****

Overflow Limit cycle oscillations:

***What are called overflow oscillations? How it can be prevented?**

- We know that the limit cycle oscillation is caused by rounding the result of multiplication.
- The limit cycle occurs due to the overflow of adder is known as overflow limit cycle oscillations.\
- Several types of limit cycle oscillations are caused by addition, which makes the filter output oscilate between maximum and minimum amplitudes.
- Let us consider 2 positive numbers n_1 & n_2

  n_1=0.111→7/8

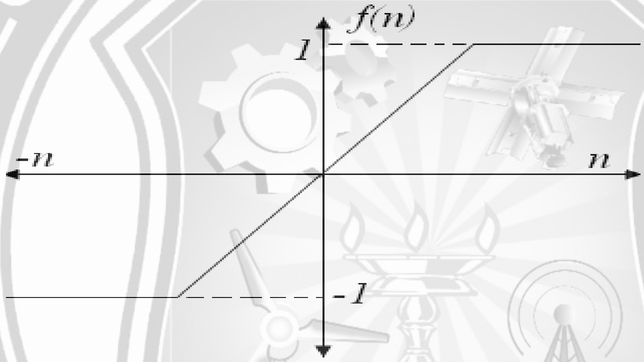
  n_2=0.110→6/8

  n_1 + n2=1.101→-5/8 in sign magnitude form.

  The sum is wrongly interpreted as a negative number.
- The transfer characteristics of an saturation adder is shown in fig below

  where n → The input to the adder

  f(n) → The corresponding output



Saturation adder transfer characteristics

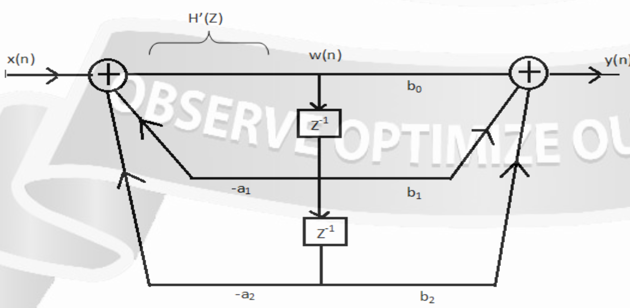
- From the transfer characteristics, we find that when overflow occurs, the sum of adder is set equal to the maximum value.

****

Signal Scaling:

***Explain how reduction of round-off errors is achieved in digital filters. [Nov/Dec-2016]**

- Saturation arithmetic eliminates limit cycles due to overflow, but it causes undeniable signal distortion due to the non linearity of the clipper.
- In order to limit the amount of non linear distortion, it is important to scale input signal and unit sample response between input and any internal summing node in the system to avoid overflow.



Realization of a second order IIR Filter

- Let us consider a second order IIR filter as shown in the above figure. Here a scale factor S_0 is introduced between the input x(n) and the adder 1 to prevent overflow at the output of adder 1.
- Now the overall input-output transfer function is

Now the transfer function

$$H(z) = S_0 \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

$$= S_0 \frac{N(z)}{D(z)}$$

From figure

$$H^{'}(z) = \frac{W(z)}{X(z)} = \frac{S_0}{1 + a_1 z^{-1} + a_2 z^{-2}} = \frac{S_0}{D(z)}$$

$$W(z) = \frac{S_0 X(z)}{D(z)} = S_0 \, \mathrm{S(z)\,x(Z)}$$

Where $S(z) = \frac{1}{D(z)}$

we have

$$w(n) = \frac{S_0}{2\pi} \int S(e^{j\theta}) X(e^{j\theta})(e^{jn\theta}) \, d\theta$$

$$\mathrm{w(n)}\ ^2 = \frac{S_0^2}{2\pi^2} \left| \int S(e^{j\theta}) X(e^{j\theta})(e^{jn\theta}) \, d\theta \right|^2$$

Using Schwartz inequality

$$\mathrm{w(n)}\ ^2 \le S_0^2 \left[\int_{2\pi} \left| S(e^{j\theta}) \right|^2 d\theta \right] \left[\int_{2\pi} \left| X(e^{j\theta}) \right|^2 d\theta \right]$$

