DESIGN OF FIR FILTERS USING FOURIER SERIES METHOD

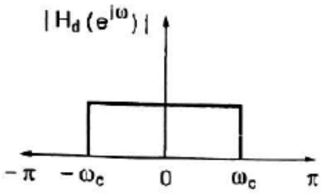
STEPS TO DESIGN FIR FILTER USING FOURIER SERIES METHOD:

$$\text{If } \left| H_d\left(e^{j\omega}\right) \right| = \{1 \quad 0 \le |\omega| \le \omega_c$$

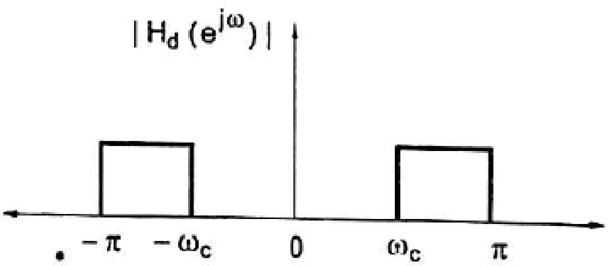
$$=\{0 \quad \text{otherwise}$$ is given specification.

Step-1: Draw the graph for given specification.

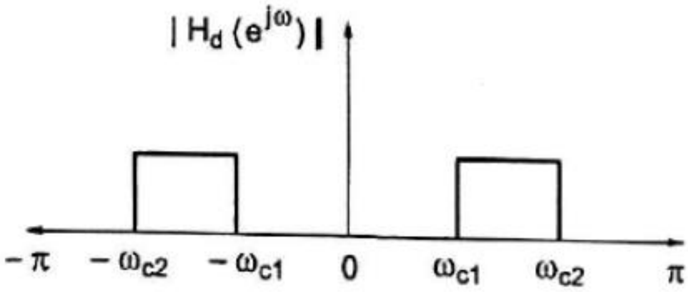
LPF:



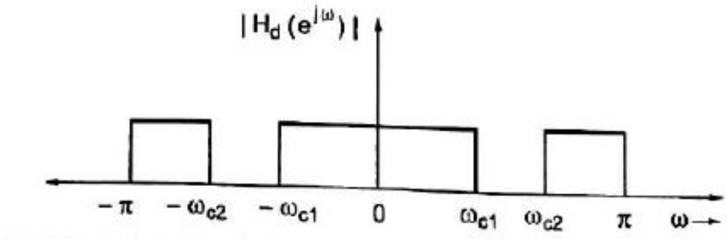
HPF:



BPF:



BSF:



Step-2: Find α

$$\alpha = \frac{N-1}{2} \quad \text{If frequency response of } \omega_c \text{ is given}$$

$$\alpha = h_d\left(e^{j\omega}\right) \ \text{coefficient}$$

Step-3: Find $h_d(n)$

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})e^{j\omega}d\omega$$

Filter coefficient hd(n) for different type of filters

LPF-

$$h_d(n) = \frac{\omega_c}{\pi} \quad \text{for } n=\alpha$$

$$h_d(n) = \frac{\sin\omega_c(n-\alpha)}{\pi(n-\alpha)} \quad \text{for } n=\alpha$$

HPF-

$$h_d(n) = 1 - \frac{\omega_c}{\pi} \quad \text{for } n=\alpha$$

$$h_d(n) = \frac{\sin(n-\alpha)\pi - \sin\omega_c(n-\alpha)}{\pi(n-\alpha)} \quad \text{for } n=\alpha$$

BPF-

$$h_d(n) = \frac{\omega_{c2}-\omega_{c1}}{\pi} \quad \text{for } n=\alpha$$

$$h_d(n) = \frac{\sin\omega_{c2}(n-\alpha) - \sin\omega_{c1}(n-\alpha)}{\pi(n-\alpha)} \quad \text{for } n=\alpha$$

BSF-

$$h_d(n) = 1 - \frac{\omega_{c2}-\omega_{c1}}{\pi} \quad \text{for } n=\alpha$$

$$h_d(n) = \frac{\sin\omega_{c1}(n-\alpha) - \sin\omega_{c2}(n-\alpha) + \sin\pi(n-\alpha)}{\pi(n-\alpha)} \quad \text{for } n=\alpha$$

