EXISTENCE CONDITIONS-LAPLACE TRANSFORM

Let $f(t)$ be a function of t defined for all $t \geq 0$ .then the Laplace transform of $f(t)$, denoted by $L[f(t)]$ is defined by

$$L[f(t)] = \int_0^\infty e^{-st} f(t)dt$$

Provided that the integral exists, "s" is a parameter which may be real or complex. Clearly $L[f(t)]$ is a function of s and is briefly written as $F(s)$ $(i.e.)$ $L[f(t)] = F(s)$

Piecewise continuous function

A function $f(t)$ is said to be piecewise continuous is an interval $a \leq t \leq b$, if the interval can be sub divided into a finite number of intervals in each of which the function is continuous and has finite right and left hand limits.

Exponential order

A function $f(t)$ is said to be exponential order if $\lim_{t\to\infty} e^{-st} f(t)$ is a finite quantity, where $s > 0$(exists).

Example: Show that the function $f(t) = e^{t^3}$ is not of exponential order.

Solution:

$$\lim_{t\to\infty} e^{-st} e^{t^3} = \lim_{t\to\infty} e^{-st+t^3} = \lim_{t\to\infty} e^{t^3-st}$$

$$= e^\infty = \infty, \text{ not a finite quantity.}$$

Hence $f(t) = e^{t^3}$ is not of exponential order.

Sufficient conditions for the existence of the Laplace transform

The Laplace transform of $f(t)$ exists if

i) $f(t)$ is piecewise continuous in the interval $a \leq t \leq b$

ii) $f(t)$ is of exponential order.

Note: The above conditions are only sufficient conditions and not a necessary condition.

Example: Prove that Laplace transform of e^{t^2} does not exist.

Solution:

$$\lim_{t\to\infty} e^{-st} e^{t^2} = \lim_{t\to\infty} e^{-st+t^2} = \lim_{t\to\infty} e^{t^2-st}$$

$$= e^\infty = \infty \text{ ,not a finite quantity.}$$

$\therefore e^{t^2}$ is not of exponential order.

Hence Laplace transform of e^{t^2} does not exist.