Introduction:

- A digital filter is just a filter that operates on the digital signals.

Types:

- FIR filter design
- IIR filter design

FIR filter:

The digital filter which designed using finite number of response co-efficient is called as finite impulse response filters. $h_o(n), h_1(n)....................h_{(N-1)}(n)$

Advantages:

1. FIR filters have exact linear phase.
2. FIR filters are always stable.
3. FIR filters can be realized in both recursive and non-recursive structure.
4. FIR filters with any arbitrary magnitude response can be tackled using FIR sequence.

Disadvantages:

1. For the same filter specification the order of the FIR filter design can be as high as 5 to 10 times that of an IIR filter.
2. Large storage requirement needed.
3. Powerful computational facilities required for the implementation.

Linear Phase (LP) FIR Filters:

Derive the condition for Linear Phase (LP) FIR Filters. [Nov/Dec-2009]

The transfer function of a FIR causal filter is given by

$$H(\text{z}) = \sum_{n=0}^{N-1} h(n)z^{-n}$$

Where *h(n)* is the impulse response of the filter.

The Fourier transform of h(n) is

$$H(\text{e}^{j\omega}) = \sum_{n=0}^{N-1} h(n)e^{-j\omega n},$$

Which is periodic in frequency with period 2π.

$$H(\text{e}^{j\omega}) = \pm|H(\text{e}^{j\omega})|e^{-j\theta(\omega)}$$

Where $H(\text{e}^{j\omega})$ is magnitude response and $\theta(\omega)$ is phase response.

We define the phase delay and group delay of a filter as

$$\tau_p = \frac{-\theta(\omega)}{\omega} and; \tau_g = \frac{-d\theta(\omega)}{d\omega} \text{ --------------------------------------->(1)}$$

For FIR filters with linear phase we can define

$$\theta(\omega) = -\alpha\omega; -\pi \le \omega \le \pi \text{ -->(2)}$$

Where α is a constant phase delay in samples.

Substitute: equation 2 in 1, we have $\tau_p = \tau_g = \alpha$, which means that α is independent of frequency. We can write,

$$\sum_{n=0}^{N-1} h(n)e^{-j\omega n} = \pm|H(\text{e}^{j\omega})|e^{-j\theta(\omega)}$$

Which gives us,

$$\sum_{n=0}^{N-1} h(n)\cos\omega n = \pm|H(\text{e}^{j\omega})|\cos\theta(\omega) \text{ ------------------------------>(3)}$$

and $$-\sum_{n=0}^{N-1} h(n)\sin\omega n = \pm|H(e^{j\omega})|\sin\theta(\omega) \text{ ---------------------------------->(4)}$$

By taking ratio of equation (3) to equation (4), we obtain

$$\frac{\sum_{n=0}^{N-1} h(n)\sin\omega n}{\sum_{n=0}^{N-1} h(n)\cos\omega n} = \frac{\sin\alpha\omega}{\cos\alpha\omega};[\theta(\omega) = -\alpha\omega] \text{ ------------------------------>(5)}$$

After simplifying equation (5) we have

$$\sum_{n=0}^{N-1} h(n)\sin(\alpha - n)\omega = 0 \text{ --->(6)}$$

Equation (6) will be zero when

$$h(n) = h(N-1-n) \text{ -->(7)}$$

And $$\alpha = \frac{N-1}{2} \text{ --->(8)}$$

Therefore, FIR filters will have constant phase and group delays when the impulse response is symmetrical about $\alpha = \frac{N-1}{2}$

The impulse response satisfying equation (7) & (8) for odd and even values of N. When N=7 the centre of symmetry of the sequence occurs at third sample and when N=6, the filter delay is $2\frac{1}{2}$ samples.