Applying parsevals theorem

$$\mathrm{w(n)}\ ^2 \le S_0^2 \sum_{n=0}^{\infty} x^2(n) \frac{1}{2\pi} \int_{2\pi} \left| S(e^{j\theta}) \right|^2 d\theta$$

if $z = e^{j\theta}$ *then* $dz = je^{j\theta} \, d\theta$

which gives

$$d\theta = \frac{dz}{jz}$$

By substituting all values

$$\mathrm{w(n)}\ ^2 \le S_0^2 \sum_{n=0}^{\infty} x^2(n) \frac{1}{2\pi j} \int_c \left| S(z) \right|^2 z^{-1} dz$$

$$\mathrm{w(n)}\ ^2 \le S_0^2 \sum_{n=0}^{\infty} x^2(n) \frac{1}{2\pi j} \int_c S(z) S(z^{-1}) z^{-1} dz$$

$$w^2(n) \le \sum_{n=0}^{\infty} x^2(n) \quad when$$

$$S_0^2 \frac{1}{2\pi j} \int_c S(z) S(z^{-1}) \, dz = 1$$

Which gives us,

$$S_0^2 = \frac{1}{\frac{1}{2\pi j} \int_c S(z) S(z^{-1}) z^{-1} dz}$$

$$= \frac{1}{\frac{1}{2\pi j} \int_c \frac{z^{-1} dz}{D(z) D(z^{-1})}}$$

$$S_0^2 = \frac{1}{I}$$

Where I=

$$\frac{1}{2\pi j}\int_c \frac{z^{-1}dz}{D(z)D(z^{-1})}$$

Note:

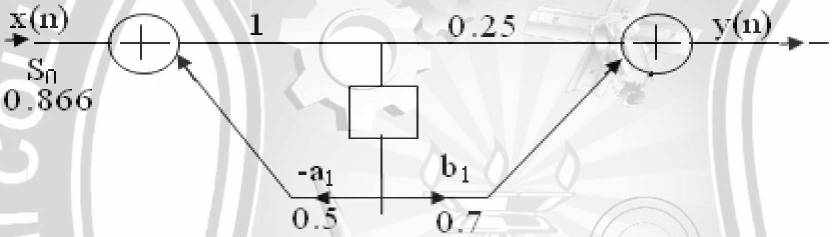
- Because of the process of scaling, the overflow is eliminated. Here so is the scaling factor for the first stage.
- Scaling factor for the second stage = S_{01} and it is given by $S_{01}^2 = \frac{1}{S_0^2 I_2}$

Where $I_2 = \frac{1}{2\pi j}\oint_c \frac{H_1(Z)H_1(Z^{-1})Z^{-1}}{D_2(Z)D_2(Z^{-1})}dZ$

****

For the given transfer function, $H(Z) = \frac{0.25+0.7Z^{-1}}{1-0.5Z^{-1}}$, find scaling factor so as to avoid ********

overflow in
the adder '1' of the filter.



Given:
D(Z) = 1-0.5Z^{-1}
D(Z^{-1}) = 1-0.5 Z
Solution:

$$I = \frac{1}{2\pi j}\oint_c \frac{1}{D(Z)D(Z^{-1})}\frac{dZ}{Z}$$

$$= \frac{1}{2\pi j}\oint_c \frac{1}{(1-0.5Z^{-1})(1-0.5Z)}\frac{dZ}{Z}$$

$$= \frac{1}{2\pi j}\oint_c \frac{Z}{(Z-0.5)}\frac{1}{(1-0.5Z)}\frac{dZ}{Z}$$

Residue of $\left.\frac{Z}{(Z-0.5)}\frac{1}{(1-0.5Z)}\right|_{Z=0.5} + 0$

I=1.3333

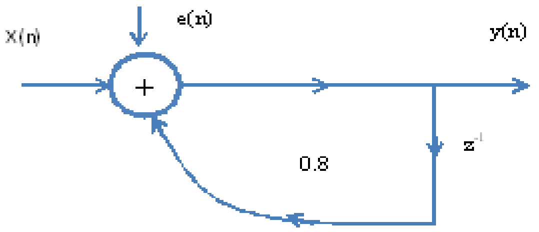
$S_0 = \frac{1}{\sqrt{I}}$

$S_0 = \frac{1}{\sqrt{1.333}}$

= 0.866

****

Consider the recursive filter shown in fig. The input x(n) has a range of values of ±100V, represented by 8 bits. Compute the variance of output due to A/D conversion process. (6)



Solution:

Given the range is ±100V

The difference equation of the system is given by $y(n) = 0.8y(n-1) + x(n)$, whose impulse response h(n) can be obtained as

$$h(n) = (0.8)^n u(n)$$

$$quantization\ step\ size = \frac{range\ of\ the\ signal}{No.of\ quantization\ levels}$$

$$= \frac{200}{2^8}$$

$$= 0.78125$$

Variance of the error signal

$$\sigma_e^2 = \frac{q^2}{12}$$

$$= \frac{(0.78125)^2}{12}$$

$$\sigma_e^2 = 0.05086$$

Variance of output

$$\sigma_{\not\in}^2 = \sigma_e^2 \sum_{n=0}^{\alpha} h^2(n)$$

$$= (0.05086)\sum_{n=0}^{\alpha}(0.8)^{2n}$$

$$= \frac{0.05086}{1-(0.8)^2} = 0.14128$$

****

The input to the system y(n)=0.999y(n-1)+x(n) is applied to an ADC. What is the power produced by the quantization noise at the output of the filter if the input is quantized to a) 8 bits b)16 bits. May-07

Solution:

y(n)=0.999y(n-1)+x(n)

Taking z-transform on both sides

$Y(z)=0.999z^{-1}Y(z)+X(z)$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1-0.999z^{-1}}$$

$$H(z)H(z^{-1})z^{-1} = (\frac{z}{z-0.999})(\frac{z^{-1}}{z^{-1}-0.999})z^{-1}$$

$$= \frac{z^{-1}}{(z-0.999)(-0.999)(z-\frac{1}{0.999})}$$

$$= \frac{-0.001}{(z-0.999)(z-0.001)}$$

$$\left.\begin{matrix}output\ noise\ power\ due\\ to\ input\ quantization\end{matrix}\right\}\sigma_{eoi}^2 = \sigma_e^2 \frac{1}{2\pi j}\int_c H(z)H(z^{-1})z^{-1}dz$$

$$= \sigma_e^2 \sum_{i=1}^{N} \operatorname{Res}\left[H(z)H(z^{-1})z^{-1}\right]\Big|_{z=p_i}$$

$$= \sigma_e^2 \sum_{i=1}^{N} \left[(z = p_i)H(z)H(z^{-1})z^{-1}\right]\Big|_{z=p_i}$$

Where $p_1, p_2, \ldots p_N$ are poles of $H(z)H(z^{-1})z^{-1}$, that lies inside the unit circle in z-plane.

$$\sigma_{eoi^2} = \sigma_{e^2}(z-0.999)\left(\frac{0.001}{(z-0.999)(z-0.001)}\right)\Bigg|_{z=0.999}$$

$$= \sigma_{e^2} 500.25$$

a) b+1=8 bits(Assuming including sign bit)

$$\sigma_\varepsilon^2 = \frac{2^{2(7)}}{12}(500.25) = 2.544\times10^{-3}$$

b) b+1=16 bits

$$\sigma_\varepsilon^2 = \frac{2^{2(15)}}{12}(500.25) = 3.882\times10^{-8}$$

****

Find the effect of coefficient quantization on pole locations of the given second order IIR system, when it is realized in direct form I and in cascade form. Assume a word length of 4 bits through truncation.

$$H(z) = \frac{1}{1-0.9z^{-1}+0.2z^{-2}}$$

Solution:

<u>Direct form I</u>

Let b=4 bits including a sign bit

$(0.9)_{10} = (0.111001...)_2$

Integer part

$\frac{0.9\times2}{1.8} \mapsto$ 1 ↓

$\frac{0.8\times2}{1.6} \mapsto$ 1

$\frac{0.6\times2}{1.2} \mapsto$ 1

$\frac{0.2\times2}{0.4} \mapsto$ 0

$\frac{0.4\times2}{0.8} \mapsto$ 0

$\frac{0.8\times2}{1.6} \mapsto$ 1

After truncation we get

$(0.111)_2=(0.875)_{10}$

$(0.2)_{10}=(0.00110\ldots)_2$

$(0.2)_{10} = \frac{0.2\times 2}{0.4}$

$\mapsto$ 0 $\downarrow$

$\frac{0.4\times 2}{0.8}$

$\mapsto$ 0

$\frac{0.8\times 2}{1.6}$

$\mapsto$ 1

$\frac{0.6\times 2}{1.2}$

$\mapsto$ 1

$\frac{0.2\times 2}{0.4}$

$\mapsto$ 0

After truncation we get

$(0.001)_2=(0.125)_{10}$

The system function after coefficient quantization is

$$H(z) = \frac{1}{1-0.875z^{-1}+0.125z^{-2}}$$

Now the pole locations are given by

$z_1=0.695$

$z_2=0.178$

If we compare the Poles of H(z) and $\overline{H}(z)$ we can observe that the poles of $\overline{H}(z)$ deviate very much from the original poles.

