Behaviour of Plane waves at the inteface of two media:

We consider ε, μ, σ the propagation of uniform plane waves in an unbounded homogeneous medium. In practice, the wave will propagate in bounded regions where several values of will be present. When plane wave travelling in one medium meets a different medium, it is partly reflected and partly transmitted. In this section, we consider wave reflection and transmission at planar boundary between two media as shown in figure 1.1.

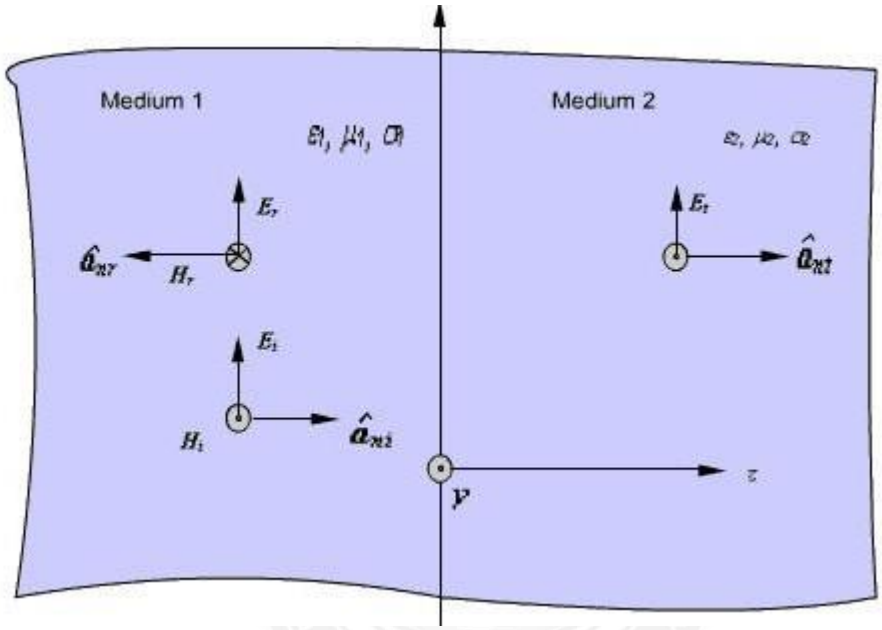

Fig 1.1 : Normal Incidence at a plane boundary
(www.brainkart.com/subject/Electromagnetic-Theory_206/)

Case1: Let z = 0 plane represent the interface between two media.

Medium 1is $(\varepsilon_1, \mu_1, \sigma_1)$ and medium 2 is characterized by $(\varepsilon_2, \mu_2, \sigma_2)$.

Let the subscripts '*i*' denotes incident, '*r*' denotes reflected and '*t*' denotes transmittedfield components respectively.

The incident wave is assumed to be a plane wave polarized along *x* and travelling inmedium

1 along $\hat{a}_z$ direction. From equation (5.24) we can write

$$\vec{E}_i(z) = E_{io} e^{-\gamma_1 z} \hat{a}_x \quad \text{.................(5.49.a)}$$

$$\vec{H}_i(z) = \frac{1}{\eta_i} \hat{a}_z \times E_{io}(z) = \frac{E_o}{\eta_i} e^{-\gamma_1 z} \hat{a}_y \quad \text{......................(5.49.b)}$$

where $\gamma_1 = \sqrt{j\omega\mu_1(\sigma_1 + j\omega\varepsilon_1)}$ and $\eta_1 = \sqrt{\dfrac{j\omega\mu_1}{\sigma_1 + j\omega\varepsilon_2}}$.

Because of the presence of the second medium at z =0, the incident wave will undergo partial reflection and partial transmission.
The reflected wave will travel along $\hat{a}_z$ in medium

1. The reflected field components are:

$$\vec{E}_r = E_{ro} e^{\gamma_1 z} \hat{a}_x \quad \text{...(5.50a)}$$

$$\vec{H}_r = \frac{1}{\eta_1}\left(-\hat{a}_z\right) \times E_{ro} e^{\gamma_1 z} \hat{a}_x = -\frac{E_{ro}}{\eta_1} e^{\gamma_1 z} \hat{a}_y \quad \text{.........(5.50b)}$$

The transmitted wave will travel in medium 2 along $\hat{a}_z$ for which the field components are

$$\vec{E}_t = E_{t0} e^{-\gamma_2 z} \hat{a}_x \quad \text{...(5.51a)}$$

$$\vec{H}_t = \frac{E_{t0}}{\eta_2} e^{-\gamma_2 z} \hat{a}_y \quad \text{...(5.51b)}$$

where $\gamma_2 = \sqrt{j\omega\mu_2(\sigma_2 + j\omega\varepsilon_2)}$ and $\eta_2 = \sqrt{\dfrac{j\omega\mu_2}{\sigma_2 + j\omega\varepsilon_2}}$