If only constant group delay is required, and not the phase delay we can write

$$\theta(\omega) = \beta - \alpha\omega$$

Now we have $H(e^{j\omega}) = \pm|H(e^{j\omega})|e^{j(\beta-\alpha\omega)}$

Equation (9) can be expressed as

$$\sum_{n=0}^{N-1} h(n)e^{-j\omega n} = \pm|H(e^{j\omega})|e^{j(\beta-\alpha\omega)} \text{ --------------------------------------->(9)}$$

which gives us

$$\sum_{n=0}^{N-1} h(n)\cos\omega n = \pm|H(e^{j\omega})|\cos(\beta-\alpha\omega) \text{ ---------------------------->(10)}$$

and $$-\sum_{n=0}^{N-1} h(n)\sin\omega n = \pm|H(e^{j\omega})|\sin(\beta-\alpha\omega) \text{ ------------------------------>(11)}$$

By taking ratio of equation (11) to (10), we get

$$\frac{-\sum_{n=0}^{N-1} h(n)\sin\omega n}{\sum_{n=0}^{N-1} h(n)\cos\omega n} = \frac{\sin(\beta-\alpha\omega)}{\cos(\beta-\alpha\omega)}$$

From which we obtain

$$\sum_{n=0}^{N-1} h(n)\sin\left[\beta-(\alpha-n)\omega\right] = 0 \text{ -->(12)}$$

If $\beta = \frac{\pi}{2}$, Equation (12) becomes,

$$\sum_{n=0}^{N-1} h(n)\cos(\alpha-n)\omega = 0 \text{ -->(13)}$$

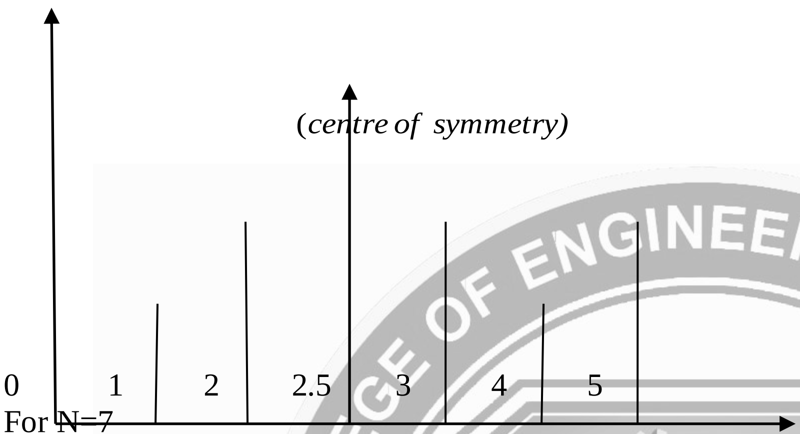
The equation 13 will be satisfied when $h(n) = h(N-1-n)$

And $\alpha = \frac{N-1}{2}$

Therefore, FIR filters have constant group delay, τ_g and not constant phase delay when the impulse response is anti-symmetrical about $\alpha = \frac{N-1}{2}$.

Example:

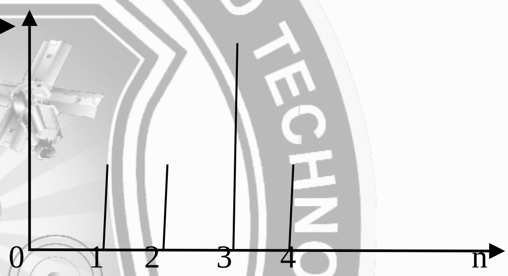
For N=6 $\alpha = \frac{N-1}{2} = \frac{6-1}{2} = 2\frac{1}{2}$ *(centre of symmetry)*



For N=7

For N=7 $\alpha = \frac{N-1}{2} = \frac{7-1}{2} = 3$ *(centre of symmetry)*

centre of symmetry



Linear Phase FIR Filter:

An FIR filter has linear phase if its unit sample response satisfies the condition

$$h(n) = \pm h(M-1-n); \qquad n = 0,1,2........N-1$$

<u>Case (i): Symmetric impulse response for "N is ODD":</u>

Determine the frequency response of FIR filter with symmetric impulse response and the order of the filter is "N is Odd".

