4.5 HELICAL SPRING.

Helical springs are the thick spring wires coiled into a helix.

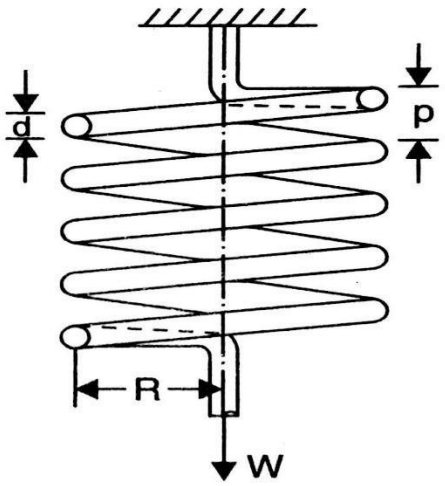
They are of two types :

1. Close-coiled helical springs and
2. Open-coiled helical springs.

Close-coiled helical springs. Close- coiled helical springs are the springs in which helix angle is very small or in other words the pitch between two adjacent turns is small. A closecoiled helical spring carrying an axial load is shown in Fig. As the helix angle in case of Close-coiled helical springs are small, hence the bending effect on the spring is ignored and we assume that the coils of a close-coiled helical springs are to stand purely torsional stresses.

Expression For Max. Shear Stress Induced In Wire.

The below shows a close-coiled helical springs subjected to an axial load.



Let d = Diameter of spring wire

P = Pitch of the helical spring

n = Number of coils

R = Mean radius of spring coil

W = Axial load on spring

C = Modulus of rigidity

τ = Max. shear stress induced in the wire

θ = Angle of twist in spring wire, and

δ = Deflection of spring due to axial load l

= Length of wire

Now twisting moment on the wire,

$$T = W \times R \qquad \text{.... (1)}$$

But twisting moment is also given by

$$T = \frac{\pi}{16}\tau d^3 \qquad \text{.... (2)}$$

Equating equations (1) and (2), we get

$$W \times R = \frac{\pi}{16}\tau d^3 \text{ or } \tau = \frac{16\,W \times R}{\pi d^3} \qquad \text{.... (3)}$$

Equation (3) gives the max. shear stress induced in the wire

Expression for deflection of spring

Now length of one coil $= \pi D$ or $2\pi R$

∴ Total Length of the wire = Length of one coil ×No. of coils or $l = 2\pi R \times n$.

As the every section of the wire is subjected to torsion, hence the strain energy stored by the spring due to torsion is given by equation(16.20). ∴ Strain energy stored by the spring,

$$U = \frac{\tau^2}{4C}.\text{ Volume}$$

$$= \left(\frac{16W\,.\,R}{\pi d^3}\right)^2 \times \frac{1}{4C} \times \left(\frac{\pi}{4}d^2 \times 2\pi R.n\right)$$

$$\left(\because \tau = \frac{16WR}{\pi d^3} \text{ and } Volume = \frac{\pi}{4d^2} \times Total\ Length\ of\ wire\right)$$

$$= \frac{32W^2R^2}{Cd^4}.\,R\,.\,n = \frac{32W^2R^3\,.\,n}{Cd^4} \qquad \text{.... (5)}$$

Work done on the spring = Average load × Deflection

$= \frac{1}{2}W \times \delta$

Equating the work done on spring to the energy stored, we get

$$\frac{1}{2}W\,.\delta = \frac{32W^2R^3\,.\,n}{Cd^4}$$

$$\therefore \delta = \frac{64WR^3\,.\,n}{Cd^4} \qquad \text{.... (6)}$$

Expression for stiffness of spring

The stiffness of spring, s = Load per unit deflection

$$= \frac{W}{\delta} = \frac{W}{\frac{64.WR^3 \,.\, n}{Cd^4}} = \frac{Cd^4}{64 \,.\, R^3 \,.\, n} \qquad \text{.... (7)}$$

Note. The solid length of the spring means the distance between the coils when the coils are touching each other. There is no gap between the coils. The solid length is given by Solid length = Number of coils × Dia. Of wire = n × d

Problem 4.5.1. A closely coiled helical spring is to carry a load of 500 N. Its means coil diameter is to be 10 times that of the wire diameter. Calculate these diameters if the maximum shear stress in the material of the spring is to be 80 N/mm^2 .

