Problem-1

- Find the DFT of a sequence x(n)={1,1,0,0}
- Given: x(n)={1,1,0,0} $\longrightarrow$ N=4

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{\frac{-j2\pi nk}{N}}, \; 0 \le k \le N-1$$

$$X(k) = \sum_{n=0}^{3} x(n) e^{\frac{-j2\pi kn}{4}}, \; 0 \le k \le 3$$

- K=0
- $X(0) = \sum_{n=0}^{3} x(n) = x(0) + x(1) + x(2) + x(3)$ X(0)=2

Problem-1 cont..

- K=1,

$$X(1) = \sum_{n=0}^{3} x(n) e^{\frac{-j2\pi(1)n}{4}} = \sum_{n=0}^{3} x(n) e^{\frac{-j\pi n}{2}}$$

$$X(1) = x(0) + x(1)e^{\frac{-j\pi 1}{2}} + x(2)e^{\frac{-j\pi 2}{2}} + x(3)e^{\frac{-j\pi 3}{2}}$$

$$= 1 + \cos\frac{\pi}{2} - j\sin\frac{\pi}{2}$$

- X(1)=1-j

Problem-1 cont..

- K=2,

$$X(2)=\sum_{n=0}^{3}x(n)e^{\frac{-j2\pi(2)n}{4}}=\sum_{n=0}^{3}x(n)e^{-j\pi n}$$

$$X(2)=x(0)+x(1)e^{-j\pi}+x(2)e^{-j2\pi}+x(3)e^{-j3\pi}$$

$$=1+\cos\pi-j\sin\pi$$

- X(2)=1-1=0
- X(2)=0

Problem-1 cont..

- K=3,

$$X(3) = \sum_{n=0}^{3} x(n) e^{\frac{-j2\pi(3)n}{4}} = \sum_{n=0}^{3} x(n) e^{\frac{-3\pi n}{2}}$$

$$X(3) = x(0) + x(1) e^{-j\frac{3(1)\pi}{2}} + x(2) e^{-j\frac{3(2)\pi}{2}} + x(3) e^{-j\frac{3(3)\pi}{2}}$$

$$= 1 + \cos\frac{3\pi}{2} - j\sin\frac{3\pi}{2}$$

- X(3)=1+j
- X(k)={2,1-j,0,1+j}

Problem-2

- Find IDFT of X(K)={1,0,1,0}

$$x(n)=\frac{1}{N}\sum_{k=0}^{N-1}X(k)e^{j2\pi nk/N},\ 0\leq n\leq N-1$$

$$x(n)=\frac{1}{4}\sum_{k=0}^{3}X(k)e^{j2\pi nk/4},\ 0\leq n\leq 3$$

- n=0, $x(0)=\frac{1}{4}\sum_{k=0}^{3}X(k)=\frac{1}{4}[X(0)+X(1)+X(2)+X(3)]$
- x(0)=0.5

Problem-2 **cont..**

- n=1, $$x(1) = \frac{1}{4}\sum_{k=0}^{3} X(k)e^{j\frac{2\pi k}{4}}$$

$$= \frac{1}{4}[X(0) + X(1)e^{j\frac{\pi}{2}} + X(2)e^{j\frac{2\pi}{2}} + X(3)e^{j\frac{3\pi}{2}}]$$

$$x(1) = \frac{1}{4}[1 + \cos\pi + j\sin\pi]$$

- x(1)=0

Problem-2 cont..

- n=2, $$x(2)=\frac{1}{4}\sum_{k=0}^{3}X(k)e^{j\frac{2(2)\pi k}{4}}$$

$$=\frac{1}{4}[X(0)+X(1)e^{j\pi}+X(2)e^{j2\pi}+X(3)e^{j3\pi}]$$

$$x(2)=\frac{1}{4}[1+\cos 2\pi+j\sin 2\pi]$$

- x(2)=0.5

Problem-2 **cont..**

- n=3, $x(3)=\frac{1}{4}\sum_{k=0}^{3}X(k)e^{j\frac{2(3)\pi k}{4}}$

$$=\frac{1}{4}[X(0)+X(1)e^{j\frac{3\pi}{2}}+X(2)e^{j\frac{6\pi}{2}}+X(3)e^{j\frac{9\pi}{2}}]$$

