Higher order linear differential equations with constant coefficient

General form of a linear differential equation of the n^{th} order with constant coefficient is $\frac{d^n y}{dx^n} + k_1 \frac{d^{n-1}y}{dx^{n-1}} + k_2 \frac{d^{n-2}y}{dx^{n-2}} \dots k_n y = x \; \dots (1)$

Where $k_1, k_2, \dots, k_n$ are constants it will be convenient to denote the operation $\frac{d}{dx}$ by a single letter D.

$Dy = \frac{dy}{dx}$ similarly $D^2 y = \frac{d^2y}{dx^2}, D^3 y = \frac{d^3y}{dx^3}$ etc

The equation (1) above can be written as

$$(D^n + k_1 D^{n-1} + \dots + k_n)y = x$$

$$ie\ f(D)y = x$$

Note:

1. $\frac{1}{D}x = \int x dx$
2. $\frac{1}{D-a}x = e^{ax}\int x e^{-ax} dx$
3. $\frac{1}{D+a}x = e^{-ax}\int x e^{ax} dx$

Result:

1. $\frac{1}{D-a}\varphi(x) = e^{ax}\int e^{-ax}\varphi(x) dx$
2. $\frac{1}{D+a}\varphi(x) = e^{-ax}\int e^{ax}\varphi(x) dx$

(i)**The general form of the linear differential equation of second order is**

$$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$$

Where P and Q are constants and R is a function of x or constant

(ii) **Differential Operators**

The symbol D stands for the operation of differential

$$Dy = \frac{dy}{dx},\ D^2y = \frac{d^2y}{dx^2}$$

$\frac{1}{D}$ stands for the operation of integration

$\frac{1}{D^2}$ stands for the operation of integration twice

$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$ can be written in the form

$$(D^2 + PD + Q)y = R$$

(iii) Complete solution is y = complementary function + Particular Integral

(iv)To find the complementary function

	Roots of A.E	C.F
1.	Roots are real and different m_1, m_2 ($m_1 \neq m_2$)	$Ae^{m_1 x} + Be^{m_2 x}$
2.	Roots are real and equal $m_1 = m_2 = m$(say)	$(Ax + B)e^{mx}$ or $(A + Bx)e^{mx}$
3.	Roots are imaginary $\propto \pm i\beta$	$e^{\propto x}[A\cos\beta x + B\sin\beta x]$
4.	Roots are $\propto \pm i\beta$ (twice)	$e^{\propto x}[(c_1 + c_2 x)\cos\beta x + (c_3 + c_4 x)\sin\beta x]$

(V) To find the particular integral

P.I.$= \frac{1}{f(D)} x$

	x	P.I
1	e^{ax}	P.I.$= \frac{1}{f(D)} e^{ax} = e^{ax} \frac{1}{f(a)}; f(a) \neq 0$ $= x e^{ax} \frac{1}{f'(a)}, f(a) = 0; f'(a) \neq 0$ $= x^2 e^{ax} \frac{1}{f''(a)}; f(a) = 0, f'(a) = 0\ f''(a) \neq 0$
2	x^n	P.I.$= \frac{1}{f(D)} x^n$ $= [f(D)]^{-1} x^n$ Expand $[f(D)]^{-1}$ and then operate
3	$\sin ax (\text{or}) \cos ax$	P.I.$= \frac{1}{f(D)} [\cos ax\ (\text{or}) \sin ax]$ Replace D^2 by-a^2
4	$e^{ax}\varphi(x)$	P.I.$= \frac{1}{f(D)} e^{ax\varphi(x)}$ $= e^{ax} \frac{1}{f(D+a)} \varphi(x)$

Problem Based on R.H.S of the given differential equation is zero

Example:

Solve $(D^2 + 2D + 1)y = 0$

Solution:

Auxiliary Equation is $m^2 + 2m + 1 = 0$

$$m = -1, -1$$

$$y = C.F$$

$$= (Ax + B)e^{-x}$$

Example:

Solve $(D^2 + 1)y = 0$ given $y(0) = 0, y'(0 = 1$

Solution:

Auxiliary Equation is $m^2 + 1 = 0$

$$m^2 = -1$$

$$m = \pm i$$

$$y = A\cos x + B\sin x$$

$$y(x) = A\cos x + B\sin x \;... (1)$$

$$y'(x) = -A\sin x + B\cos x \;... (2)$$

Given $y(0) = 0$

(1) $\Rightarrow$ $A = 0$

Given $y'(0) = 1$

(2) $\Rightarrow$ $B = 1$

(1) $\Rightarrow$ $y(x) = \sin x$

Type I : Problems Based on P.I = $\frac{1}{f(D)}e^{ax}$ ⟶ Replace D by a

Example :