Given :

Load on spring, W = 500 N

Max. shear stress, $\tau = 80$ N/mm2

Let d = Diameter of wire

D = Mean diameter of coil

∴ D = 10 d.

Using equation (4),$\tau = \frac{16\,WR}{\pi d^3}$

or $$80 = \frac{16 \times 500 \times \left(\frac{D}{2}\right)}{\pi d^3} \left(R = \frac{D}{2}\right)$$

$$= \frac{8000 \times \left(\frac{10d}{2}\right)}{\pi d^3}$$

or $80 \times \pi d^3 = 8000 \times 5d$

$$d^2 = \frac{8000 \times 5}{80 \times \pi} = 159.25$$

∴ d = √159.25 = 12.6 mm = 1.26 cm. **Ans.**

∴ D = 10× d = 10 × 1.26 = 12.6 cm. **Ans.**

Problem 4.5.2. In above problem if the stiffness of the spring is 20 N per mm deflection and modulus of rigidity = 8.4×10^4 N/mm^2, find the number of coils in the closely coiled helical spring .

Given :

Stiffness, s = 20 N/mm

Modulus of rigidity, C = 8.4×10^4 N/mm^2

From problem 5, W = 500 N, τ= 80 N/mm^2

d = 12.6 mm and D = 126 mm

∴ R = D/2 = 126/2 = 63 mm

Let n = Number of coils in the spring

We know, stiffness $= \frac{Load}{\delta}$

$$20 = \frac{500}{\delta}$$

∴ $\delta = \frac{500}{20} = 25$ mm

Using equation (6),

$$\delta = \frac{64WR^3 \,.\, n}{Cd^4}$$

$$25 = \frac{64 \times 500 \times 63^3 \times n}{8.4 \times 10^4 \times 12.6^4}$$

$$n = \frac{25 \times 8.4 \times 10^4 \times 12.6^4}{64 \times 500 \times 63^3} = 6.6 \text{ say } 7.0$$

∴ n = 7. **Ans.**

Problem 4.5.3: A closely coiled helical spring of round steel wire 10 mm in diameter having 10 complete turns with a mean diameter of 12 cm is subjected to an axial load of 200 N. Determine :

1) The deflection of the spring
2) Maximum shear stress in the wire,
3) Stiffness of the spring. Take C = 8×10^4 N/mm^2.

Given :

Dia. of wire, d = 10 mm

No. of turns, n = 10

Mean dia. of coil, D = 12 cm = 120 mm

∴ Radius of coil, R = D/2 = 60 mm

Axial load, W = 200 N

Modulus of rigidity, C = 8×10^4 N/mm^2

Let δ = Deflection of the spring

τ = Max. shear stress in the wire

s = Stiffness of the spring.

1) Using equation (6),

$$\delta = \frac{64WR^3 \cdot n}{Cd^4} = \frac{64 \times 200 \times 60^3 \times 10}{8 \times 10^4 \times 10^4} = 34.5 \text{ mm. } \textbf{Ans.}$$

2) Using equation (4),

$$\tau = \frac{16\,WR}{\pi d^3} = \frac{16 \times 200 \times 60}{\pi \times 10^3} = 61.1 \text{ N/mm}^2. \textbf{ Ans.}$$

3) Stiffness of the spring,

$$s = \frac{W}{\delta} = \frac{200}{34.5} = 5.8 \text{ N/mm. } \textbf{Ans.}$$

Problem 4.5.4. A close coiled helical spring of 10 cm mean diameter is made up of 1 cm diameter rod and has 20 turns. The spring carries an axial load of 200 N. Determine the shearing stress. Taking the value of modulus of rigidity = 8.4×10^4 N/mm^2, determine the deflection when carrying this load. Also calculate the stiffness of the spring and the frequency of free vibration for a mass hanging from it.

Given :

Mean diameter of coil, D = 10 cm = 10 mm

Mean radius of coil, R = 5 cm = 50 mm

Diameter of rod, d = 1cm = 10 mm

Number of turns, n = 20

Axial load, W = 200 N

Modulus of rigidity, C = 8.4×10^4 N/mm^2

Let τ = Shear stress in the material of the spring

δ = Deflection of the spring due to axial load

s = Stiffness of spring

τ = Frequency of free vibration.