In medium 1,

$$\vec{E}_1 = \vec{E}_i + \vec{E}_r \text{ and } \vec{H}_1 = \vec{H}_i + \vec{H}_r$$

and in medium 2,

$$\vec{E}_2 = \vec{E}_t \text{ and } \vec{H}_2 = \vec{H}_t$$

Applying boundary conditions at the interface z = 0, i.e., continuity of tangential field components and noting that incident, reflected and transmitted field components are tangential at the boundary, we can write

$$\vec{E}_i(0) + \vec{E}_r(0) = \vec{E}_i(0)$$

$$\& \quad \vec{H}_i(0) + \vec{H}_r(0) = \vec{H}_t(0)$$

From equation 5.49 to 5.51 we get,

$$E_{io} + E_{ro} = E_{to}$$ ....(5.52a)

$$\frac{E_{io}}{\eta_1} - \frac{E_{ro}}{\eta_1} = \frac{E_{to}}{\eta_2}$$ ....(5.52b)

Eliminating
Eto ,

$$\frac{E_{io}}{\eta_1} - \frac{E_{ro}}{\eta_1} = \frac{1}{\eta_2}\left(E_{io} + E_{ro}\right)$$

or, $$E_{io}\left(\frac{1}{\eta_1} - \frac{1}{\eta_2}\right) = E_{ro}\left(\frac{1}{\eta_1} + \frac{1}{\eta_2}\right)$$

or, $$E_{ro} = \tau E_{io}$$

...............(5.53)

$$\tau = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

is called the reflection coefficient.

From equation (5.52), we can write

$$2E_{io} = E_{to}\left[1 + \frac{\eta_1}{\eta_2}\right]$$

or, $$T = \frac{2\eta_2}{\eta_1 + \eta_2}$$ ....(5.54)

$$E_{to} = \frac{2\eta_2}{\eta_1 + \eta_2}E_{io} = TE_{io}$$

is called the transmission coefficient. We observe that,

$$T = \frac{2\eta_2}{\eta_1 + \eta_2} = \frac{\eta_2 - \eta_1 + \eta_1 + \eta_2}{\eta_1 + \eta_2} = 1 + \tau$$ ....(5.55)

The following may be noted

Let us now consider specific cases:

Case I: Normal incidence on a plane conducting boundary

The medium 1 is perfect dielectric $(\sigma_1 = 0)$ and medium 2 is perfectly conducting $(\sigma_2 = \infty)$.

$$\therefore \eta_1 = \sqrt{\frac{\mu_1}{\epsilon_1}}$$

$$\eta_2 = 0$$

$$\gamma_1 = \sqrt{(j\omega\mu_1)(j\omega\epsilon_1)}$$
$$= j\omega\sqrt{\mu_1\epsilon_1} = j\beta_1$$

From (5.53) and (5.54)

$$\tau = -1$$

and T =0

Hence the wave is not transmitted to medium 2, it gets reflected entirely from the interface to the medium 1.

$$\therefore \vec{E}_1(z) = E_{io}e^{-j\beta_1 z}\hat{a}_x - E_{io}e^{j\beta_1 z}\hat{a}_x = -2jE_{io}\sin\beta_1 z\,\hat{a}_x$$

&

$$\therefore \vec{E}_1(z,t) = \mathrm{Re}\left[-2jE_{io}\sin\beta_1 z e^{j\omega t}\right]\hat{a}_x = 2E_{io}\sin\beta_1 z\sin\omega t\,\hat{a}_x \quad \text{.................................(5.55)}$$

Proceeding in the same manner for the magnetic field in region 1, we can show that,

$$\vec{H}_1(z,t) = \hat{a}_y\frac{2E_{io}}{\eta_1}\cos\beta_1 z\cos\omega t \quad \text{...(5.57)}$$

The wave in medium 1 thus becomes a **standing wave** due to the super position of a forward travelling wave and a backward travelling wave. For a given *' t'*, both $\vec{E}_1$ and $\vec{H}_1$ vary sinusoidally with distance measured from z = 0. This is shown in figure 1.2.