Cascade form

$$H(z) = \frac{1}{1-0.5z^{-1}(1-0.4z^{-1})}$$

$(0.5)_{10} = (0.1000)_2$

After truncation we get

$(0.100)_2=(0.5)_{10}$

After truncation we get

$(0.011)_2=(0.375)_{10}$

$(0.4)_{10} = \frac{0.4\times 2}{0.8}$

$\mapsto$ 0 $\downarrow$

$\frac{0.8\times 2}{1.6}$

$\mapsto$ 0

$\frac{0.6\times 2}{1.2}$

$\mapsto$ 1

$\frac{0.2\times 2}{0.4}$

$\mapsto$ 1

$\frac{0.4\times 2}{0.8}$

$\mapsto$ 0

$(0.4)_{10}=(0.01100\ldots)_2$

The system function after coefficient quantization is

$$H(z) = \frac{1}{(1-0.5z^{-1})(1-0.375z^{-1})}$$

The pole locations are given by

$z_1=0.5$

z_2=0.375
on comparing the poles of the cascade system with original poles we can say that one of the poles is same and other pole is very close to original pole.
****

A LTI system is characterized by the difference equation y(n)=0.68y(n-1)+0.5x(n). The input signal x(n) has a range of -5V to +5V, represented by 8-bits. Find the quantization step size, variance of the error signal and variance of the quantization noise at the output.

Solution:

Given

Range R=-5V to +5V = 5-(-5) =10
Size of binary, B= 8 bits (including sign bit)
Quantization step size,

$$q = \frac{R}{2^8} = \frac{10}{2^8} = 0.0390625$$

$$\text{variance of error signal}, \sigma_e^2 = \frac{q^2}{12} = \frac{0.0390625^2}{12} = 1.27116*10^{-4}$$

The difference equation governing the LTI system is
Y (n) =0.68y (n-1) +0.15x (n)
On taking z transform of above equation we get

$$Y(z) = 0.68z^{-1}Y(z) + 0.15X(z)$$

$$Y(z) - 0.68z^{-1}Y(z) = 0.15X(z)$$

$$Y(z)[1-0.68z^{-1}] = 0.15X(z)$$

$$\frac{Y(z)}{X(z)} = \frac{0.15}{1-0.68z^{-1}}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{0.15}{1-0.68z^{-1}}$$

$$H(z)H(z^{-1})z^{-1} = \frac{0.15}{1-0.68z^{-1}} * \frac{0.15}{1-0.68z} * z^{-1}$$

$$H(z)H(z^{-1})z^{-1} = \frac{0.225z^{-1}}{\left(1-\frac{0.68}{z}\right)(-0.68)\left(z-\frac{1}{0.68}\right)}$$

$$H(z)H(z^{-1})z^{-1} = \frac{-0.0331z^{-1}}{\left(\frac{z-0.68}{z}\right)(z-1.4706)} = \frac{-0.0331z^{-1}}{(z-0.68)(z-1.4706)}$$

Now, poles of H (z) H (z^{-1}) z^{-1} are p_1=0.68, p_2=1.4706
Here, p_1=0.68 is the only pole that lies inside the unit circle in z-plane
Variance of the input quantization noise at the output.

$$\sigma_{eoi}^2 = \sigma_e^2 \frac{1}{2\pi j} \oint_c H(z)H(z^{-1})z^{-1}dz$$

$$\sigma_{eoi}^2 = \sigma_e^2 \sum_{i=1}^{N} \left[\text{Res}\, H(z)H(z^{-1})z^{-1} \right]\Big|_{z=pi}$$

$$\sigma_{eoi}^2 = \sigma_e^2 \sum_{i=1}^{N} \left[(z-p_i)H(z)H(z^{-1})z^{-1} \right]\Big|_{z=pi}$$

$$\sigma_{eoi}^2 = \sigma_e^2 (z-0.68) * \frac{-0.0331}{(z-0.68)(z-1.4706)}\Big|_{z=0.68}$$

$$\sigma_{eoi}^2 = \sigma_e^2 * \frac{-0.0331}{(0.68-1.4706)}\Big| = 0.0419\sigma_e^2$$