Using equation (16.24),

$$\tau = \frac{16\,WR}{\pi d^3} = \frac{16 \times 200 \times 50}{\pi \times 10^3} = 50.93 \text{ N/mm}^2.$$ **Ans.**

Deflection of the spring

Using equation (16.26),

$$\delta = \frac{64WR^3 \,.\, n}{Cd^4} = \frac{64 \times 200 \times 50^3 \times 20}{8.4 \times 10^4 \times 10^4} = 38.095 \text{ mm.}$$ **Ans.**

Stiffness of the spring

$$\text{Stiffness} = \frac{Load on spring}{Deflection of spring} = \frac{200}{38.095} = 5.25 \text{ N/mm.}$$ **Ans.**

Frequency of free vibration

$$\delta = 38.095 \text{ mm} = 3.8095 \text{ cm}$$

Using the relation, $$\tau = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}} = \frac{1}{2\pi}\sqrt{\frac{981}{3.8095}} = 2.55 \text{ cycles/sec.}$$ **Ans.**

Problem 4.5.5: A close coiled helical spring of mean diameter 20 cm is made of 3 cm diameter rod and has 16 turns. A weight of 3 kN is dropped on this spring. Find the height by which the weight should be dropped before striking the spring so that the spring may be compressed by 18 cm. Take C = 8 × 10^4 N/mm^2.

Sol. Given :

Mean dia of coil, D =20 cm = 200 mm

Mean radius of coil, R = 200/2 = 100 mm

Dia. of spring rod, d = 3 cm = 30 mm

Number of turns, n =16

Weight dropped, W = 3 kN = 3000 N

Compression of the spring, δ = 18 cm = 180 mm

Modulus of rigidity , C =8 × 10^4 N/mm^2

Let h = Height through which the weight W is dropped

W = Gradually applied load which produces the compression of spring equal to 180 mm. Now using

deflection equation,

$$\delta = \frac{64WR^3 \,.\, n}{Cd^4}$$

or $$180 = \frac{64 \times W \times 100^3 \times 16}{8 \times 10^4 \times 30^4}$$

or $W = \frac{180 \times 8 \times 10^4 \times 30^4}{64 \times 100^3 \times 16} = 11390$ N

Work done by the falling weight on spring

= Weight falling (h + δ) = 3000 (h + 180) N-mm Energy

stored in the spring = $\frac{1}{2} \times 11390 \times$ 180 = 1025100 N-mm.

Equating the work done by the falling weight on the spring to the energy stored in the spring, we

get

$$3000 (h + 180) = 1025100$$

$$h + 180 = \frac{1025100}{3000} = 341.7 \text{ mm}$$

∴ h = 341.7 – 180 = 161.7 mm. **Ans.**

Problem 4.5.6. The stiffness of a close coiled helical spring is 1.5 N/mm of compression under a maximum load of 60 N. The maximum shearing stress produced in the wire of the spring is 125 N/mm^2. The solid length of the spring (when the coils are touching) is given as 5 cm . Find : 1) Diameter of wire,(2) mean diameter of the coils and (3) number of coils required. Take C = 4.5 × 10^4 N/mm^2.

Sol. Given :

Stiffness of spring, s = 1.5 N/mm

Load on spring, W = 60 N

Maximum shear stress, τ = 125 N/mm^2

Solid length of spring, = 5 cm = 50 mm

Modulus of rigidity, C = 4.5 × 10^4 N/mm^2

Let d = Diameter of wire,

D = Mean dia, of coil, and

R = Mean radius of coil = $\frac{D}{2}$

n = Number of coils.

Using stiffness equation

$$s = \frac{Cd^4}{64 \,.\, R^3 \,.\, n} \text{ or } 1.5 = \frac{4.5 \times 10^4 \times d^4}{64 \times R^3 \times n}$$

$$\therefore \quad d^4 = \frac{1.5 \times 64 \times R^3 \times n}{4.5 \times 10^4} = 0.002133R^3 \times n \qquad \ldots. (1)$$

Using shear stress equation,

$$\tau = \frac{16\,W\,R}{\pi d^3} \text{ or } 125 = \frac{16 \times 60 \times R}{\pi d^3}$$

$$\therefore \quad R = \frac{125 \times \pi d^3}{16 \times 60} = 0.40906\ d^3 \ldots. (2)$$