The frequency response of impulse response can be written as,

$$H(e^{j\omega}) = \sum_{n=0}^{\frac{M-3}{2}} h(n)e^{-j\omega n} + h\left(\frac{M-1}{2}\right)e^{-\frac{j\omega(M-1)}{2}} + \sum_{n=\frac{N+1}{2}}^{M-1} h(n)e^{-j\omega n} \text{ ------------------------->(1)}$$

$$Let\ n = M-1-n, where\ z = e^{j\omega} \Big| H(e^{j\omega}) = H(z) \Big| z = e^{j\omega}$$

$$H(e^{j\omega}) = \sum_{n=0}^{\frac{M-3}{2}} h(n)e^{-j\omega n} + h\left(\frac{M-1}{2}\right)e^{-\frac{j\omega(M-1)}{2}} + \sum_{n=0}^{\frac{M-3}{2}} h(M-1-n)\, e^{-j\omega(M-1-n)} \text{ --------->(2)}$$

For a symmetrical impulse response, $h(n) = h(M-1-n)$, *substituting this relation in above equation (2)*

$$H(e^{j\omega}) = e^{-\frac{j\omega(M-1)}{2}} \left[\sum_{n=0}^{\frac{M-3}{2}} h(n)e^{j\omega\left(\frac{M-1}{2-n}\right)} + \sum_{n=0}^{\frac{M-1}{2}} h(n)\, e^{-\frac{j\omega(M-1)}{2-n}} + h\left(\frac{M-1}{2}\right) \right]$$

$$H(e^{j\omega}) = e^{-\frac{j\omega(M-1)}{2}} \left[\left[2\sum_{n=0}^{\frac{M-3}{2}} h(n)\cos\omega\left(\frac{M-1}{2}\right) - n \right] + h\left(\frac{M-1}{2}\right) \right]$$

The polar form of $H(e^{j\omega})$ *can be expressed as*

$$H(e^{j\omega}) = \left|H(e^{j\omega})\right| e^{j\angle H(e^{j\omega})}$$

$\therefore$ *Magnitude of* $H(e^{j\omega})$ *is given as* $\quad \left|H(e^{j\omega})\right| = h\left(\frac{M-1}{2}\right) + 2\sum_{n=0}^{\frac{M-3}{2}} h(n)\cos\omega\left(\frac{M-1}{2}-n\right)$

Angle of $H(e^{j\omega})$ *is given as* $\quad \angle H(e^{j\omega}) = \begin{cases} -\omega\left(\frac{M-1}{2}\right), \ for \left|H(e^{j\omega})\right| > 0 \\ -\omega\left(\frac{M-1}{2}\right) + \pi, \ for \left|H(e^{j\omega})\right| < 0 \end{cases}$

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Case (ii) : Symmetric Impulse Response For –"N is EVEN":

Determine the frequency response of FIR filter with symmetric impulse response and the order of the filter N is Even. [Nov/Dec-2013]

The frequency response of impulse response can be written as,

$$H\left(e^{j\omega}\right) = \sum_{n=0}^{\frac{M-2}{2}} h(n)e^{-j\omega n} + \sum_{n=0}^{\frac{M-2}{2}} h(n)e^{-j\omega n} \text{ --->(1)}$$

Let $n = M-1-n$, *where* $z = e^{j\omega}\left|H(e^{j\omega}) = H(z)\right|z = e^{j\omega}$

$$H\left(e^{j\omega}\right) = \sum_{n=0}^{\frac{M-2}{2}} h(n)e^{-j\omega n} + \sum_{n=0}^{\frac{M-2}{2}} h(M-1-n)\, e^{-j\omega(M-1-n)} \text{ ------------------------------->(2)}$$