$$x(3)=\frac{1}{4}[1+\cos 3\pi+j\sin 3\pi]$$

- x(3)=0
- x(n)={0.5,0,0.5,0}

Problem-3

- Find DFT of the sequence
- x(n)={1/4,1/4,1/4}

$$x(n)=\begin{cases}\frac{1}{4};0\le n\le 2\\ 0;otherwise\end{cases}$$

$$X(k)=\sum_{n=0}^{N-1}x(n)e^{\frac{-j2\pi nk}{N}},\ 0\le k\le N-1$$

$$X(k)=\sum_{n=0}^{2}x(n)e^{\frac{-j2\pi kn}{3}},\ 0\le k\le 2$$

$$X(k)=\frac{1}{4}\left[1+e^{-j\omega}+e^{-j2\omega}\right]\Big|_{\omega=\frac{2\pi k}{3}}$$

$$X(k)=\frac{1}{4}e^{-j\omega}\left[e^{j\omega}+1+e^{-j\omega}\right]\Big|_{\omega=\frac{2\pi k}{3}}$$

$$X(k)=\frac{1}{4}e^{-j\omega}\left[1+2\cos\omega\right]\Big|_{\omega=\frac{2\pi k}{3}}$$

$$\boxed{X(k)=\frac{1}{4}e^{-j\frac{2\pi k}{3}}\left[1+2\cos\left(\frac{2\pi k}{3}\right)\right]}$$

Problem-4

- Find DFT of the sequence $x(n) = \begin{cases} \frac{1}{5}; -1 \le n \le 1 \\ 0; otherwise \end{cases}$

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{\frac{-j2\pi nk}{N}}, \ 0 \le k \le N-1$$

$$X(k) = \sum_{n=-1}^{1} x(n) e^{\frac{-j2\pi kn}{3}}$$

$$X(k) = \frac{1}{5}\left[e^{-j\omega} + 1 + e^{j\omega}\right]\Bigg|_{\omega=\frac{2\pi k}{3}}$$

$$X(k) = \frac{1}{5}\left[1 + 2\cos\omega\right]\Bigg|_{\omega=\frac{2\pi k}{3}}$$

$$\boxed{X(k) = \frac{1}{5}\left[1 + 2\cos\left(\frac{2\pi k}{3}\right)\right]}$$

Problem-5

- Find DFT of the sequence: $x(n)=a^n$ for $0<a<1$

$$X(k)=\sum_{n=0}^{N-1}x(n)e^{\frac{-j2\pi nk}{N}},\ 0\leq k\leq N-1$$

$$X(k)=\sum_{n=0}^{N-1}a^n e^{\frac{-j2\pi nk}{N}}$$

$$\sum_{n=0}^{N-1}a^n=\frac{1-a^N}{1-a}$$

$$X(k)=\sum_{n=0}^{N-1}\left(ae^{\frac{-j2\pi k}{N}}\right)^n=\frac{1-\left(ae^{\frac{-j2\pi k}{N}}\right)^N}{1-ae^{\frac{-j2\pi k}{N}}}$$

$$\boxed{X(k)=\frac{1-a^N}{1-ae^{\frac{-j2\pi k}{N}}}}$$

Problem-6

- Find 4-point DFT of the sequence:

$$x(n) = \cos\left(\frac{n\pi}{4}\right)$$

- x(n)={1,0.707,0,-0.707}

- X(k)={1,1-j1.414,1,1+j1.414}

Problem-7

- Determine the 8-point DFT of the sequence x(n)={1,1,1,1,1,1,0,0} here N=8

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{\frac{-j2\pi nk}{N}}, \; 0 \le k \le N-1$$

$$X(k) = \sum_{n=0}^{7} x(n) e^{\frac{-j2\pi nk}{8}}, \; 0 \le k \le 7$$

$$X(k) = \sum_{n=0}^{7} x(n) e^{\frac{-j\pi nk}{4}}, \; 0 \le k \le 7$$

Problem-7 cont..