Substituting the value of R in equation (1) , we get

$$d^4 = 0.002123 \times (\ 0.40906 d^3)^3 \times n$$

$$= 0.002123 \times (0.40906^3) \times d^9 \times n = 0.00014599 \times d^9 \times n$$

or $$\frac{d^9\ .\ n}{d^4} = \frac{1}{0.00014599} \text{ or } d^5\ .\ n = \frac{1}{0.00014599} \quad \ldots. (3)$$

Now

Solid length $= n \times d$ or $50 = n \times d$

$$n = \frac{50}{d} \quad \ldots. (4)$$

Substituting this value of n in equation (3), we get

$$d^5 \times \frac{50}{d} = \frac{1}{0.00014599}$$

$$d^4 = \frac{1}{0.00014599} \times \frac{1}{50} = 136.99$$

$$d = (136.99)^{1/4} = 3.42 \text{ mm. } \textbf{Ans.}$$

Substituting this value in equation (4)

$$n = \frac{50}{d} = \frac{50}{3.42} = 14\ .62 \text{ say } 15. \textbf{ Ans.}$$

Also from equation (2),

$$R = 0.40906\ d^3 = 0.40906 \times (3.42)^3 = 16.36 \text{ mm}$$

i.e., Mean dia. of coil, $D = 2R = 2 \times 16.36 = 32.72$ mm. Ans.

Problem 4.5.7: A close coiled helical spring has a stiffness of 10 N/mm. Its length when fully compressed, with adjacent coils touching each other is 40 cm. The modulus of rigidity of the material of the spring is 0.8×10^5 N/mm^2.

1) Determine the wire diameter and mean coil diameter if their ratio is $\frac{1}{10}$.
2) If the gap between any two adjacent coil is 0.2 cm, what maximum load can be applied before the spring becomes solid, i.e., adjacent coils touch.
3) What is the corresponding maximum shear stress in the spring

Sol. Given :

Stiffness of spring, s = 10 N/mm

Length of spring when fully compressed i.e., solid length

= 40 cm = 400 mm

Modulus of rigidity, $C = 0.8 \times 10^5$ N/mm^2

Let d = Diameter of wire of spring

D = Mean coil diameter n = Number of turns

W = Maximum load applied when spring becomes solid τ = Maximum shear stress induced in the wire.

Now $\frac{d}{D} = \frac{1}{10}$

Gap between any two adjacent coil = 0.2 cm = 2.0 mm

∴ Total gap in coils = Gap between two adjacent coil × Number of turns

= 2 × n mm.

When spring is fully compressed, there is no gap in the coils and hence maximum compression of the coil will be equal to the total gap in the coil.

∴ Maximum compression, $\delta = 2\times n$ mm Now

using equation (16.27),

$$s = \frac{Cd^4}{64 \,.\, R^3 \,.\, n} \text{ or } 10 = \frac{0.8 \times 10^5 \times d^4}{64 \,.\, R^3 \,.\, n}$$

$$d^4 = \frac{10 \times 64 \times R^3 \times n}{0.8 \times 10^5} = \left(\frac{8}{10^3}\right) R^3 \times n \qquad \text{.... (1)}$$

But from equation (16 .28),

Solid length = n × d or 400 = n × d

$$n = \frac{400}{d} \qquad \text{.... (2)}$$

Substituting the value of n in equation (1),

$$d^4 = \left(\frac{8}{10^3}\right) \times R^3 \times \frac{400}{d} \qquad = 3.2 \times \frac{R^3}{d}$$

But mean coil radius,

$$R = \frac{D}{2}$$

$$\therefore \qquad d^5 = 3.2 \times \left(\frac{D}{2}\right)^3 = \frac{3.2 \times D^3}{8} = 0.4\, D^3$$

$$\frac{d^5}{D^3} = 0.4 \text{ or } \frac{d^3}{D^3} \cdot d^2 = 0.4$$

$$\left(\frac{1}{10}\right)^3 \cdot d^2 = 0.4 \qquad \left(\frac{d}{D} = \frac{1}{10}\right)$$

