIR digital filter involve the design of equivalent analog filter and then converting the analog filter to digital filter. Hence to design a Butterworth IIR digital filter, first an analog Butterworth filter transfer function is determined using the given specifications. Then the analog filter transfer function is converted to a digital filter transfer function by using either impulse invariant transformation or bilinear transformation.

The filter passes all frequencies below Ωc. This is called pass band of the filter. Also the filter blocks all the frequencies above Ωc. This is called stop band of the filter. Ωc is called cutoff frequency or critical frequency.

No Practical filters can provide the ideal characteristic. Hence approximations of the ideal characteristic are used. Such approximations are standard and used for filter design. Butterworth filters are defined by the property that the magnitude response is maximally flat in the pass band.

Let ω_p = pass band edge digital frequency in rad/sample.

ω_s = Stop band edge digital frequency in rad/sample

$T = \frac{1}{F_s}$ = Sampling time in sec

Where F_s = Sampling frequency in Hz.

A_p = Gain at a pass band frequency ω_p

A_s = Gain at a pass band frequency ω_s

1. Choose either bilinear or impulse invariant transformation, and determine the specifications of analog filter. The gain or attenuation of analog filter is same as digital filter. The band edge frequencies are calculated using the following equations.

Let Ω_p = Pass band edge analog frequency corresponding to ω_p

Ω_s = Stop band edge analog frequency corresponding to ω_s

For bilinear transformation

$$\Omega p = \frac{2}{T} \tan \frac{\omega_p}{2}$$

$$\Omega s = \frac{2}{T} \tan \frac{\omega_s}{2}$$

For impulse invariant transformation

$$\Omega p = \frac{\omega_p}{T}$$

$$\Omega s = \frac{\omega_s}{T}$$

2. Decide the order N of the filter. In order to estimate the order N, calculate the parameter N_1 using the following equation

$$N_1 = \frac{1}{2}\frac{\log\left[\frac{(1/A_s^2)-1}{(1/A_p^2)-1}\right]}{\log\frac{\Omega s}{\Omega p}}$$

Choose N such that $\geq N_1$.Usually N is chosen as nearest integer just greater than N_1.

3. Determine the normalized transfer function, $H(s_n)$ of the analog low pass filter.

When N is even,

$$H(s_n) = \prod_{k=1}^{\frac{N}{2}} \frac{1}{s_n^2+b_k s_n+1}$$

When N is odd

$$H(s_n) = \frac{1}{s_n+1} \prod_{k=1}^{\frac{N}{2}} \frac{1}{s_n^2+b_k s_n+1}$$

Where $b_k = 2\sin\left[\frac{(2k-1)\pi}{2N}\right]$

4. Calculate the analog cutoff frequency, Ω c.

Cutoff frequency, $\Omega c. = \frac{\Omega s}{\left[\left(\frac{1}{A_s{}^2}\right)-1\right]^{\frac{1}{2N}}}$

5. Determine the un normalized analog transfer function H(s) of the low pass filter.

$$H(s) = H(s_n)|_{sn = s/\ \Omega c}$$

When the order N is even, H(s) is obtained by letting $s_n \rightarrow {}_{s/}\ \Omega c$

$H(s) = \prod_{k=1}^{N/2} \frac{1}{s_n^2+ b_k s_n+1} |_{\ sn = s/\ \Omega c}$

$= \prod_{k=1}^{N/2} \frac{\Omega_c^2}{s^2+ b_k \Omega_c s+\Omega_c^2}$

When the order N is odd, H(s) is obtained by letting $_{sn = s/}\ \Omega c$

In above equation

$H(s) = \frac{1}{s_n+1}\prod_{k=1}^{N-1/2} \frac{1}{s_n^2+ b_k s_n+1} |_{\ sn = s/\ \Omega c}$

$= \frac{\Omega c}{s+\Omega c}\prod_{k=1}^{N-1/2} \frac{1}{s^2+s\ b_k\Omega c+\Omega c^{\,2}}$

6.Determine the transfer function of digital filter, H(z).Using the chosen transformation in step-1 transform H(s) into H(Z).When impulse invariant transformation is employed ,if T<1,then multiply H(Z) by T to normalize the magnitude.

7. Realize the digital filter transfer function H(z) by a suitable structure.

8. Verify the design by sketching the frequency response $H(e^{j\omega})$

$$H\,(e^{j\omega}) = H(z)\,|_{z=}\,e^{j\omega}$$

The basic filter design is low pass filter design. The high pass, band pass or band stop filters are obtained from low pass filter design by frequency transformation.