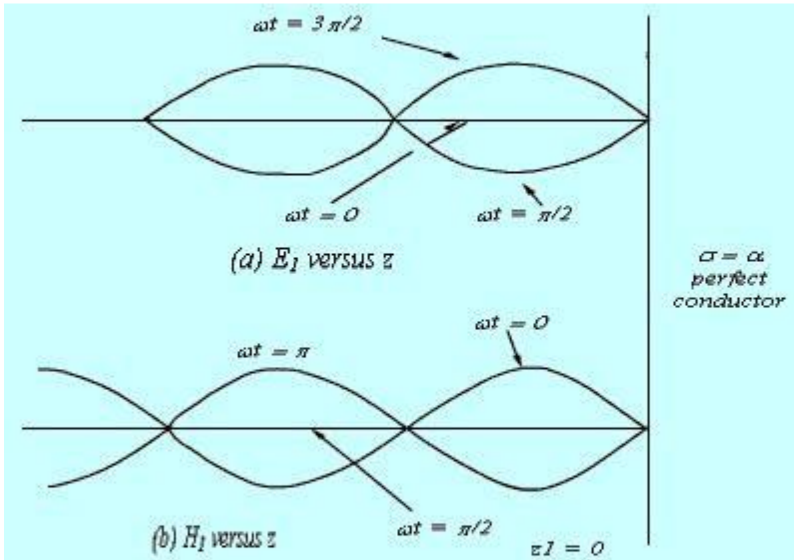

Figure 1.2: Generation of standing wave
(www.brainkart.com/subject/Electromagnetic-Theory_206/)

Zeroes of $E1(z,t)$ and Maxima of $H1(z,t)$.

Maxima of $E1(z,t)$ and zeroes of $H1(z,t)$.

$$\left.\begin{matrix}\text{occur at } \beta_1 z = -n\pi \quad \text{or } z = -n\dfrac{\lambda}{2} \\ \text{occur at } \beta_1 z = -(2n+1)\dfrac{\pi}{2} \quad \text{or } z = -(2n+1)\dfrac{\lambda}{4}, \quad n = 0,1,2,\ldots\end{matrix}\right\}$$

.......(5.58)

Case2: Normal incidence on a plane dielectric boundary

If the medium 2 is not a perfect conductor (i.e. $\sigma_2 \neq \infty$) partial reflection will result. There will be a reflected wave in the medium 1 and a transmitted wave in the medium 2.Because of the reflected wave, standing wave is formed in medium 1.
From equation (5.49(a)) and equation (5.53) we can write

$$\vec{E}_1 = E_{io}\left(e^{-\gamma_1 z} + \Gamma e^{\gamma_1 z}\right)\hat{a}_x$$ ....(5.59)

Let us consider the scenario when both the media are dissipation less i.e. perfect dielectrics ($\sigma_1 = 0, \sigma_2 = 0$)

$$\gamma_1 = j\omega\sqrt{\mu_1\varepsilon_1} = j\beta_1 \qquad \eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}}$$

$$\gamma_2 = j\omega\sqrt{\mu_2\varepsilon_2} = j\beta_2 \qquad \eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}}$$ ....(5.50)

In this case both η_1 and η_1 become real numbers.

$$\begin{aligned}\vec{E}_1 &= \hat{a}_x E_{io}\left(e^{-j\beta_1 z} + \Gamma e^{j\beta_1 z}\right)\\ &= \hat{a}_x E_{io}\left((1+T)e^{-j\beta_1 z} + \Gamma\left(e^{j\beta_1 z} - e^{-j\beta_1 z}\right)\right)\\ &= \hat{a}_x E_{io}\left(Te^{-j\beta_1 z} + \Gamma\left(2j\sin\beta_1 z\right)\right)\end{aligned}$$ ....(5.51)

From (5.51), we can see that, in medium 1 we have a traveling wave component with amplitude TE_{io} and a standing wave component with amplitude $2JE_{io}$.

The location of the maximum and the minimum of the electric and magnetic field components in the medium 1from the interface can be found as follows.

The electric field in medium 1 can be written as

$$\vec{E}_1 = \hat{a}_x E_{io} e^{-j\beta_1 z}\left(1 + \Gamma e^{j2\beta_1 z}\right)$$ ....(5.52)

If $\eta_2 > \eta_1$ i.e. $\Gamma > 0$

The maximum value of the electric field is

$$\left|\vec{E}_1\right|_{max} = E_{io}\left(1+T\right)$$ ....(5.53)

and this occurs when

$$2\beta_1 z_{max} = -2n\pi$$

or $$z_{max} = -\frac{n\pi}{\beta_1} = -\frac{n\pi}{2\pi/\lambda_1} = -\frac{n}{2}\lambda_1$$, n = 0, 1, 2, 3 .... (5.54)