∴ $d^2 = 0.4 \times 10^3 = 400$

∴ $d = \sqrt{400} = 20 \text{ mm} = 2\text{cm}$. **Ans.**

But $\frac{d}{D} = \frac{1}{10}$

$D = 10 \times d = 10 \times 2 = 20.0$ cm. **Ans.**

Let us find first number of turns. From equation (2), we have

$$n = \frac{400}{d} = \frac{400}{20} = 20 \qquad (d=20)$$

$$\delta = 2 \times n = 2 \times 20 = 40\ mm$$

We know, stiffness of spring is given by

$$s = \frac{W}{\delta} \text{ or } 10 = \frac{W}{40}$$

$W = 10 \times 40 = 400\ N$. **Ans.**

Using eqution (16.24), we have

$$\tau = \frac{16WR}{\pi d^3}$$

$$= \frac{16 \times 400 \times 100}{\pi \times 20^3} \qquad \left(R = \frac{D}{2} = \frac{20}{2} = 10 \text{ cm} = 100 \text{ mm}\right)$$

$= 25.465 \text{ N/mm}^2$. **Ans.**

Problem 4.5.8. Two close- colied concentric helical springs of the same length, are wound out of the same wire, circular in cross- section and supports a compressive load 'P'. The inner spring consists of 20 turns of mean diameter 16 cm and the outer spring has 18 turns of mean diameter 20 cm. Calculate the maximum stress produced in each spring if the diameter of wire = 1cm and P = 1000 N.

Sol. Given :

Total load supported, P = 1000 N

Both the springs are of the same length of the same material and having same dia . of wire.

Hence values of L,C and 'd' will be same.

For inner spring

No. of turns, $n_i = 20$

Mean dia.., $D_i = 16\ cm = 160$ mm $\therefore\ R_i = \frac{160}{2} = 80\ mm$

Dia of wire, $d_i = 1$ cm = 10 mm

For outer spring

No. of turns, $n_o = 18$

Mean dia.., $D_o = 20$ cm = 200 mm $\therefore R_o = \frac{200}{2} = 100$ mm

Dia. of wire, $d_o = 1$ cm = 10 mm

Let W_i = Load carried by inner spring

W_o = Load carried by outer spring

τ_i = Max. shear stress produced in inner spring

τ_o = Max. shear stress produced in outer spring.

Now $W_i + W_o$= Total load carried = 1000 …. (1)

Since there are two close-coiled concentric helical springs, hence deflection of both the springs will be same.

$\delta_o = \delta_i$ where δ_o = Deflection of outer spring

δ_i = Deflection of inner spring.

The deflection of close-coiled spring is given by equation (16.26) as

$$\delta = \frac{64WR^3 \cdot n}{Cd^4}$$

Hence for outer spring , we have

$$\delta_o = \frac{64W_o \times R_o \times n_o}{C \times d_o^4} = \frac{64W_o \times 100^3 \times 18}{C \times 10^4} \qquad (R_o = 100,\ d_o = 10)$$

Similarly for inner spring , we have

$$\delta_i = \frac{64W_i \times R_i \times n_i}{C \times d_i^4} = \frac{64W_i \times 80^3 \times 20}{C \times 10^4}$$

(Material of wires is same. Hence value of C will be same.)

But

$$\delta_o = \delta_i$$

∴

$$\frac{64W_o \times 100^3 \times 18}{C \times 10^4} = \frac{64W_i \times 80^3 \times 20}{C \times 10^4}$$

$$W_o \times 100^3 \times 18 = W_i \times 80^3 \times 20$$

$$W_o = \frac{W_i \times 80^3 \times 20}{100^3 \times 18} = 0.569\ W_i$$

Substituting the value of W_o in equation (1), we get W_i

$+ 0.569\ W_i = 1000$ or $1.569\ W_i = 1000$

∴ $$W_i = \frac{1000}{1.596} = 637.3 \text{ N.}$$

But from equation (1), $W_i + W_o = 1000$

∴ $$W_o = 1000 - W_i = 1000 - 637.3 = 362.7 \text{ N.}$$

The maximum shear stress produced is given by equation (16.24) as

$$\tau = \frac{16WR}{\pi d^3}$$

For outer spring, the maximum shear stress produced is given by,

$$\tau_0 = \frac{16W_o \times R_o}{\pi \times d_0^3} = \frac{16 \times 362.7 \times 100}{\pi \times 10^3}$$

$= 184.72$ N/mm^2. **Ans.**

Similarly for inner spring, the maximum shear stress produced is given by,

$$\tau_i = \frac{16W_i \times R_i}{\pi \times d_i^3} = \frac{16 \times 637.3 \times 80}{\pi \times 10^3}$$

$= 259.66$ N/mm^2. **Ans.**

Problem 4.5.8. A closely coiled helical spring made of 10 mm diameter steel wire has 15 coils of 100 mm mean diameter. The spring is subjected to an axial load of 100 N. Calculate:

i. The maximum shear stress induced,
ii. The deflection, and iii. Stiffness of the spring n Take modulus of rigidity, $C = 8.16 \times 10^4$ N/mm^2.