Solve $(D^2 - D - 6)y = 3e^{4x} + 5$

Solution:

Auxiliary Equation is $m^2 - m - 6 = 0$

$$(m-3)(m+2) = 0$$

$$m_1 = 3,\; m_2 = -2$$

C.F $= Ae^{3x} + Be^{-2x}$

P.I$_1$ $= \frac{1}{D^2-D-6} 3e^{4x}$ Replace D by 4

$= \frac{1}{16-4-6} 3e^{4x}$

$= \frac{1}{6} 3e^{4x} \quad = \frac{1}{2} e^{4x}$

P.I$_2$ $= \frac{1}{D^2-D-6} 5$

$= \frac{1}{D^2-D-6} 5e^{0x}$ Replace D by 0

$= \frac{-1}{6} 5 \quad = \frac{-5}{6}$

The general solution is y = C.F + P.I

$$y = Ae^{3x} + Be^{-2x} + \frac{1}{2} e^{4x} - \frac{5}{6}$$

Example:

Solve $(D^2 + 7D + 12)y = 14e^{-3x}$

Solution:

Auxiliary Equation is $m^2 + 7m + 12 = 0$

$m = -3, \ m = -4$

C.F $= Ae^{-3x} + Be^{-4x}$

P.I $= \frac{1}{D^2+7D+12} 14e^{-3x}$

$= 14 \frac{1}{9-21+12} e^{-3x}$ Replace D by -3

$= \frac{1}{0} e^{-3x}$ (fails)

$= 14x \frac{1}{2D+7} e^{-3x}$

$= 14x \frac{1}{-6+7} e^{-3x}$

$= 14xe^{-3x}$

The general solution is y = C.F + P.I

$$y = Ae^{-3x} + Be^{-4x} + 14xe^{-3x}$$

Example:

Find the P.I of $(D^2 - 1)\, y = (e^x + 1)^2$

Solution:

Given $(D^2 - 1)y = (e^x + 1)^2$

$= (e^x)^2 + 1 + 2e^x$

$= e^{2x} + e^0 + 2e^x$

$P.I_1 = \frac{1}{D^2-1} e^{2x}$ Replace D by 2

$= \frac{1}{4-1} e^{2x}$

$= \frac{1}{3} e^{2x}$

$P.I_2 = \frac{1}{D^2-1} e^{0x}$

$= \frac{1}{-1} e^{0x}$ Replace D by 0

$= -e^{0x}$

$= -1$

$P.I_3 = \frac{1}{D^2-1} 2e^x$

$= 2 \frac{1}{D^2-1} e^x$ Replace D by 1

$= 2 \frac{1}{1-1} e^x$ (fails)

$= 2x \frac{1}{2D} e^x$ Replace D by 1

$= 2x \frac{1}{2} e^x$

$= xe^x$

$P.I = P.I_1 + P.I_2 + P.I_3$

$= \frac{1}{3} e^{2x} - 1 + xe^x$

Type II:

Problems Based on P.I = $\frac{1}{f(D)}$ sin ax (or) $\frac{1}{f(D)}$ cos ax → Replace D^2 by $-a^2$

Example :

Solve $(D^2 - 4D + 4)y = e^{2x} + \sin^2 x$

Solution:

Auxiliary Equation is $m^2 - 4m + 4 = 0$

$(m-2)(m-2) = 0$

$m = 2, 2$

$C.F = (Ax + B)e^{2x}$

$$\text{P.I} = \frac{1}{D^2-4D+4}[e^{2x} + \sin^2 x]$$

$$= \frac{1}{D^2-4D+4}\left[e^{2x} + \frac{1-\cos 2x}{2}\right]$$

$$\text{P.I}_1 = \frac{1}{D^2-4D+4} e^{2x} \qquad \text{Replace D by 2}$$

$$= \frac{1}{4-8+4} e^{2x}$$

$$= \frac{1}{0} e^{2x} \quad \text{(fails)}$$

$$= x.\frac{1}{2D-4} e^{2x} \qquad \text{Replace D by 2}$$

$$= x.\frac{1}{4-4} e^{2x}$$

$$= x\frac{1}{0} e^{2x} \quad \text{(fails)}$$

$$= x^2 \frac{1}{2} e^{2x}$$

$$\text{P.I}_2 = \frac{1}{D^2-4D+4}\left(\frac{1}{2}\right) e^{0x} \qquad \text{Replace D by 0}$$