The minimum value of $|\vec{E}_1|$ is

$$|\vec{E}_1|_{min} = E_{io}(1-\Gamma)$$ ....(5.55)

And this occurs when

$$2\beta_1 z_{min} = -(2n+1)\pi$$

or $$z_{min} = -(2n+1)\frac{\lambda_1}{4}$$, n = 0, 1, 2,

Γ $\eta_2 < \eta_1$ i.e. <3.............................(5.55) For

The maximum value of $|\vec{E}_1|$ is $E_{io}(1-\Gamma)$ which occurs at the z_{min} locations and the minimum

value of $|\vec{E}_1|$ is $E_{io}(1+\Gamma)$ which occurs at z_{max} locations as given by the equations (5.54) and (5.55).

From our discussions so far we observe that $\frac{|E|_{max}}{|E|_{min}}$ can be written as

$$S = \frac{|E|_{max}}{|E|_{min}} = \frac{1+|\Gamma|}{1-|\Gamma|}$$ ....(5.57)

The quantity S is called as the standing wave ratio.

As $0 \le |\Gamma| \le 1$ the range of S is given $1 \le S \le \infty$ by

From (5.52), we can write the expression for the magnetic field in medium 1 as

$$\vec{H}_1 = \hat{a}_y \frac{E_{io}}{\eta_1} e^{-j\beta_1 z}\left(1-\Gamma e^{j2\beta_1 z}\right)$$ ....(5.58)

From (5.58) we find that $|\vec{H}_1|$ will be maximum at locations where $|\vec{E}_1|$ is minimum and vice versa.
In medium 2, the transmitted wave propagates in the + z direction.

Oblique Incidence of EM wave at an interface

So far we have discuss the case of normal incidence where electromagnetic wave traveling in a lossless medium impinges normally at the interface of a second medium. In this section we shall consider the case of oblique incidence. As before, we consider two cases

i. When the second medium is a perfect conductor.
ii. When the second medium is a perfect dielectric.

A plane incidence is defined as the plane containing the vector indicating the direction of propagation of the incident wave and normal to the interface. We study two specific cases when the incident electric field $\vec{E}_i$ is perpendicular to the plane of incidence (perpendicular polarization) and $\vec{E}_i$ is parallel to the plane of incidence (parallel polarization). For a general case, the incident wave may have arbitrary polarization but the same can be expressed as a linear combination of these two individual cases.

Oblique Incidence at a plane conducting boundary i. Perpendicular Polarization

The situation is depicted in figure 1.3.

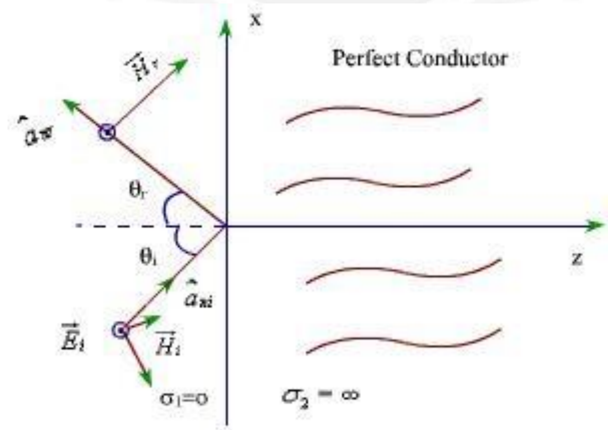

Figure1.3:Perpendicularpolarization
(www.brainkart.com/subject/Electromagnetic-Theory_206/)

As the EM field inside the perfect conductor is zero, the interface reflects the incident plane wave. $\hat{a}_{ni}$ and $\hat{a}_{nr}$ respectively represent the unit vector in the direction of propagation of the incident and reflected waves, θ_i is the angle of incidence and θ_r is the angle of reflection.

We find that

$$\hat{a}_{ni} = \hat{a}_z \cos\theta_i + \hat{a}_x \sin\theta_i$$
$$\hat{a}_{nr} = -\hat{a}_z \cos\theta_r + \hat{a}_x \sin\theta_r \quad \text{.............................(5.59)}$$

Since the incident wave is considered to be perpendicular to the plane of incidence, which for the present case happens to be xz plane, the electric field has only y-component.

$$\vec{E}_i(x,z) = \hat{a}_y E_{io} e^{-j\beta_1 \bar{a}_{ni}\cdot\bar{r}}$$
$$= \hat{a}_y E_{io} e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$$