Sol. Given :

Dia. of wire, d = 10 mm

Number of coils, n =15

Mean dia. of coil, D = 100 mm

Mean radius of coil, $R = \frac{100}{2} = 50$ mm

Axial load, W = 100 N

Modulus of rigidity, C = 8.16 × 10^4 N/mm^2.

i. Maximum shear stress induced

Wkt $\tau = \frac{16WR}{\pi d^3} = \frac{16 \times 100 \times 50}{\pi \times 10^3}$ = 24.46 N/mm^2. **Ans.**

The deflection (δ) Wkt

$$\delta = \frac{64WR^3 \,.\, n}{Cd^4} = \frac{64 \times 100 \times 50^3 \times 15}{8.16 \times 10^4 \times 10^4} = 14.7 \text{ mm. } \textbf{Ans.}$$

Stiffness of the spring

$$\text{Stiffness} = \frac{Load\ on\ spring}{Deflection\ of\ spring}$$

$$= \frac{100}{14.7} = 6.802 \text{ N/mm. } \textbf{Ans.}$$

IMPORTANT TERMS

TORSION OF SHAFT		
Torsion Equation	$\frac{T}{J} = \frac{C\theta}{l} = \frac{\tau}{R}$	T = Toque (T_{max}) l = length of shaft θ = Angle of twist in radian J = $= \frac{\pi}{32} D^4$ Polar moment of Inertia For solid shaft $\frac{\pi}{32}(D^4 - d^4)$ for hollow shaft
Toque on shaft(T)	$T = \frac{\pi}{16} \tau D^3$	D = Outer diameter of shaft d = inner diameter of hollow shaft
	$T = \frac{\pi}{16} \tau \left(\frac{D^4 - d^4}{D}\right)$	τ = shear stress C = Modulus of rigidity of shaft
Torsional rigidity	= C X J	

Power transmission	$P = \frac{2\pi NT}{60}$	N = Speed in rpm T = Torque (T_{mean})
% of saving material	$= \frac{W_s - W_h}{W_s} \ x\ 100$	$W_s = Weight\ of\ solid\ shaft$ $W_h = Weight\ of\ hollow\ shaft$
% of saving material for same material and same length	$= \frac{A_s - A_h}{A_s} \ x\ 100$	$A_s = Area\ of\ solid\ shaft$ $A_h = Area\ of\ hollow\ shaft$

Find suitable diameter of shaft	Find diameter of shaft in two method 1. Based on shear stress $T = \frac{\pi}{16}\tau D^3$ 2. Based on angle of twist $\frac{T}{J} = \frac{C\theta}{l}$	Largest diameter is the suitable diameter of shaft Same procedure followed for hollow shaft
SERIES SHAFT (Shaft with varying cross section)	1. Angle of twist $\theta = \theta_1 + \theta_2$ 2. $Length\ L = l_1 + l_2$ 3. $Torque\ T = T_1 = T_2$	
PARALLEL SHAFT (Composite shaft)	1. Angle of twist same $\theta = \theta_1 = \theta_2$ 2. $Length\ L = l_1 = l_2$ 3. $Torque\ T = T_1 + T_2$	
COMBINED BENDING AND TORSION ACT ON A SHAFT		
Major Principal Stress	$= \frac{16}{\pi D^3}(M + \sqrt{M^2 + T^2})$ $= \frac{16D}{\pi(D^4 - d^4)}(M + \sqrt{M^2 + T^2})$	M = Bending moment T = Torque V = Volume = Area x length for solid shaft $= \frac{\pi D^3}{4} l$ $= \frac{\pi}{4}(D^2 - d^2)l$ for hollow shaft
Minor Principal Stress	$= \frac{16}{\pi D^3}(M - \sqrt{M^2 + T^2})$ $= \frac{16D}{\pi(D^4 - d^4)}(M - \sqrt{M^2 + T^2})$	
Maximum shear stress	$= \frac{16}{\pi D^3}(\sqrt{M^2 + T^2})$ $= \frac{16D}{\pi(D^4 - d^4)}(\sqrt{M^2 + T^2})$	
Strain energy stored in a body due to torsion	$U = \frac{}{4C} V \tau^2$	
SPRING (CLOSED COIL HELICAL SPRING)		
Deflection (δ)	$\delta = \frac{}{Cd^4} 64WR^3n$	W = axial load on the spring R = mean radius of coil spring
Energy Stored in spring (U)	= Average load x deflection = W x δ/2 $= \frac{32W\ R\ n}{Cd^4}$	n = number of turn in coil C = modulus of rigidity of spring d = diameter of spring wire δ = deflection of spring
Spring stiffness (s)	= W /δ $= \frac{Cd}{64\ R^3 n}$ 4	
Shear stress induced in spring (τ)	$\tau = \frac{16\ WR}{}$ 3	