- k=0

$$X(0) = \sum_{n=0}^{7} x(n)e^{-j(0)} = x(0) + x(1) + \ldots + x(7)$$

$$X(0) = 1+1+1+1+1+1+0+0$$

$$\boxed{X(0) = 6}$$

Problem-7 cont..

- k=1

$$X(1)=\sum_{n=0}^{7}x(n)e^{\frac{-j\pi n(1)}{4}}$$

$$X(1)=x(0)+x(1)e^{\frac{-j\pi}{4}}+x(2)e^{\frac{-j2\pi}{4}}+x(3)e^{\frac{-j3\pi}{4}}+x(4)e^{\frac{-j4\pi}{4}}$$

$$+x(5)e^{\frac{-j5\pi}{4}}+\cancel{x(6)e^{\frac{-j6\pi}{4}}}+\cancel{x(7)e^{\frac{-j7\pi}{4}}}$$

$$X(1)=1+0.707-j0.707-j-0.707-j0.707-1-0.707+j0.707$$

$$\boxed{X(1)=-0.707-j0.707}$$

Problem-7 cont..

- k=2

$$X(2)=\sum_{n=0}^{7}x(n)e^{\frac{-j\pi n(2)}{4}}=\sum_{n=0}^{7}x(n)e^{\frac{-j\pi n}{2}}$$

$$X(2)=x(0)+x(1)e^{\frac{-j\pi}{2}}+x(2)e^{\frac{-j2\pi}{2}}+x(3)e^{\frac{-j3\pi}{2}}+x(4)e^{\frac{-j4\pi}{2}}$$

$$+x(5)e^{\frac{-j5\pi}{2}}+\cancel{x(6)e^{\frac{-j6\pi}{2}}}+\cancel{x(7)e^{\frac{-j7\pi}{2}}}$$

$$X(2)=1-j-1+j+1-j$$

$$\boxed{X(2)=1-j}$$

Problem-7 cont..

- k=3

$$X(3) = \sum_{n=0}^{7} x(n) e^{\frac{-j\pi n(3)}{4}}$$

$$X(3) = x(0) + x(1)e^{\frac{-j3\pi}{4}} + x(2)e^{\frac{-j6\pi}{4}} + x(3)e^{\frac{-j9\pi}{4}} + x(4)e^{\frac{-j12\pi}{4}}$$

$$+ x(5)e^{\frac{-j15\pi}{4}} + \cancel{x(6)e^{\frac{-j18\pi}{4}}} + \cancel{x(7)e^{\frac{-j21\pi}{4}}}$$

$$\boxed{X(3) = 0.707 + j0.293}$$

Problem-7 cont..

- k=4

$$X(4) = \sum_{n=0}^{7} x(n) e^{\frac{-j\pi n(4)}{4}} = \sum_{n=0}^{7} x(n) e^{-j\pi n}$$

$$X(4) = x(0) + x(1)e^{-j\pi} + x(2)e^{-j2\pi} + x(3)e^{-j3\pi} + x(4)e^{-j4\pi}$$
$$+ x(5)e^{-j5\pi} + x(6)e^{-j6\pi} + x(7)e^{-j7\pi}$$

$$\boxed{X(4) = 0}$$

Problem-7 cont..

- k=5

$$X(5)=\sum_{n=0}^{7}x(n)e^{\frac{-j\pi n(5)}{4}}$$

$$X(5)=x(0)+x(1)e^{\frac{-j5\pi}{4}}+x(2)e^{\frac{-j10\pi}{4}}+x(3)e^{\frac{-j15\pi}{4}}+x(4)e^{\frac{-j20\pi}{4}}$$

$$+x(5)e^{\frac{-j25\pi}{4}}+\cancel{x(6)e^{\frac{-j30\pi}{4}}}+\cancel{x(7)e^{\frac{-j35\pi}{4}}}$$

$$\boxed{X(5)=0.707-j0.293}$$

Problem-7 cont..