$$= \frac{1}{8}$$

$$\text{P.I}_3 = \frac{1}{D^2-4D+4}\left(\frac{-\cos 2x}{2}\right) \qquad \text{Replace } D^2 \text{ by -4}$$

$$= \frac{1}{-4-4D+4}\left(\frac{-\cos 2x}{2}\right)$$

$$= \frac{1}{-4D}\left(\frac{-\cos 2x}{2}\right)$$

$$= \frac{1}{8D}\cos 2x$$

$$= \frac{1}{8}\int \cos 2x \, dx$$

$$= \frac{1}{8}\left(\frac{\sin 2x}{2}\right)$$

$$= \frac{1}{16}\sin 2x$$

The general solution y = C.F + P.I_1 + P.I_2 + P.I_3

$$y = (Ax + B)e^{2x} + \frac{x^2}{2}e^{2x} + \frac{1}{8} + \frac{1}{16}\sin 2x$$

Example :

Find the P.I of $(D^2+4)\,y = \cos 2x$

Solution:

$$\text{P.I} = \frac{1}{D^2+4}\cos 2x \qquad \text{Replace } D^2 \text{ by } -4$$

$$= \frac{1}{-4+4}\cos 2x$$

$$= \frac{1}{0}\cos 2x \qquad \text{(fails)}$$

$$= x\frac{1}{2D}\cos 2x$$

$$= \frac{x}{2D}\cos 2x$$

$$= \frac{x}{2}\int \cos 2x\, dx$$

$$= \frac{x}{2}\frac{\sin 2x}{2} = \frac{x}{4}sin2x$$

$$P.I = \frac{x}{4}sin2x$$

Example :

Find the P.I of $\frac{d^3y}{dx^3} + 4\frac{dx}{dx} = sin2x$

Solution:

$$\text{P.I} = \frac{1}{D^3+4D}\sin 2x$$

$$= \frac{1}{D(D^2+4)}\sin 2x \qquad \text{Replace } D^2 \text{ by } -4$$

$$= \frac{1}{D(-4+4)}\sin 2x \qquad \text{(fails)}$$

$$= x\frac{1}{3D^2+4}\sin 2x \qquad \text{Replace } D^2 \text{ by } -4$$

$$= x\frac{1}{3(-4)+4}\sin 2x$$

$$= x\frac{1}{-12+4}\sin 2x$$

$$= \frac{-x}{8}\sin 2x$$

$$P.I = \frac{-x}{8}\sin 2x$$

Type III: Problems Based on R.H.S= $e^{ax} + \sin ax\ (or)\ e^{ax} + \cos ax$

Example :

Solve $(D^2 - 3D + 2)y = 2\cos(2x+3) + 2e^x$

Solution:

Auxiliary Equation is $m^2 - 3m + 2 = 0$

$$m = 1, m = 2$$

$$\text{C.F} = Ae^x + Be^{2x}$$

P.I$_1$ = $\frac{1}{D^2-3D+2} 2e^x$

$= 2 \; \frac{1}{1-3+2} e^x$ Replace D by 1

$= 2 \; \frac{1}{0} e^x$ (fails)

$= 2x \; \frac{1}{2D-3} e^x$

$= 2x \; \frac{1}{2-3} e^x$ Replace D by 1

$= -2xe^x$

P.I$_2$ = $\frac{1}{D^2-3D+2} 2\cos(2x+3)$

$= \; 2 \frac{1}{-4-3D+2} \cos(2x+3)$ Replace D^2 by -4

$= 2 \frac{1}{-3D-2} \cos(2x+3)$

$= 2 \frac{1}{-3D-2} \; \frac{-3D+2}{-3D+2} \cos(2x+3)$

$= 2 \; \frac{-3D+2}{9D^2-4} \cos(2x+3)$ Replace D^2 by -4

$= 2 \; \frac{-3D+2}{-40} \cos(2x+3)$

$= \; \frac{-3D+2}{-20} \cos(2x+3)$

$= \; 6\sin(2x+3) + 2\cos(2x+3) \,/ -20$

$= \; -\frac{1}{10}\cos(2x+3) - \frac{3}{10}\sin(2x+3)$

The general solution is y = C.F + P.I$_1$ + P.I$_2$

$$y = Ae^x + Be^{2x} - 2xe^x - \frac{1}{10}\cos(2x+3) - \frac{3}{10}\sin(2x+3)$$

Type IV : Problems Based on R.H.S = Polynomial in x

Binomial expression

$(1+x)^{-1} = 1 - x + x^2 - x^3 + \cdots$ …………..