Step-4: Find h(n)

$$h(n) = h_d(n) \quad \text{for } -\infty \le n \le \infty$$
$$= 0 \quad \text{otherwise}$$

Step-5: Find H(z) transfer function

$$H(z)=\sum_{n=0}^{\frac{N-1}{2}} h(n)\,[z^{-n} + z^{n}] + h(\alpha) \quad \text{if } \alpha = 0$$
$$= h(\alpha) + \sum_{n=0}^{N-1} h(n)\, z^{n} \quad \text{if } \alpha \ne 0$$

Step-6: Find the transfer function of the realizable filter is

$$H(z)=z^{\frac{-N-1}{2}} H(z)$$

Step-7: Find magnitude of H(z).

1. **Design a FIR LPF with cut-off frequency of 1 kHz and sampling frequency of 4 kHz with 11 samples using Fourier series method:**

Given

f_c=1khz

f_s=4khz

N=11

$$\omega_c = \omega_c T = \frac{\Omega_c}{f_s} = \frac{2\pi*1*10^3}{4*10^3} = \frac{\pi}{2} = 0.5\pi \text{ rad/sec}$$

The desired frequency response $H_d(e^{j\omega})$ of low pass filter is

$$H_d(e^{j\omega}) = 1 \text{ for } -\omega_c \le \omega \le +\omega_c$$
$$= 0 \quad \text{for} -\pi \le \omega \le -\omega_c \text{ and } \omega_c \le \omega \le \pi$$

The desired impulse response $h_d(n)$ of the LPF is

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n} d\omega$$

$$=\frac{1}{2\pi}\left[\frac{e^{j\omega_c n}}{jn}-\frac{e^{-j\omega_c n}}{jn}\right]$$

$$=\frac{1}{2\pi}\left[\frac{e^{j\omega_c n}-e^{j\omega_c n}}{jn}\right]$$

$$h_d(n)=\frac{1}{\pi n}\left[\frac{e^{j\omega_c n}-e^{j\omega_c n}}{2j}\right]=\frac{1}{\pi n}\sin\omega_c n$$

The impulse response h(n) of FIR filter,

$H_d(n)=\frac{\sin\omega_c n}{\pi n}$ for $n=-\frac{N-1}{2}$ to $\frac{N-1}{2}$

$=\frac{\omega_c}{\pi}$ for n=0

N=1 $\frac{N-1}{2}=\frac{11-1}{2}=5$

Hence, calculate h(n) for n=-5 to +5

When n=0;

$h(0)=\frac{\omega_c}{\pi}=\frac{0.5\pi}{\pi}=0.5$

When n=1;

$h(1)=\frac{\sin\omega_c n}{\pi n}=\frac{\sin(0.5\pi*1)}{\pi*1}=0.3183$

When n=2;

$h(2)=\frac{\sin\omega_c n}{\pi n}=\frac{\sin(0.5\pi*2)}{\pi*2}=0$

When n=3;

$$h(3)=\frac{\sin\omega_c n}{\pi n}=\frac{\sin(0.5\pi*3)}{\pi*3}=-0.1061$$

When n=4;

$$h(4)=\frac{\sin\omega_c n}{\pi n}=\frac{\sin(0.5\pi*4)}{\pi*4}=0$$

When n=5;

$$h(5)=\frac{\sin\omega_c n}{\pi n}=\frac{\sin(0.5\pi*5)}{\pi*5}=0.0637$$

When n=-1; h(-1)=h(1)=0.3183

n=-2; h(-2)=h(2)=0

n=-3; h(-3)=h(3)=-0.1061

n=-4; h(-4)=h(4)=0

n=-5; h(-5)=h(5)=0.0637

The transfer function H(z) of the digital LPF is given by,

$$H(z)=z^{-\frac{N-1}{2}} z\{h(n)\}$$

$$= z^{-\frac{N-1}{2}} \sum_{n=-\frac{N-1}{2}}^{\frac{N-1}{2}} h(n)\, z^{-n}$$

$$=z^{-5} \sum_{n=-5}^{5} h(n)\, z^{-n}$$

$$=z^{-5}[h(-5)z^{5} + h(-4)z^{4} + h(-3)z^{3} + h(-2)z^{2} + h(-1)z^{1} + h(0)z^{0} + h(1)z^{-1} + h(2)z^{-2} + h(3)z^{-3}+h(4)\, z^{-4} + h(5)z^{-5}]$$

