

CE short circuit current gain using hybrid- π model:

Figure 4.6.1 shows the Hybrid- π model for a single transistor with a resistive load R_L .

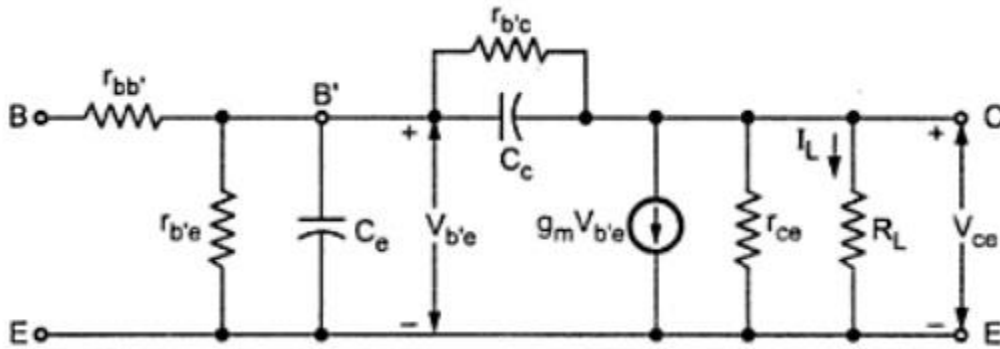


Figure 4.6.1 Hybrid- π model for a single transistor with a resistive load R_L

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Miller capacitance is $C_M = C_{b'c} (1 + g_m R_L)$

Here, $R_L = 0$

$$\therefore C_M = C_{b'c} (C_e)$$

Parallel combination of $r_{b'e}$, and $(C_e + C_c)$ is given as

$$\begin{aligned} Z &= \frac{r_{b'c} \times \frac{1}{j\omega(C_e + C_c)}}{r_{b'e} + \frac{1}{j\omega(C_e + C_c)}} \\ &= \frac{r_{b'e}}{1 + j\omega r_{b'e} (C_e + C_c)} \end{aligned}$$

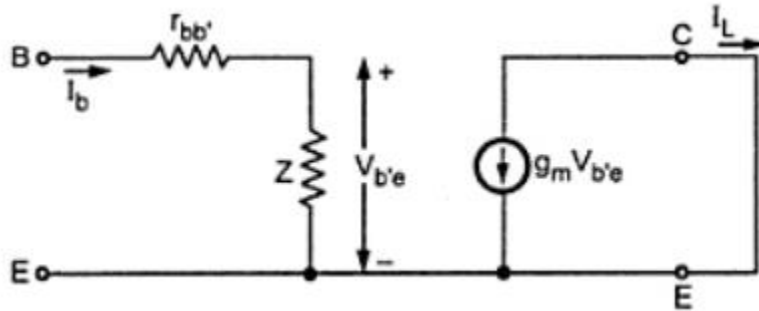


Figure 4.6.2 Simplified Hybrid pi model

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$$V_{b'e} = I_b Z$$

$$Z = V_{b'e} / I_b$$

The current gain for the circuit figure 4.6.2 is,

$$A_i = \frac{I_L}{I_b} = \frac{-g_m V_{b'e}}{I_b} \quad \because I_L = -g_m V_{b'e}$$

$$A_i = -g_m Z$$

$$= \frac{-g_m r_{b'e}}{1 + j\omega r_{b'e} (C_e + C_c)}$$

$$A_i = \frac{-h_{fe}}{1 + j\omega r_{b'e} (C_e + C_c)}$$

Figure 4.6.3 shows the Frequency Vs Current Gain

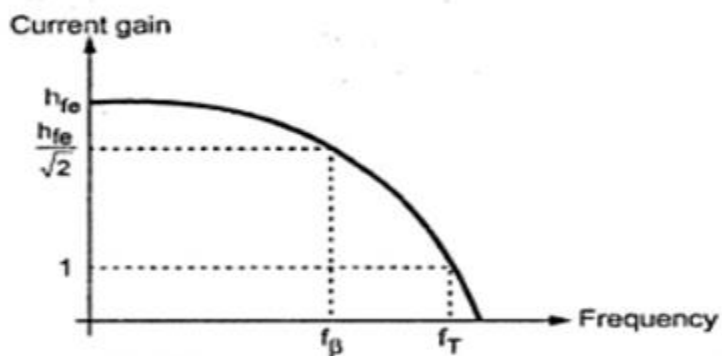


Figure 4.6.3 Frequency Vs Current Gain

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$$f_{\beta} = \frac{1}{2\pi r_{b'e} (C_e + C_c)}$$

$$A_i = \frac{-h_{fe}}{1 + j\frac{f}{f_{\beta}}}$$

$$|A_i| = \frac{h_{fe}}{\sqrt{1 + \left(\frac{f}{f_{\beta}}\right)^2}}$$

 f_{β} (Cutoff frequency):

It is the frequency at which the transistor short circuit CE current gain drops by 3dB or $1/\sqrt{2}$ times from its value at low frequency. It is given as,

$$f_{\beta} = \frac{1}{2\pi r_{b'e} (C_e + C_c)}$$

or

$$= \frac{g_{b'e}}{2\pi (C_e + C_c)}$$

or

$$= \frac{1}{h_{fe}} \frac{g_m}{2\pi (C_e + C_c)} \quad \because g_{b'e} = \frac{1}{r_{b'e}} = \frac{g_m}{h_{fe}}$$

 f_{α} (Cut-off frequency):

It is the frequency at which the transistor short circuit CB current gain drops by 3dB or $1/\sqrt{2}$ times from its value at low frequency.