The corresponding magnetic field is given by

$$\vec{H}_i(x,z) = \frac{1}{\eta_1}\left[\hat{a}_n \times \vec{E}_i(x,z)\right]$$
$$= \frac{1}{\eta_1}\left[-\cos\theta_i \hat{a}_x + \sin\theta_i \hat{a}_z\right] E_{io} e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)} \quad \text{...........................(5.70)}$$

Similarly, we can write the reflected waves as

$$\vec{E}_r(x,z) = \hat{a}_y E_{ro} e^{-j\beta_1 \bar{a}_{nr}\cdot\bar{r}}$$
$$= \hat{a}_y E_{ro} e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)} \quad \text{..(5.71)}$$

Since at the interface z=o, the tangential electric field is zero.

$$E_{io} e^{-j\beta_1 x\sin\theta_i} + E_{ro} e^{-j\beta_1 x\sin\theta_r} = 0 \quad \text{...(5.72)}$$

Consider in equation (5.72) is satisfied if we have

$$E_{ro} = -E_{io}$$
$$\text{and } \theta_i = \theta_r \quad \text{..................................(5.73)}$$

The condition $\theta_i = \theta_r$ is Snell's law of reflection.

$$\therefore \vec{E}_r(x,z) = -\hat{a}_y E_{io} e^{-j\beta_1(x\sin\theta_i - z\cos\theta_i)} \quad \text{..................................(5.74)}$$

The total electric field is given by

$$\vec{E}_1(x,z) = \vec{E}_i(x,z) + \vec{E}_r(x,z)$$
$$= -\hat{a}_y 2jE_{io}\sin(\beta_1 z\cos\theta_i)e^{-j\beta_1 x\sin\theta_i} \qquad \text{..................................(5.75)}$$

$$\text{and } \vec{H}_r(x,z) = \frac{1}{n_1}\left[\hat{a}_{nr}\times\vec{E}_r(x,z)\right] \qquad \text{....................(5.76)}$$

Sim

$$= \frac{E_{io}}{n_1}\left[-\hat{a}_x\cos\theta_i - \hat{a}_z\sin\theta_i\right]e^{-j\beta_1(x\sin\theta_i - z\cos\theta_i)} \qquad \text{............................(5.77)}$$

$$\vec{H}_1(x,z) = -2\frac{E_{io}}{n_1}\left[\hat{a}_x\cos\theta_i\cos(\beta_1 z\cos\theta_i)e^{-j\beta_1 x\sin\theta_i} + \hat{a}_z j\sin\theta_i\sin(\beta_1 z\cos\theta_i)e^{-j\beta_1 x\sin\theta_i}\right] \qquad \text{..............................(5.78)}$$

The wave propagating along the x direction has its amplitude varying with z and hence constitutes a **non uniform** plane wave. Further, only electric field $\hat{E}_1$ is perpendicular to the direction of propagation (i.e. x), the magnetic field has component along the direction of propagation. Such waves are called transverse electric or TE waves.

ii. **Parallel Polarization**:

In this case also $\hat{a}_{ni}$ and $\hat{a}_{nr}$ are given by equations (5.59). Here $\vec{H}_1$ and $\vec{H}_r$ have only y component.

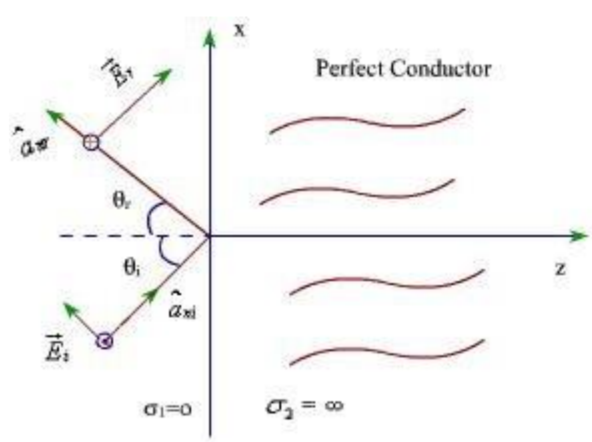

Figure 1.4: Parallel *Polarization*

(www.brainkart.com/subject/Electromagnetic-Theory_206/)

With reference to fig (1.4), the field components can be written

as: Incident field components:

$$\vec{E}_i(x,z) = E_{io}\left[\cos\theta_i \hat{a}_x - \sin\theta_i \hat{a}_z\right] e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$$

$$\vec{H}_i(x,z) = \hat{a}_y \frac{E_{io}}{n_1} e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$$

............................(5.79)

Reflected field components:

$$\vec{E}_r(x,z) = E_{ro}\left[\hat{a}_x \cos\theta_r + \hat{a}_z \sin\theta_r\right] e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}$$

$$\vec{H}_r(x,z) = -\hat{a}_y \frac{E_{ro}}{n_1} e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}$$

............................(5.80)

Since the total tangential electric field component at the interface is zero.

$$E_i(x,0) + E(x,0) = 0$$

Which leads to $E_{io} = -E_{ro}$ and $\theta_i = \theta_r$ as before.