- k=6

$$X(6)=\sum_{n=0}^{7} x(n)e^{\frac{-j\pi n(6)}{4}} = \sum_{n=0}^{7} x(n)e^{\frac{-j\pi n(3)}{2}}$$

$$X(6)=x(0)+x(1)e^{\frac{-j3\pi}{2}}+x(2)e^{\frac{-j6\pi}{2}}+x(3)e^{\frac{-j9\pi}{2}}+x(4)e^{\frac{-j12\pi}{2}}$$

$$+x(5)e^{\frac{-j15\pi}{2}}+\cancel{x(6)e^{\frac{-j18\pi}{2}}}+\cancel{x(7)e^{\frac{-j21\pi}{2}}}$$

$$\boxed{X(6)=1+j}$$

Problem-7 cont..

- k=7

$$X(7) = \sum_{n=0}^{7} x(n) e^{\frac{-j\pi n(7)}{4}}$$

$$X(7) = x(0) + x(1)e^{\frac{-j7\pi}{4}} + x(2)e^{\frac{-j14\pi}{4}} + x(3)e^{\frac{-j21\pi}{4}} + x(4)e^{\frac{-j28\pi}{4}}$$
$$+ x(5)e^{\frac{-j35\pi}{4}} + \cancel{x(6)e^{\frac{-j42\pi}{4}}} + \cancel{x(7)e^{\frac{-j49\pi}{4}}}$$

$$\boxed{X(7) = -0.707 - j0.293}$$

Problem-7 cont..

- Final answer:
- X(k)={6,

  -0.707-j1.707,

  1-j,

  0.707+j0.293,

  0,

  0.707-j0.293,

  1+j,

  -0.707+j1.707}
- X(k)={6,-0.707-j1.707,1-j, 0.707+j0.293, 0,0.707-j0.293,1+j, -0.707+j1.707}

Problem-8

- Find IDFT of the sequence

  X(k)={5,0,1-j,0,1,0,1+j,0} here N=8

$$x(n)=\frac{1}{N}\sum_{k=0}^{N-1}X(k)e^{j\frac{2\pi nk}{N}},\ 0\leq n\leq N-1$$

$$x(n)=\frac{1}{8}\sum_{k=0}^{7}X(k)e^{j\frac{2\pi kn}{8}},\ 0\leq n\leq 7$$

Problem-8 Cont..

- n=0

$$x(0)=\frac{1}{8}\left[X(0)+X(1)+X(2)+X(3)+X(4)+X(5)+X(6)+X(7)\right]$$

$$x(0)=\frac{1}{8}\left[5+0+1-j+0+1+0+1+j+0\right]$$

$$x(0)=\frac{1}{8}\left[8\right] \qquad \boxed{x(0)=1}$$

Problem-8 Cont..

- n=1,

$$x(1)=\frac{1}{8}\sum_{k=0}^{7}X(k)e^{j\frac{2\pi k(1)}{8}} = \frac{1}{8}\sum_{k=0}^{7}X(k)e^{j\frac{\pi k}{4}}$$

$$x(1)=\frac{1}{8}\begin{bmatrix} X(0)+X(1)e^{j\frac{\pi}{4}}+X(2)e^{j\frac{2\pi}{4}}+X(3)e^{j\frac{3\pi}{4}}+X(4)e^{j\frac{4\pi}{4}} \\ +X(5)e^{j\frac{5\pi}{4}}+X(6)e^{j\frac{6\pi}{4}}+X(7)e^{j\frac{7\pi}{4}} \end{bmatrix}$$

$$\boxed{x(1)=0.75}$$

Problem-8 **Cont..**

- Similarly $\boxed{x(2)=0.5}$ $\boxed{x(3)=0.25}$ $\boxed{x(4)=1}$

  $\boxed{x(5)=0.75}$ $\boxed{x(6)=0.5}$ $\boxed{x(7)=0.25}$

- Final Answer:
- x(n)={1,0.75,0.5,0.25,1,0.75,0.5,0.25}