Expression for Back E.M.F or Induced E.M.F. per Phase in synchronous motor

Case i) Under excitation, $E_{bph} < V_{ph}$.

$Z_s = R_a + j\,X_s = |\,Z_s\,| \angle \theta\ \Omega$ $\theta = \tan^{-1}(X_s/R_a)$

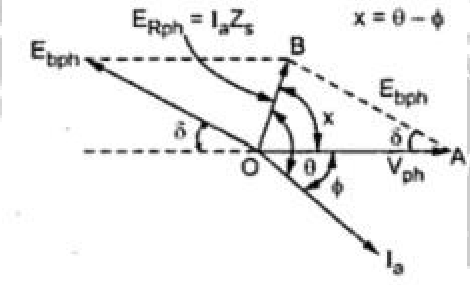
$E_{Rph} \wedge I_{aph} = \theta$, I_a lags always by angle θ. V_{ph} = Phase voltage applied

E_{Rph} = Back e.m.f. induced per phase

$$E_{Rph} = I_a \times Z_s \text{ V} \qquad \text{... per phase}$$

Let p.f. be $\cos\Phi$, lagging as under excited, $V_{ph} \wedge I_{aph} = \Phi$

Phasor diagram is shown in the Fig. 1.



Applying cosine rule to Δ OAB,

$(E_{bph})^2 = (V_{ph})^2 + (E_{Rph})^2 - 2V_{ph}\,E_{Rph} \times (V_{ph} \wedge E_{Rph})$ but $V_{ph} \wedge E_{Rph} = x = \theta - \Phi$

$(E_{bph})^2 = (V_{ph})^2 + (E_{Rph})^2 - 2V_{ph}\,E_{Rph} \times (\theta - \Phi)$..(1)

where $E_{Rph} = I_{aph} \times Z_s$ Applying sine rule to Δ OAB, $E_{bph}/\sin x = E_{Rph}/\sin\delta$

$$\therefore \quad \boxed{\sin\delta = \frac{E_{Rph}\,\sin(\theta - \phi)}{E_{bph}}} \qquad \text{... (2)}$$

So once E_{bph} is calculated, load angle δ can be determined by using sine rule.

Case ii) Over excitation, $E_{bph} > V_{ph}$

p.f. is leading in nature.

$E_{Rph} \wedge I_{aph} = \theta$

$V_{ph} \wedge I_{aph} = \Phi$

The phasor diagram is shown in the Fig. 2.

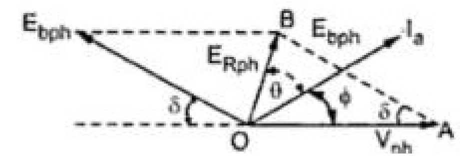


Figure 2.10. Phasor diagram for overexcited condition

Applying cosine rule to Δ OAB,

$(E_{bph})^2 = (V_{ph})^2 + (E_{Rph})^2 - 2V_{ph}\ E_{Rph}$ x $\cos(V_{ph} \wedge E_{Rph})$ $V_{ph} \wedge E_{Rph} = \theta + \Phi$

∴ $(E_{bph})^2 = (V_{ph})^2 + (E_{Rph})^2 - 2\ V_{ph}\ E_{Rph} \cos(\theta + \Phi)$ (3)

But $\theta + \Phi$ is generally greater than 90^o

∴ $\cos(\theta + \Phi)$ becomes negative, hence for leading p.f., $E_{bph} > V_{ph}$.

Applying sine rule to Δ OAB,

$E_{bph}/\sin(E_{Rph} \wedge V_{ph}) = E_{Rph}/\sin\delta$

∴ $$\boxed{\sin\delta = \frac{E_{Rph}\ \sin(\theta + \phi)}{E_{bph}}} \quad \text{... (4)}$$

Hence load angle δ can be calculated once E_{bph} is known.

Case iii) Critical excitation

In this case $E_{bph} \approx V_{ph}$, but p.f. of synchronous motor is unity.

∴ cos = 1 ∴ $\Phi = 0^o$

i.e. V_{ph} and I_{aph} are in phase and

$E_{Rph} \wedge I_{aph} = \theta$

Phasor diagram is shown in the Fig. 3.

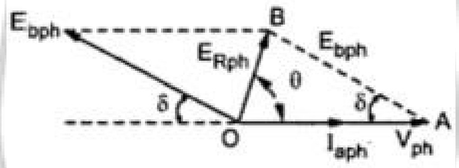


Figure 2.11. Phasor diagram for unity p.f. condition

Applying cosine rule to OAB,

$(E_{bph})^2 = (V_{ph})^2 + (E_{Rph})^2 - 2V_{ph}\ E_{Rph} \cos\theta$......................................(5)

Applying sine rule to OAB,

$E_{bph}/\sin\theta = E_{Rph}/\sin\delta$

∴ $$\boxed{\sin\delta = \frac{E_{Rph}\ \sin\theta}{E_{bph}}} \quad \text{... (6)}$$

where $E_{Rph} = I_{aph}$ x Z_s V

Thus in general the induced e.m.f. can be obtained by,

$$(E_{bph})^2 = (V_{ph})^2 + (E_{Rph})^2 - 2\ V_{ph}\ E_{Rph} \cos(\theta \pm \phi)$$

+ sign for lagging p.f. while – sign for leading p.f.