Substituting these quantities in (5.79) and adding the incident and reflected electric and magnetic field components the total electric and magnetic fields can be written as

$$\vec{E}_i(x,z) = -2E_{io}\left[\hat{a}_x j\cos\theta_i \sin(\beta_1 z\cos\theta_i) + \hat{a}_z \sin\theta_i \cos(\beta_1 z\cos\theta_i)\right] e^{-j\beta_1 x\sin\theta_i}$$

and $$\vec{H}_i(x,z) = \hat{a}_y \frac{2E_{io}}{n_1} \cos(\beta_1 z\cos\theta_i) e^{-j\beta_1 x\sin\theta_i}$$

...........................(5.81)

Once again, we find a standing wave pattern along z for the x and y components of $\vec{E}$ and $\vec{H}$, while a non uniform plane wave propagates along x with a phase velocity given $v_{p1x} = \frac{vp_1}{\sin\theta_i}$ by where $v_{p1} = \frac{\omega}{\beta_1}$. Since, for this propagating wave, magnetic field is in transverse direction, such waves are called transverse magnetic or TM waves.

Oblique incidence at a plane dielectric interface

We continue our discussion on the behavior of plane waves at an interface; this time we consider a plane dielectric interface. As earlier, we consider the two specific cases, namely parallel and perpendicular polarization.

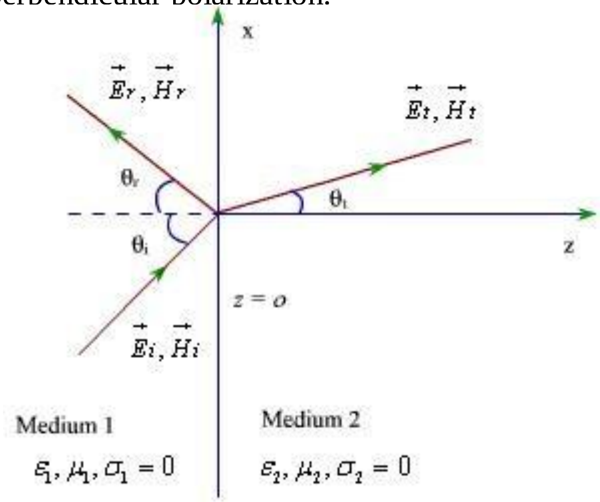

Fig 1.5: Oblique incidence at a plane dielectric interface
(www.brainkart.com/subject/Electromagnetic-Theory_206/)

For the case of a plane dielectric interface, an incident wave will be reflected partially and transmitted partially.

In Fig(1.5), θ_i, θ_0 and θ_t corresponds respectively to the angle of incidence, reflection and transmission.

1. **Parallel Polarization**

As discussed previously, the incident and reflected field components can be written as

$$\vec{E}_i(x,z) = E_{io}\left[\cos\theta_i \hat{a}_x - \sin\theta_i \hat{a}_z\right] e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$$

$$\vec{H}_i(x,z) = \hat{a}_y \frac{E_{io}}{\eta_1} e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$$

.... (5.82)

$$\vec{E}_r(x,z) = E_{ro}\left[\hat{a}_x \cos\theta_r + \hat{a}_z \sin\theta_r\right] e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}$$

$$\vec{H}_r(x,z) = -\hat{a}_y \frac{E_{ro}}{\eta_1} e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}$$

..........................(5.83)

In terms of the reflection coefficient Γ

$$\vec{E}_r(x,z) = \Gamma E_{io}\left[\hat{a}_x \cos\theta_r + \hat{a}_z \sin\theta_r\right] e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}$$

$$\vec{H}_r(x,z) = -\hat{a}_y \frac{\Gamma E_{io}}{n_1} e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}$$

. ...(5.84)

The transmitted filed can be written in terms of the transmission coefficient *T*

$$\vec{E}_t(x,z) = TE_{io}\left[\hat{a}_x \cos\theta_t - \hat{a}_z \sin\theta_t\right] e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}$$

$$\vec{H}_t(x,z) = \hat{a}_y \frac{TE_{io}}{n_2} e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}$$

..........................(5.85)