For a symmetrical impulse response, $h(n) = h(M-1-n)$, *substituting this relation in above equation (2)*

$$H\left(e^{j\omega}\right) = \sum_{n=0}^{\frac{M-3}{2}} h(n)e^{-j\omega n} + h\left(\frac{M-1}{2}\right)e^{-\frac{j\omega(M-1)}{2}} + \sum_{n=0}^{\frac{M-1}{2}} h(n)\, e^{-j\omega(M-1-n)} \text{ ----------------->(3)}$$

$$H\left(e^{j\omega}\right) = e^{-\frac{j\omega(M-1)}{2}}\left[\sum_{n=0}^{\frac{M-2}{2}} h(n)e^{j\omega\left(\frac{M-1}{2}-n\right)} + \sum_{n=0}^{\frac{M-2}{2}} h(n)\, e^{-j\omega\left(\frac{M-1}{2}-n\right)}\right]$$

$$H\left(e^{j\omega}\right) = e^{-\frac{j\omega(M-1)}{2}}\left[2\sum_{n=0}^{\frac{M-2}{2}} h(n)\cos\omega\left(\frac{M-1}{2}-n\right)\right]$$

The polar form of $H(e^{j\omega})$ *can be expressed as* $H\left(e^{j\omega}\right) = \left|H\left(e^{j\omega}\right)\right|e^{j\angle H(e^{j\omega})}$

$\therefore$ *Magnitude* $\left|H(e^{j\omega})\right| = 2\sum_{n=0}^{\frac{M-2}{2}} h(n)\cos\omega\left(\frac{M-1}{2}-n\right)$

Angle of $H(e^{j\omega})$ *is given as* $\angle H(e^{j\omega}) = \begin{cases} -\omega\left(\frac{M-1}{2}\right), \ for \left|H(e^{j\omega})\right| > 0 \\ -\omega\left(\frac{M-1}{2}\right) + \pi, \ for \left|H(e^{j\omega})\right| < 0 \end{cases}$

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Case (iii) : Antisymmetric for "N is ODD":

Determine the frequency response of FIR filter with Antisymmetric impulse response and the order of the filter N is Odd.

For this type of sequence

$$h\left(\frac{M-1}{2}\right) = 0$$

$$H(e^{j\omega}) = \sum_{n=0}^{M-1} h(n)e^{-j\omega n}$$

The frequency response of impulse response can be written as,

$$H\left(e^{j\omega}\right) = \sum_{n=0}^{\frac{M-3}{2}} h(n)e^{-j\omega n} + \sum_{n=0}^{\frac{M-3}{2}} h(M-1-n)e^{-j\omega(M-1-n)}$$

$$\textit{for antisymmetric impulse response}, h(n) = -h(M-1-n)$$

$$H(e^{j\omega}) = \sum_{n=0}^{\frac{M-3}{2}} h(n)e^{-j\omega n} - \sum_{n=0}^{\frac{M-3}{2}} h(n)e^{-j\omega(M-1-n)}$$

$$= e^{-j\omega\left(\frac{M-1}{2}\right)}\left[\sum_{n=0}^{\frac{M-3}{2}} h(n)e^{j\omega\left(\frac{M-1}{2}-n\right)} - \sum_{n=0}^{\frac{M-3}{2}} h(n)e^{-j\omega\left(\frac{M-1}{2}-n\right)}\right]$$

$$= e^{-j\omega\left(\frac{M-1}{2}\right)} j\left[2\sum_{n=0}^{\frac{M-3}{2}} h(n)\sin\omega\left(\frac{M-1}{2}-n\right)\right]$$

$$= e^{-j\omega\left(\frac{M-1}{2}\right)} e^{j\frac{\pi}{2}}\left[2\sum_{n=0}^{\frac{M-3}{2}} h(n)\sin\omega\left(\frac{M-1}{2}-n\right)\right]$$

$$H\left(e^{j\omega}\right) = e^{j\left[\frac{\pi}{2}-\omega\left(\frac{M-1}{2}\right)\right]}\left[2\sum_{n=0}^{\frac{M-3}{2}} h(n)\sin\omega\left(\frac{M-1}{2}-n\right)\right]$$

$$\textit{Magnitude of } H(e^{j\omega}) \textit{ is given as,} \qquad \left|H(e^{j\omega})\right| = 2\sum_{n=0}^{\frac{M-3}{2}} h(n)\sin\omega\left(\frac{M-1}{2}-n\right)$$

$$\textit{Angle of } H(e^{j\omega}) \textit{ is given as,} \qquad \angle H(e^{j\omega}) = \begin{cases} \frac{\pi}{2} - \omega\left(\frac{M-1}{2}\right) & \textit{for } \left|H(e^{j\omega})\right| > 0 \\ \frac{3\pi}{2} - \omega\left(\frac{M-1}{2}\right) & \textit{for } \left|H(e^{j\omega})\right| < 0 \end{cases}$$

