3.7 METHODS OF CORRELATES

Method of correlation involves in determining the multiples or the individual constants, which can be further used for finding the probable values of unknowns. For finding these, a lot of conditions were established.

PROBLEM -1

The following angles were measured at a station O as to close the horizon.

$\angle AOB = 83°42'28''.75$	**weight 3**
$\angle BOC = 102°15'43''.26$	**weight 2**
$\angle COD = 94°38'27''.22$	**weight 4**
$\angle DOA = 79°23'23''.77$	**weight 2**

Adjust the angles by method of Correlates.

Solution:

$\angle AOB = 83°42'28''.75$	Weight 3
$\angle BOC = 102°15'43''.26$	Weight 2
$\angle COD = 94°38'27''.22$	Weight 4
$\angle DOA - 79°23'23''.77$	Weight 2

Sum = $360°00'03.''00$

Hence, the total correction E = 360° - (360°0'3")

= -3"

Let e_1, e_2, e_3 and e_4 be the individual corrections to the four angles respectively. Then by the condition equation, we get

$$e_1 + e_2 + e_3 + e_4 = -3'' \quad \text{-------- (1)}$$

Also, from the least square principle, $\Sigma(we^2)$ = a minimum

$$3e_1^2 + 2e_2^2 + 4e_3^2 + 2e_4^2 = \text{a minimum} \quad \text{------- (2)}$$

Differentiating (1) and (2), we get

$$\delta e_1 + \delta e_2 + \delta e_3 + \delta e_4 = 0 \quad \text{--------- (3)}$$

$$3e_1\delta e_1 + 2e_2\delta e_2 + 4e_3\delta e_3 + 2e_4\delta e_4 = 0 \qquad \text{---------} (4)$$

Multiplying equation (3) by –λ and adding it to (4), we get

$$\delta e_1(3e_1 - \lambda) + \delta e_2(2e_2-\lambda) + \delta e_3(4e_3-\lambda) + \delta e_4(2e_4-\lambda) = 0 \text{-----------}(5)$$

Since the coefficients of $\delta e_1, \delta e_2, \delta e_3, \delta e_4$ must vanish independently, we have

$$3e_1 - \lambda = 0 \text{ or } e_1 = \frac{\lambda}{3}$$

$$2e_1 - \lambda = 0 \text{ or } e_2 = \frac{\lambda}{2} \qquad \text{----------} (6)$$

$$4e_1 - \lambda = 0 \text{ or } e_3 = \frac{\lambda}{4}$$

$$2e_1 - \lambda = 0 \text{ or } e_4 = \frac{\lambda}{2}$$

Substituting these values in (1), we get

$$\frac{\lambda}{3} + \frac{\lambda}{2} + \frac{\lambda}{4} + \frac{\lambda}{2} = -3$$

$$\lambda\left(\frac{19}{12}\right) = -3$$

$$\lambda = -\frac{3*12}{19}$$

Hence

$$e_1 = \frac{1}{3} * \frac{3*12}{19} = -\frac{12}{19} = -0.63''$$

$$e_2 = \frac{1}{2} * \frac{3*12}{19} = -\frac{18}{19} = -0.95''$$

$$e_3 = \frac{1}{4} * \frac{3*12}{19} = -\frac{9}{19} = -0.47''$$

$$e_4 = \frac{1}{2} * \frac{3*12}{19} = -\frac{18}{19} = -0.95''$$

Sum = -3.0”

BOC = 102°15'43".26 – 0".95 = 102°15'42".31

COD = 94°38'27".22 – 0".47 = 94°38'26".75

DOA = 79°23'23".77 – 0".95 = 79°23'22".82

360°00'00".00

The following round of angles was observed from central station to surrounding station of a triangulation survey.

A = 93°43'22" **weight 3**

B = 74°32'39" **weight 2**

C = 101°13'44" **weight 2**

D = 90°29'50" **weight 3**

In addition, one angle $\overline{(A+B)}$ was measured separately as comb angle with a mean value of 168°16'06" (wt 2).

Determine the most probable values of the angles A, B, C and D.

Solution:

A + B+C+D = 359°59'35".