We can now enforce the continuity of tangential field components at the boundary i.e. *z=0*

$$\cos\theta_i e^{-j\beta_1 x\sin\theta_i} + \Gamma\cos\theta_r e^{-j\beta_1 x\sin\theta_r} = T\cos\theta_t e^{-j\beta_2 x\sin\theta_t}$$

$$\text{and } \frac{1}{n_1} e^{-j\beta_1 x\sin\theta_i} - \frac{\Gamma}{n_1} e^{-j\beta_1 x\sin\theta_r} = \frac{T}{n_2} e^{-j\beta_2 x\sin\theta_t}$$

..........................(5.85)

If both E_x and H_y are to be continuous at *z=0* for all *x* , then form the phase matching we have

$$\beta_1 \sin\theta_i = \beta_1 \sin\theta_r = \beta_2 \sin\theta_t$$

$\therefore$ We find that

$$\theta_i = \theta_r$$

$$\text{and } \beta_1 \sin\theta_i = \beta_2 \sin\theta_t$$

..........................(5.87)

Further, from equations (5.85) and (5.87) we have

$$\cos\theta_i + \Gamma\cos\theta_i = T\cos\theta_t$$

$$\text{and } \frac{1}{n_1} - \frac{\Gamma}{n_1} = \frac{T}{n_2}$$

..........................(5.88)

$$\therefore \quad \cos\theta_i(1+\Gamma) = T\cos\theta_t$$

$$\text{and } \frac{1}{n_1}(1-\Gamma) = \frac{T}{n_2}$$

$$\therefore T = \frac{n_2}{n_1}(1-\Gamma)$$

$$\cos\theta_i(1+\Gamma) = \frac{n_2}{n_1}(1-\Gamma)\cos\theta_t$$

$$\therefore (n_1\cos\theta_i + n_2\cos\theta_t)\Gamma = n_2\cos\theta_t - n_1\cos\theta_i$$

or $$\Gamma = \frac{n_2\cos\theta_t - n_1\cos\theta_i}{n_2\cos\theta_t + n_1\cos\theta_i}$$ ....(5.89)

and $$T = \frac{n_2}{n_1}(1-\Gamma)$$

$$= \frac{2n_2\cos\theta_i}{n_2\cos\theta_t + n_1\cos\theta_i}$$ ....(5.90)

From equation (5.90) we find that there exists specific angle $\theta_i = \theta_b$ for which $\Gamma = 0$ such that

$$n_2\cos\theta_t = n_1\cos\theta_b$$

or $$\sqrt{1-\sin^2\theta_t} = \frac{n_1}{n_2}\sqrt{1-\sin^2\theta_b}$$ ....(5.91)

Further, $$\sin\theta_t = \frac{\beta_1}{\beta_2}\sin\theta_b$$

$\mu_1 = \mu_2 = \mu_0$ ....(5.92)

For non magnetic material
Using this condition

$$1-\sin^2\theta_t = \frac{\varepsilon_1}{\varepsilon_2}(1-\sin^2\theta_b)$$

and $$\sin^2\theta_t = \frac{\varepsilon_1}{\varepsilon_2}\sin^2\theta_b$$ ....(5.93)

From equation (5.93), solving for $\sin\theta_b$ we get

$$\sin\theta_b = \frac{1}{\sqrt{1+\frac{\varepsilon_1}{\varepsilon_2}}}$$

This angle of incidence for which $\Gamma = 0$ is called Brewster angle. Since we are dealing with parallel polarization we represent this angle by $\theta_{b\parallel}$ so that

$$\sin\theta_{b\|} = \frac{1}{\sqrt{1+\frac{\varepsilon_1}{\varepsilon_2}}}$$

2. Perpendicular Polarization

For this case

$$\vec{E}_i(x,z) = \hat{a}_y E_{io} e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$$

$$\vec{H}_i(x,z) = \frac{E_{io}}{\eta_1}\left[-\hat{a}_x\cos\theta_i + \hat{a}_z\sin\theta_i\right]e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)} \quad \text{.........................(5.94)}$$

$$\vec{E}_r(x,z) = \hat{a}_y \Gamma E_{io} e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}$$

$$\vec{H}_r(x,z) = \frac{\Gamma E_{io}}{\eta_1}\left[\hat{a}_x\cos\theta_r + \hat{a}_z\sin\theta_r\right]e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)} \quad \text{.........................(5.95)}$$

$$\vec{E}_t(x,z) = \hat{a}_y T E_{io} e^{-j\beta_1(x\sin\theta_t + z\cos\theta_t)}$$

$$\vec{H}_t(x,z) = \frac{TE_{io}}{\eta_2}\left[-\hat{a}_x\cos\theta_t + \hat{a}_z\sin\theta_t\right]e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)} \quad \text{.........................(5.95)}$$