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Case (iv) : Antisymmetric For –"N is EVEN":

Determine the frequency response of FIR filter with Antisymmetric impulse response and the order of the filter N is Even. [Nov/Dec-2013]

The frequency response of impulse response can be written as,

$$H\left(e^{j\omega}\right) = \sum_{n=0}^{\frac{M-2}{2}} h(n)e^{-j\omega n} + \sum_{n=0}^{\frac{M-2}{2}} h(M-1-n)e^{-j\omega(M-1-n)}$$

$$\textit{for antisymmetric impulse response}, h(n) = -h(M-1-n)$$

$$H(e^{j\omega}) = \sum_{n=0}^{\frac{M-2}{2}} h(n)e^{-j\omega n} - \sum_{n=0}^{\frac{M-2}{2}} h(n)e^{-j\omega(M-1-n)}$$

$$= e^{-j\omega\left(\frac{M-1}{2}\right)} j\left[2\sum_{n=0}^{\frac{M-2}{2}} h(n)\sin\omega\left(\frac{M-1}{2}-n\right)\right]$$

$$H(e^{j\omega}) = e^{-j\omega\left(\frac{M-1}{2}\right)} e^{j\frac{\pi}{2}} \left[2\sum_{n=0}^{\frac{M-2}{2}} h(n) \sin \omega\left(\frac{M-1}{2} - n\right) \right]$$

$$H(e^{j\omega}) = e^{j\left[\frac{\pi}{2} - \omega\left(\frac{M-1}{2}\right)\right]} \left[2\sum_{n=0}^{\frac{M-2}{2}} h(n) \sin \omega\left(\frac{M-1}{2} - n\right) \right]$$

$$= e^{-j\omega\left(\frac{M-1}{2}\right)} j \left[2\sum_{n=0}^{\frac{M-2}{2}} h(n) \sin \omega\left(\frac{M-1}{2} - n\right) \right]$$

$$= e^{j\left[\frac{\pi}{2} - \omega\left(\frac{M-1}{2}\right)\right]} \left[2\sum_{n=0}^{\frac{M-2}{2}} h(n) \sin \omega\left(\frac{M-1}{2} - n\right) \right]$$

Magnitude of $H(e^{j\omega})$ *is given as,* $\left|H(e^{j\omega})\right| = 2\sum_{n=0}^{\frac{M-2}{2}} h(n) \sin \omega\left(\frac{M-1}{2} - n\right)$

Angle of $H(e^{j\omega})$ *is given as,* $\angle H(e^{j\omega}) = \begin{cases} \frac{\pi}{2} - \omega\left(\frac{M-1}{2}\right) & for \left|H(e^{j\omega})\right| > 0 \\ \frac{3\pi}{2} - \omega\left(\frac{M-1}{2}\right) & for \left|H(e^{j\omega})\right| < 0 \end{cases}$

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Structures of FIR Filters:

Explain with neat sketches the Structure of FIR filters. [Nov/Dec-2012]

The realization of FIR filter is given by

- Transversal structure.
- Linear phase realization
- Polyphase realization.

Transversal structure:

It contains two forms of realization such as,

- Direct form realization
- Cascade form realization.

Direct form realization:

The system function of an FIR filter can be written as

$$H(z) = \sum_{n=0}^{N-1} h(n)\, z^{-n}$$

$$= h(0) + h(1)\, z^{-1} + h(2)\, z^{-2} + ... + h(N-1)\, z^{-(N-1)} \qquad eq(1)$$

$$Y(z) = h(0)X(z) + h(1)z^{-1} X(z) + h(2)z^{-2} X(z) + ... + h(N-1)\, z^{-(N-1)}\, X(z) \qquad eq(2)$$

This structure is known as direct form realization. It requires N multipliers, N-1 adders, and N-1 delay elements.