Total correction E = 360° - (359° 59' 35")

= + 25°

Similarly, $\overline{(A+B)}$ = (A+B)

Hence correction E' = A + B - $\overline{(A+B)}$

= 168°16'01" – 168°16'06"

= -5"

Let e_1 ,e_2, e_3 ,e_4 and e_5 be the individual corrections to A, B, C, D $\overline{(A+B)}$ respectively. Then by the condition equation, we get

$e_1 + e_2 + e_3 + e_4 = -25"$ -------- (1(a))

$e_5 - e_1 - e_2 = -5"$ -------- (1(b))

Also, from the least square principle, $\Sigma(we^2)$ = a minimum

$3e_1^2 + 2e_2^2 + 2e_3^2 + 3e_4^2 + 2e_5^2$ = a minimum ------- (2)

Differentiating (1a) (1b) and (2), we get

$\delta e_1 + \delta e_2 + \delta e_3 + \delta e_4 = 0$ --------- (3a)

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$\delta e_5 - \delta e_1 - \delta e_2 = 0$ ---------- (3b)

$$3e_1\delta e_1 + 2e_2\delta e_2 + 2e_3\delta e_3 + 3e_4\delta e_4 + 2e_5\delta e_5 = 0 \qquad \text{---------} \ (4)$$

Multiplying equation (3a) by $-\lambda_1$, (3b) by $-\lambda_2$ and adding it to (3), we get

$\delta e_1(3e_1-\lambda_1+\lambda_2) + \delta e_2(2e_2 - \lambda_1 + \lambda_2) + \delta e_3(2e_3 - \lambda_1) + \delta e_4(3e_4 - \lambda_1)$

$+\delta e_5(-\lambda_2 + 2e_5) = 0$ --------- (5)

Since the coefficients of $\delta e_1, \delta e_2, \delta e_3, \delta e_4$ etc. must vanish independently, we have

$-\lambda_1 + \lambda_2 + 3e_1 = 0$ or $e_1 = \frac{\lambda_1}{3} - \frac{\lambda_2}{3}$

$-\lambda_1 + \lambda_2 + 2e_2 = 0$ or $e_2 = \frac{\lambda_1}{2} - \frac{\lambda_2}{2}$

$-\lambda_2 + 2e_3 = 0$ or $e_3 = \frac{\lambda_1}{2}$ ------------ (6)

$-\lambda_1 + 3e_4 = 0$ or $e_4 = \frac{\lambda_1}{3}$

$-\lambda_2 + 2e_5 = 0$ or $e_5 = -\frac{\lambda_2}{2}$

Substituting these values of e_1, e_2, e_3, e_4 and e_5 in Equations (1a) and (1b)

$$\frac{\lambda_1}{3} - \frac{\lambda_2}{3} + \frac{\lambda_1}{2} - \frac{\lambda_2}{2} + \frac{\lambda_1}{2} + \frac{\lambda_1}{3} = 25 \quad from (1a)$$

or $$5\frac{\lambda_1}{3} - \frac{5}{6}\lambda_2 = 25$$

$$\frac{\lambda_1}{3} - \frac{1}{6}\lambda_2 = 5 \qquad \text{--------} \ (I)$$

$$\frac{\lambda_2}{2} - \frac{\lambda_1}{3} + \frac{\lambda_2}{3} - \frac{\lambda_1}{2} + \frac{\lambda_2}{32} = -5 \quad from (1b)$$

$$4\frac{\lambda_2}{3} - \frac{5}{6}\lambda_1 = -5 \qquad \text{---------} \ (II)$$

Solving (I) and (II) simultaneously, we get

$$\lambda_1 = +\frac{210}{11}$$

$$\lambda_2 = +\frac{90}{11}$$

Hence $$e_1 = \frac{1}{3}.\frac{210}{11} - \frac{1}{3}.\frac{90}{11} = +\frac{40''}{11} = +3''.64$$

$$e_2 = \frac{1}{2}.\frac{210}{11} - \frac{1}{2}.\frac{90}{11} = +\frac{60''}{11} = +5''.45$$

$$e_3 = \frac{1}{2}.\frac{210}{11} = +\frac{105''}{11} = +9''.55$$

$$e_4 = \frac{1}{3}.\frac{210}{11} = +\frac{70''}{11} = +6''.36$$

Total = +25".00

Also

$$e_5 = \frac{1}{2}.\frac{90}{11} + 4''09.$$

Hence the corrected angles are

A = 93°43'22" + 3".64 = 93°43'25".64

B = 74°32'39" + 5".45 = 74°32'44".45

C = 103°13'44" + 9".55 = 101°13'53".55

D = 90°29'50" + 6".36 = 90°29'56".36

Sum = 360°00'00".00