Using symmetry condition h(n)=h(-n)

$= z^{-5}[h(5)z^{5} + h(4)z^{4} + h(3)z^{3} + h(2)z^{2} + h(1)z^{1} + h(0)z^{0} + h(1)z^{-1} + h(2)z^{-2} + h(3)z^{-3} + h(4) z^{-4} + h(5)z^{-5}$

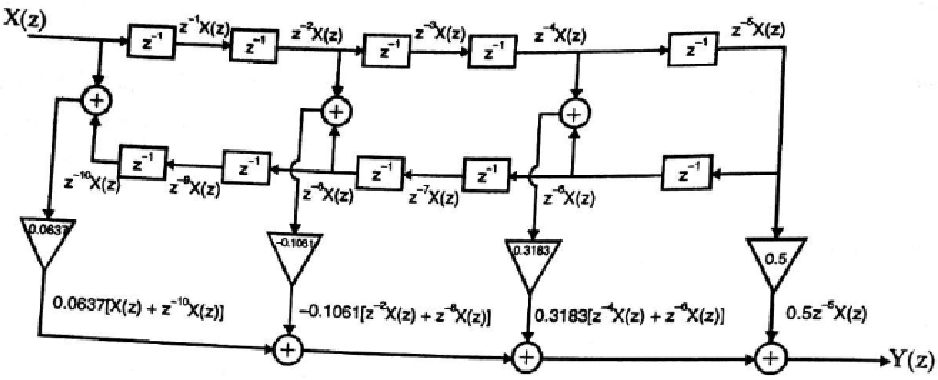
$=z^{-5}[h(0) + h(1)[z + z^{-1}] + h(2)[z^{2} + z^{-2} + h(3)[z^{3} + z^{-3}] + h(4)[z^{4} + z^{-4}] + h(5)[z^{-5} + z^{-5}]$

$H(z) = [h(0)z^{-5} + h(1)[z^{-4} + z^{-6}] + h(2)[z^{-3} + z^{-7} + h(3)[z^{-2} + z^{-8}] + h(4)[z^{-1} + z^{-9}] + h(5)[z^{0} + z^{-10}]]$

$H(z)= 0.5z^{-5} + 0.3183[z^{-4} + z^{-6}] - 0.1061[z^{-2} + z^{-8}] + 0.0637[z^{0} + z^{-10}]]$

$Y(z)= 0.5z^{-5}x(z) + 0.3183[z^{-4} + z^{-6}]x(z) - 0.1061[z^{-2} + z^{-8}]x(z) + 0.0637[z^{0} + z^{-10}]x(z)]$

LINEAR PHASE STRUCTURE OF FIR LPF:



Find the magnitude of H(z):

N→odd, symmetric

$$|H(e^{j\omega})| = h(0) + \sum_{n=1}^{\frac{N-1}{2}} 2h(n)\cos\omega n$$

$$= h(0) + \sum_{n=1}^{5} 2h(n)\cos\omega n$$

$$=h(0)+ 2h(1)\cos\omega + 2h(2)\cos\omega 2 + 2h(3)\cos\omega 3 + 2h(4)\cos\omega 4 + 2h(5)\cos\omega 5$$

$$=0.5+ 2 * 0.3183\cos\omega + 2 * 0(\cos\omega 2) + 2 * (-0.1061)\cos\omega 3 + 2 * 0\cos\omega 4 + 2 * 0.0367\cos\omega 5$$

$$|H(e^{j\omega})|=0.5+ 0.6366\cos\omega - 0.2122\cos\omega 3 + 0.1274\cos\omega 5$$

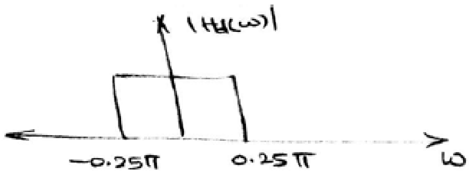
2. **A LPF is required to be designed with the desired frequency response.**