The current gain for CB configuration is given as,

$$A_i = \frac{-h_{fb}}{1 + j \frac{f}{f_\alpha}}$$

where

$$f_\alpha = \frac{1}{2\pi r_{b'e} (1 + h_{fb}) C_e}$$

$$= \frac{1 + h_{fe}}{2\pi r_{b'e} C_e} \approx \frac{h_{fe}}{2\pi r_{b'e} C_e}$$

$$|A_i| = \frac{h_{fb}}{\sqrt{1 + \left(\frac{f}{f_\alpha}\right)^2}}$$

At

$$f = f_\alpha$$

$$|A_i| = \frac{h_{fb}}{\sqrt{2}}$$

Parameter f_T :

It is the frequency at which short circuit CE current gain becomes unity.
at $f = f_T$,

$$1 = \frac{h_{fe}}{\sqrt{1 + \left(\frac{f_T}{f_\beta}\right)^2}}$$

The ratio of f_T / f_β is quite large compared to 1.

$$f_T = g_m / 2\pi C_e$$

Current gain with resistive load:

$$C_{eq} = C_e + C_c (1 + g_m R_L)$$

For further simplification in figure 4.6.4,

At output circuit value of C_c can be calculated as,

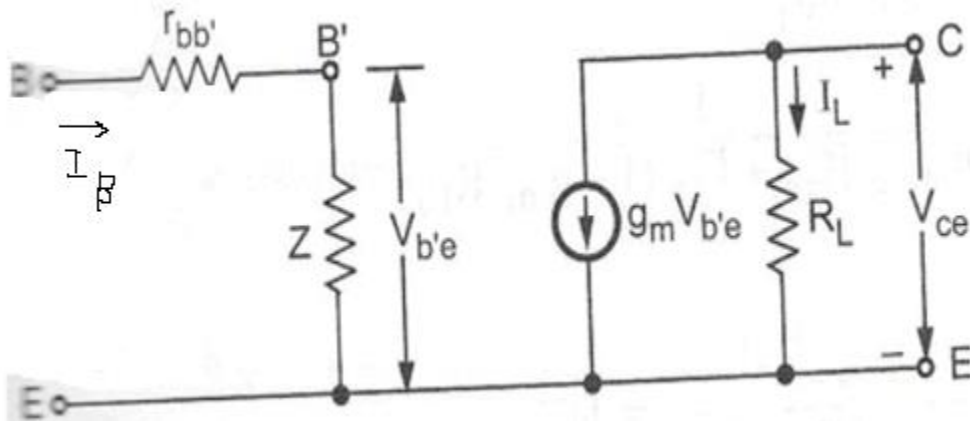


Figure 4.6.4. Simplified hybrid – π model for CE with R_L

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$$\frac{1}{\frac{j\omega C_c}{k-1}} \approx \frac{1}{j\omega C_c}$$

$$C_c \left(\frac{k}{k-1} \right) \approx C_c$$

$$Z = \frac{V_{b'e}}{I_b}$$

$$A_i = \frac{-h_{fe}}{1 + j\left(\frac{f}{f_H}\right)}$$

$$|A_i| = \frac{h_{fe}}{\sqrt{1 + \left(\frac{f}{f_H}\right)^2}}$$

At $f = f_H$
 $A_i = \frac{h_{fe}}{\sqrt{2}}$

f_H is the frequency at which the transistor gain drops by 3dB or $1/\sqrt{2}$ times from its value at low frequency in figure 4.6.5. It is given as

$$f_H = \frac{1}{2\pi r_{b'e} C_{eq}}$$

$$= \frac{1}{2\pi r_{b'e} [C_e + C_c (1 + g_m R_L)]}$$

At $R_L = 0$

$$f_H = \frac{1}{2\pi r_{b'e} [C_e + C_c]} = f_\beta$$

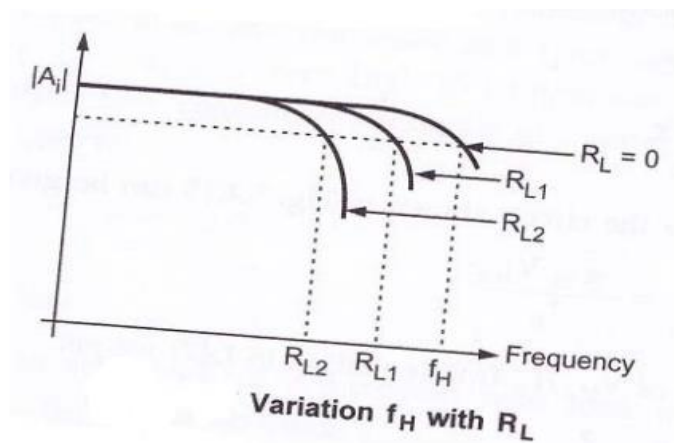


Figure 4.6.5. Variation f_H with R_L

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Current gain including source resistance:

Figure 4.6.6. shows hybrid pi Equivalent circuit with current source

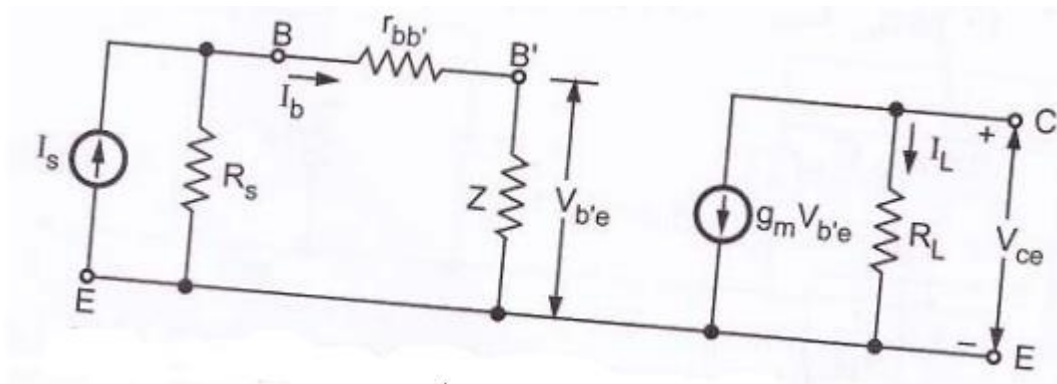


Figure 4.6.6. Equivalent circuit with current source

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$$\frac{I_L}{I_s} = \frac{-g_m r_{b'e} R_s}{R_s + r_{bb'} + r_{b'e}}$$

$$\begin{aligned} A_{is} \text{ at low frequency} &= \\ &= \frac{-h_{fe} R_s}{R_s + h_{ie}} \end{aligned}$$

Voltage gain including source resistance:

$$\begin{aligned} A_{vs} &= \frac{V_o}{V_s} = \frac{I_L}{I_s} \frac{R_L}{R_s} = \frac{-g_m Z R_s}{R_s + r_{bb'} + Z} \times \frac{R_L}{R_s} \\ &= \frac{-g_m Z R_L}{R_s + r_{bb'} + Z} \end{aligned}$$

$$\begin{aligned} A_{vs \text{ low}} &= \frac{I_L}{I_s} \frac{R_L}{R_s} = \frac{-h_{fe} R_s}{R_s + h_{ie}} \times \frac{R_L}{R_s} \\ &= \frac{-h_{fe} R_L}{R_s + h_{ie}} \end{aligned}$$

Cutoff frequency including source resistance:

$$A_{is \text{ high}} = \frac{A_{is}}{1 + j \left(\frac{f}{f_H} \right)}$$

$$A_{vs \text{ high}} = \frac{A_{vs}}{1 + j \left(\frac{f}{f_H} \right)}$$

where, $f_H = \frac{1}{2\pi R_{eq} C_{eq}}$

where, $R_{eq} = r_{b'e} \parallel (r_{bb'} + R_s)$

and $C_{eq} = C_e + C_c [1 + g_m R_L]$

For $R_L = 0$,

$$\begin{aligned} f_H &= \frac{1}{2\pi R(C_e + C_c)} \\ &= \frac{f_T}{g_m R} \quad \because f_T = \frac{g_m}{2\pi(C_e + C_c)} \\ &= \frac{h_{fe} f_\beta}{g_m R} \quad \because f_T = h_{fe} f_\beta \\ &= \frac{f_\beta}{g_{b'e} R} \quad \because g_{b'e} = \frac{g_m}{h_{fe}} \end{aligned}$$

Gain Bandwidth Product:**i. Gain Bandwidth Product for Voltage:**

$$|A_{vs \text{ low } f_H}| = |A_{vso} f_H| = \frac{-h_{fe} R_L}{R_s + h_{ie}} \times \frac{1}{2\pi R_{eq} C_{eq}}$$

$$= \frac{R_L}{R_s + r_{bb'}} * \frac{f_T}{1 + 2\pi f_T C_C R_L}$$

ii. Gain Bandwidth Product for current:

$$|A_{iso} \times f_H| = \frac{g_m R_s}{2\pi C(R_s + r_{bb'})}$$

$$= \frac{f_T}{1 + 2\pi f_T C_c R_L} \cdot \frac{R_s}{R_s + r_{bb'}}$$