Using continuity of field components at *z=0*

$$e^{-j\beta_1 x\sin\theta_i} + \Gamma e^{-j\beta_1 x\sin\theta_r} = TE_{io}e^{-j\beta_2 x\sin\theta_t}$$

$$\text{and } -\frac{1}{\eta_1}\cos\theta_i e^{-j\beta_1 x\sin\theta_i} + \frac{\Gamma}{\eta_1}\cos\theta_r e^{-j\beta_1 x\sin\theta_r} = -\frac{T}{\eta_2}\cos\theta_t e^{-j\beta_2 x\sin\theta_t} \quad \text{.........................(5.97)}$$

As in the previous case

$$\beta_1\sin\theta_i = \beta_1\sin\theta_r = \beta_2\sin\theta_t$$

$$\therefore \quad \theta_i = \theta_r$$

$$\text{and } \sin\theta_t = \frac{\beta_1}{\beta_2}\sin\theta_i \quad \text{.........................(5.98)}$$

Using these conditions we can write

$$1+\Gamma = T$$

$$-\frac{\cos\theta_i}{\eta_1} + \frac{\Gamma\cos\theta_i}{\eta_1} = -\frac{T\cos\theta_t}{\eta_2} \quad \text{.........................(5.99)}$$

From equation (5.99) the reflection and transmission coefficients for the perpendicular polarization can be computed as

$$\Gamma = \frac{n_2 \cos\theta_i - n_1 \cos\theta_t}{n_2 \cos\theta_i + n_1 \cos\theta_t}$$

$$\text{and} \quad T = \frac{2n_2 \cos\theta_i}{n_2 \cos\theta_i + n_1 \cos\theta_t} \quad \text{..........................(5.100)}$$

We observe that if $\Gamma = 0$ for an angle of incidence $\theta_i = \theta_b$

$$n_2 \cos\theta_b = n_1 \cos\theta_t$$

$$\therefore \cos^2\theta_t = \frac{n_2}{n_1}\cos^2\theta_b$$

$$= \frac{\mu_2\varepsilon_1}{\mu_1\varepsilon_2}\cos^2\theta_b$$

$$\therefore 1 - \sin^2\theta_t = \frac{\mu_2\varepsilon_1}{\mu_1\varepsilon_2}\left(1 - \sin^2\theta_b\right)$$

Again $$\sin\theta_t = \frac{\beta_1}{\beta_2}\sin\theta_b$$

$$\therefore \sin^2\theta_t = \frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}\sin^2\theta_b$$

$$\therefore \left(1 - \frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}\sin^2\theta_b\right) = \frac{\mu_2\varepsilon_1}{\mu_1\varepsilon_2} - \frac{\mu_2\varepsilon_1}{\mu_1\varepsilon_2}\sin^2\theta_b$$

or $$\sin^2\theta_b\left(\frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2} - \frac{\mu_2\varepsilon_1}{\mu_1\varepsilon_2}\right) = \left(1 - \frac{\mu_2\varepsilon_1}{\mu_1\varepsilon_2}\right)$$

or $$\sin^2\theta_b\left(\frac{\mu_1^2 - \mu_2^2}{\mu_1\mu_2\varepsilon_2}\right)\varepsilon_1 = \left(\frac{\mu_1\varepsilon_2 - \mu_2\varepsilon_1}{\mu_1\varepsilon_2}\right)$$

or $$\sin^2\theta_b = \frac{\mu_2\left(\mu_1\varepsilon_2 - \mu_2\varepsilon_1\right)}{\varepsilon_1\left(\mu_1^2 - \mu_2^2\right)} \quad \text{.. (5.101)}$$

We observe if $\mu_1 = \mu_2 = \mu_0$ i.e. in this case of non magnetic material Brewster angle does not exist as the denominator or equation (5.101) becomes zero. Thus for perpendicular polarization in dielectric media, there is Brewster angle so that Γ can be made equal to zero.

From our previous discussion we observe that for bothpolarizations

$$\sin\theta_t = \frac{\beta_1}{\beta_2}\sin\theta_i$$

$\mu_1 = \mu_2 = \mu_0$

$$\sin\theta_t = \sqrt{\frac{\varepsilon_1}{\varepsilon_2}}\sin\theta_i$$

For $\varepsilon_1 > \varepsilon_2$, $\theta_t > \theta_i$

The incidence angle $\theta_i = \theta_r$ for which $\theta_t = \frac{\pi}{2}$ i.e. $\theta_c = \sin^{-1}\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$ is called the critical angle ofincidence.