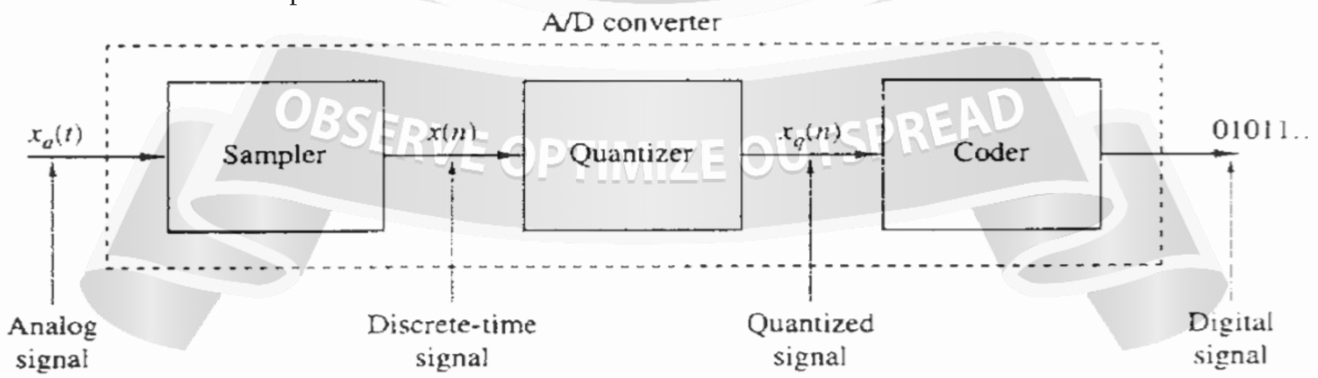
$$\sigma_{eoi}^2 = 0.0419 * 1.2716 * 10^{-4}$$

$$\sigma_{eoi}^2 = 5.328 * 10^{-6}$$

Analog to digital conversion:

10. Explain the ADC and DAC in detail.

A/D conversion has three process.



Basic parts of an analog-to digital (A/D) converter

1. Sampling
- Sampling is the conversion of a continuous- tome signal into a discrete-time signal obtained by taking the samples of continuous-time signal at discrete instants.
- Thus if $x_a(t)$ is the input to the sampler, the output is $x_a(nT) \equiv x(n)$, where T is called the sampling interval.
2. Quantisation

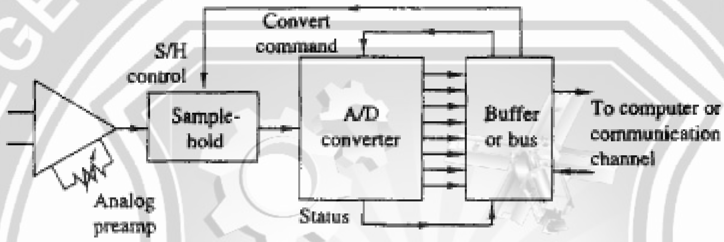
- The process of converting a discrete-time continuous amplitude signal into digital signal is called quantization.
- The value of each signal sample is represented by a value selected from a finite set of possible values.
- The difference between the unquantised sample x(n) and the quantized output $x_q(n)$ is called the quantization error or quantization noise.

$$e_q(n)= x_q(n)-x(n)$$

- To eliminate the excess bits either discard them by the process of truncation or discard them by rounding the resulting number by the process of rounding.
- The values allowed in the digital signals are called the quantization levels
- The distance Δ between two successive quantization levels is called the quantization step size or resolution.
- The quality of the output of the A/D converter is measured by the signal-to-quantization noise ratio.

3. Coding

- In the coding process, each discrete value $x_q(n)$ is represented by a b-bit binary sequence.



Block diagram of basic elements of an A/D Converter

Digital to analog conversion:

- To convert a digital signal into an analog signal, digital to analog converters are used.

Basic operations in converting a digital signal into an analog signal

- The D/A converter accepts, at its input, electrical signals that corresponds to a binary word, and produces an output voltage or current that is proportional to the value of the binary word.
- The task of D/A converter is to interpolate between samples.
- The sampling theorem specifies the optimum interpolation for a band limited signal.
- The simplest D/A converter is the zero order hold which holds constant value of sample until the next one is received.
- Additional improvement can be obtained by using linear interpolation to connect successive samples with straight line segment.
- Better interpolation can be achieved y using more sophisticated higher order interpolation techniques.
- Suboptimum interpolation techniques result in passing frequencies above the folding frequency. Such frequency components are undesirable and are removed by passing the output of the interpolator through a proper analog filter which is called as post filter or smoothing filter.
- Thus D/A conversion usually involve a suboptimum interpolator followed by a post filter.

****