$H_d(\omega) = \{e^{-j2\omega} \qquad -0.25\pi < \omega \leq 0.25\pi$

$\qquad = \{0 \qquad 0.25\pi < \omega \leq \pi$

Where N=5

Step1: Draw the graph



Step2: Find α

$$\alpha = \frac{N-1}{2} = \frac{4}{2} = 2$$

Step3:Find $h_d(n)$

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})e^{j\omega n}d\omega$$

$$= \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{-j\omega 2}e^{j\omega n}d\omega$$

$$= \frac{1}{2\pi}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} e^{j\omega(n-2)}d\omega$$

$$= \frac{1}{2\pi}\left[\frac{e^{j\frac{\pi}{4}(n-2)}}{j(n-2)} - \frac{e^{-j\frac{\pi}{4}(n-2)}}{j(n-2)}\right]$$

$$= \frac{1}{2\pi}\left[\frac{e^{\frac{j\pi(n-2)}{4}} - e^{j\frac{\pi}{4}(n-2)}}{j(n-2)}\right]$$

$$h_d(n) = \frac{1}{\pi(n-2)}\left[\frac{e^{j\frac{\pi}{4}(n-2)} - e^{j\frac{\pi}{4}(n-2)}}{2j}\right] = \frac{1}{\pi(n-2)}\sin\frac{\pi}{4}(n-2)$$

$$h_{d(n)=}\frac{1}{\pi(n-2)}\sin\frac{\pi}{4}(n-2) \quad \text{for } n \neq 2$$

$$=1/4 \qquad\qquad \text{for } \quad n=2$$

$$h_d(0) = \frac{1}{\pi(-2)}\sin\frac{\pi}{4}(-2) = \frac{-1}{-2\pi} = \frac{1}{\pi 2} = 0.1591$$

$$h_d(1) = \frac{1}{\pi(-1)}\sin\frac{\pi}{4}(-1) = \frac{-0.707}{-\pi} = 0.2550$$

$$h_d(2) = \frac{1}{4} = 0.25$$

$$h_d(3) = \frac{1}{\pi(1)}\sin\frac{\pi}{4}(1) = 0.2250$$

$$h_d(4) = \frac{1}{\pi(2)}\sin\frac{\pi}{4}(2) = \frac{1}{2\pi} = 0.1591$$

Step4: Find h(n)

$$h(n) = h_d(n)$$

Step5: Find H(z)

$$H(z) = h(\alpha) + \sum_{n=0}^{N-1} h(n)\, z^n$$

$$= h(2) + \sum_{n=0}^{4} h(n)\, z^n$$

$$H(z) = h(2) + h(0)\, z^0 + h(1)z^1 + h(2)z^2 + h(3)z^4 + h(4)z^4$$

$$H(z) = 0.25 + 0.1591 + 0.2250z^1 + 0.25z^2 + 0.2550z^3 + 0.1591z^4$$

$$H(z) = 0.25[1 + z^2] + 0.1591[1 + z^4] + 0.2550[z^1 + z^3]0.1591z^4$$

Step6: Find H'(z)

$$H'(z)= z^{-2}H(z)$$

$$H(z) = z^{-2}(0.25[1 + z^2] + 0.1591[1 + z^4] + 0.2550[z^1 + z^3])$$

$$H(z) = 0.25z^{-2} + 0.25 + 0.1591[z^{-2} + z^2] + 0.2550[z^1 + z^1]$$

Step7:Find the magnitude of H(z)

$$H(\omega)=h\left(\frac{N-1}{2}\right) + 2[\sum_{n=0}^{\frac{N-3}{2}} h(n)\cos(\omega\left(\frac{N-1}{2} - n\right))]$$

$$=h(2)+2[\sum_{n=0}^{1} h(n)\cos\omega(2 - n)]$$

$$=h(2)+2[h(0)\cos 2\omega + h(1)\cos\omega]$$

$$H(\omega) =0.25+0.3182\cos 2\omega + 0.45\cos\omega$$