	πd	
Solid length of spring	= n x d	
OPEN COIL HELICAL SPRING		
Deflection (δ) Due to axial load	$\delta = \frac{64WR^3nsec\alpha}{d^4}\left[\frac{\cos^2\alpha}{C} + \frac{2\sin^2\alpha}{E}\right]$	W = axial load on the spring R = mean radius of coil spring n = number of turn in coil C = modulus of rigidity of spring d = diameter of spring wire δ = deflection of spring $\alpha = helix\ angle\ of\ spring$ E = young's modulus
Deflection (δ) Due to axial twisting couple	$\delta = \frac{0}{d^4}\left[\frac{}{C} - \frac{}{E}\right]$ $64M\ R^2nsin\alpha\ 1 \quad 2$	
Angle of rotation (β) Due to axial load	$\beta = \frac{64WR^2nsin\alpha}{d^4}\left[\frac{1}{C} - \frac{2}{E}\right]$	
Angle of rotation (β) Due to axial twisting couple	$\beta = \frac{64M_0Rnsec\alpha}{d^4}\left[\frac{\sin^2\alpha}{C}\right.$	
Bending Stress	$\left. + \frac{2\cos^2\alpha}{E}\right]$ $f = \frac{32\ WR\ sin\alpha}{\pi d^3}$ $\tau = \frac{16\ WRcos\alpha}{\pi d^3}$	
Shear stress		

THEORETICAL PROBLEMS

1. Define the term torsion, torsional rigidity and polar moment of inertia.
2. Derive an expression for the shear stress produced in a circular shaft which is subjected to torsion. what are the assumptions made in the derivation
3. Prove that the torque transmitted by a solid shaft when subjected to torsion is given by T = $\pi/16\tau D^3$ where D =dia of shaft and τ = max shear stress
4. When a circular shaft is subjected to torsion show that the shear stress varies linearly from the axis to the surface
5. Derive the relation for circular shaft when subjected to torsion as shown

$$T/J = \tau/R = C\theta/L$$

Where T = torque transmitted

J = polar moment of inertia

τ = max shear stress

R = radius of shaft

C = modulus of rigidity

θ = angle of twist

L = length of shaft

6. Find an expression for the torque transmitted by a hollow circular shaft of ext dia D_O and int dia D_i
7. Define the term polar modulus. Find the expression for a solid shaft and hollow shaft
8. What do you mean by strength of shaft
9. Define torsional rigidity ofshaft. Prove that the torsional rigidity is the torque required to produce a twist of one radian in a unit length of shaft
10. Prove that the strain energy stored in a body due to shear stress is given by

$$U = \tau^2/2C * V$$

Where τ = shear stress

C = modulus of rigidity

V = volume of body

11. Find an expression for strain energy stored in a body due to torsion
12. A hollow shaftof ext dia D and int dia d is subjected to torsion, prove that the strain energy stored is given by

$U = \tau^2/4CD^2\ (D^2 + d^2)$ where V = volume of hollow shaft. τ = shear stress

13. What is a spring. Name the two important types of spring
14. Prove that the central deflection of the laminated spring is given by

$\delta = \sigma l^2/4Et$

Where σ = max stress

E = modulus of elasticity

L = length of leaf spring

T = thickness of each plate

15. Define helical springs. Name the two important types of helical springs.
16. Prove that the max shear stress induced in wire of close coiled helical spring is given by

$\tau = 16WR/\pi d^3$

Where τ = max shear stress

W = axial load

R = mean radius of spring coil

17. Find an expression for the strain energy stored by the close coiled helical spring when subjected to axial load W
18. Prove that the deflection of a close coiled helical spring at the centres due to axial load W is given by $\delta = 64WnR^3/Cd^4$

