PROVING LANGUAGES NOT TO BE REGULAR

Theorem

Let L be a regular language. Then there exists a constant **'c'** such that for every string **w** in **L** −

$|w| \geq c$

We can break **w** into three strings, **w = xyz**, such that −

- $|y| > 0$
- $|xy| \leq c$
- For all $k \geq 0$, the string xy^kz is also in L.

Applications of Pumping Lemma

Pumping Lemma is to be applied to show that certain languages are not regular. It should never be used to show a language is regular.

- If **L** is regular, it satisfies Pumping Lemma.
- If **L** does not satisfy Pumping Lemma, it is non-regular.

Method to prove that a language L is not regular

- At first, we have to assume that **L** is regular.
- So, the pumping lemma should hold for **L**.
- Use the pumping lemma to obtain a contradiction −
    - Select **w** such that **$|w| \geq c$**
    - Select **y** such that **$|y| \geq 1$**
    - Select **x** such that **$|xy| \leq c$**
    - Assign the remaining string to **z.**
    - Select **k** such that the resulting string is not in **L.**

Hence L is not regular.

Problem

Prove that $\mathbf{L = \{a^i b^i \mid i \geq 0\}}$ is not regular.

Solution

- At first, we assume that **L** is regular and n is the number of states.
- Let w = $a^n b^n$. Thus $|w| = 2n \geq n$.
- By pumping lemma, let w = xyz, where $|xy| \leq n$.
- Let $x = a^p$, $y = a^q$, and $z = a^r b^n$, where $p + q + r = n$, $p \neq 0$, $q \neq 0$, $r \neq 0$. Thus $|y| \neq 0$.
- Let k = 2. Then $xy^2z = a^p a^{2q} a^r b^n$.
- Number of as = $(p + 2q + r) = (p + q + r) + q = n + q$
- Hence, $xy^2z = a^{n+q} b^n$. Since $q \neq 0$, xy^2z is not of the form $a^n b^n$.
- Thus, xy^2z is not in L. Hence L is not regular.