Where R = mean radius of spring coil

N = number of coil

C = modulus of rigidity

D = dia of wire

NUMERICAL PROBLEMS

1. The shearing stress in a solid shaft is not exceed 45 N/mm^2 when the torque transmitted is 4000 Nm. Determine the min dia of the shaft. Ans = 16.49 mm
2. Find the max torque transmitted by a hollow circular shaft of ext dia 30cm and int dia 15cm if the shear stress is not to exceed 40N/mm^2. Ans = 198.8Kn
3. Two shafts of the same material and of same length are subjected to same torque if the first shaft is of a solid circular section and the second shaft is of hollow circular section, whose int dia is 0.7 times the outside dia and the max shear stress developed in each shaft is the same, compare the weight of the shaft. Ans =(1.633/1)
4. Find the max shear stress induced in a solid circular shaft of dia 20cm when the shaft transmit 187.5kW at 200rpm. Ans = 5.7N/mm^2
5. A solid circular shaft is to transmit 375kW at 150 rpm. Find the dia of shaft if the shear stress is not to exceed 65N/mm^2. What percent saving in weight would be obtained if this is replaced by a hollow shaft whose int dia equal to 2/3 of ext dia, the length, the material and max shear stress being the same. Ans = 12.29 cm and 35.71%
6. A solid shaft has to transmit 112.5 kW at 250 rpm. taking allowable shear stress as 70 N/mm^2. Find suitable dia for the shaft if the max torque transmitted at each revolution exceeds the mean by 20%. Ans = 7.20 cm
7. A hollow shaft is to transmit 337.5 kW at 100 rpm. If the shear stress is not to exceed 65N/mm^2 and the int dia is 0.6 of the ext dia, find the ext and int dia assuming the max torque is 1.3 times the mean. Ans = 15.52 cm and 9.312 cm

8. Determine the dia of solid steel shaft which will transmit 112.5 kW at 200 rpm. Also determine the length of shaft if the twist must not exceed 1.5 over the entire length. The max shear stress is limited to 55N/mm^2. Take the value of modulus of rigidity = $8*10^4 N/mm^2$. Ans = 7.9 cm and 150.4 cm
9. Determine the dia of solid shaft which will transmit 337.5 kW at 300 rpm. The max shear stress should not exceed 35N/mm^2 and twist should not be more than 1 in a shaft region of 2.5m. take modulus of rigidity = $9*10^4$ N/mm^2. Ans = 11.57 cm
10. and the coupling are equally strong in torsion. Ans = 11.6 cm and 0.848 cm
11. A hollow shaft of 1.5 m long has ext dia 60mm. It has 30mm int dia for a part of length and 40mm int dia for rest of length. If the max shear stress in it is not to exceed 85 N/mm^2 determine the max horse power transmitted at 350 rpm. If the twist produced in two portions of the shafts are equal find the length. Ans = 141.37, 808.23mm, 691.77mm
12. A leaf spring carries a central load of 2.5 kN. The leaf spring is to be made of 10 steel plates 6 cm wide and 5mm thick. If the bending stress is limited to 100 N/mm^2. Determine the length of spring and deflection at the centre of the spring. Ans = 40 cm, 0.4cm
13. A laminated leaf spring 0.9m long is made up of plates each 5cm wide and 1cm thick. If the bending stress in the plate is limited to 120 N/mm^2, how many plates would be required to enable the spring to carry a central pont load of 2.65 kN. What is the deflection under the load. Ans = 6 plates, 1.215 cm
14. A closely coiled helical spring is to carry a load of 1kN. Its mean coil dia is to be 10 times the wire dia. Calculate these dia if te max shear stress in the materiall of the spring is to be 90 N/mm^2. Ans = 16.82 cm and 1.68cm
15. In ques 16 if stiffness of spring is 20N/mm deflection and modulus of rigidity = $8.4*10^4$ N/mm^2 find the number of coils in the closely coiled helical spring. Ans = 9
16. A closely coiled helical spring of round steel wire 8mm in dia having 10 complete turns with a mean dia of 10 cm is subjected to an axial load of 250 N. Determine the deflection of spring, max shear stress in wire, stiffness of spring. Ans = 6.1cm, 124.34 N/mm^2, 4.1 N/mm^2