$(1-x)^{-1} = 1 + x + x^2 + x^3 + \cdots$ …………..

$(1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \cdots$ ………

$(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \cdots$ ………..

Example:

Solve y''+4y' $+ 5y = 3x - 2$

Solution:

Auxiliary Equation is $m^2 + 4m + 5 = 0$

$$m = \frac{-4\pm\sqrt{16-20}}{2}$$

$$= \frac{-4\pm\sqrt{-4}}{2}$$

$$= \frac{-4\pm 2i}{2} = -2 \pm i$$

$\propto = -2,\ \beta = 1$

C.F $= e^{-2x}[Acosx + Bsinx]$

P.I $= \frac{1}{D^2+4D+5}(3x-2)$

$$= \frac{1}{5\left[1+\frac{D^2+4D}{5}\right]}(3x-2)$$

$$= \frac{1}{5}\left[1+\frac{D^2+4D}{5}\right]^{-1}(3x-2)$$

$$= \frac{1}{5}\left[1-\frac{D^2+4D}{5}\right](3x-2)$$

$$= \frac{1}{5}\left[1-\frac{D^2}{5}-\frac{4D}{5}\right](3x-2)$$

$$= \frac{1}{5}\left[(3x-2)-\frac{D^2}{5}(3x-2)-\frac{4D}{5}(3x-2)\right]$$

$$= \frac{1}{5}\left[3x-2-0-\frac{4}{5}(3)\right]$$

$$= \frac{1}{5}\left[\frac{15x-10-12}{5}\right]$$

$$= \frac{1}{25}[15x-22]$$

The general solution is y = C.F +P.F

$$y = e^{-2x}[Acosx + Bsinx] + \frac{1}{25}[15x-22]$$

Example:

Solve $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = x^2 + 3$

Solution:

$$(D^2 - 5D + 6)y = x^2 + 3$$

Auxiliary Equation is $m^2 - 5m + 6 = 0$

m = 3, 2

C.F = Ae^{3x} + Be^{2x}

P.I$_1$ = $\frac{1}{D^2-5D+6}x^2$

$$= \frac{1}{6\left[1+\frac{D^2-5D}{6}\right]} x^2$$

$$= \frac{1}{6}\left[1 + \frac{D^2-5D}{6}\right]^{-1} x^2$$

$$= \frac{1}{6}\left[1 - \left(\frac{D^2-5D}{6}\right) + \left(\frac{D^2-5D}{6}\right)^2 \ldots .\right] x^2$$

$$= \frac{1}{6}\left[1 - \frac{D^2}{6} + \frac{5D}{6} + \frac{25}{36}D^2\right] x^2$$

$$= \frac{1}{6}\left[x^2 - \frac{D^2(x^2)}{6} + \frac{5D(x^2)}{6} + \frac{25}{36}D^2(x^2)\right]$$

$$= \frac{1}{6}\left[x^2 - \frac{2}{6} + \frac{5\times 2x}{6} + \frac{25}{36}(2)\right]$$

$$= \frac{1}{6}\left[x^2 + \frac{5}{3}x + \frac{19}{18}\right]$$

P.I$_2$ = $\frac{1}{D^2-5D+6} 3e^{0x}$

$$= \frac{1}{2}$$

The general solution is y =C.F + P.I$_1$ +P.I$_2$

$$y = Ae^{3x} + Be^{2x} + \frac{1}{6}\left[x^2 + \frac{5}{3}x + \frac{19}{18}\right] + \frac{1}{2}$$

Example:

Solve (D^3 +8) $y = x^4 + 2x + 1$

Solution :

Auxiliary Equation is $m^3 + 8 = 0$

$$m = -2, m^2 - 2m + 4 = 0$$

$$m = \frac{1 \pm i\sqrt{3}}{2}$$

$$= \frac{1}{2} \pm i\frac{\sqrt{3}}{2}$$

$$\alpha = \frac{1}{2} \quad \beta = \frac{\sqrt{3}}{2}$$

C.F = $Ae^{-2x} + Be^{\frac{1}{2}x}\left[Bcos\frac{\sqrt{3}}{2}x + Ccos\frac{\sqrt{3}x}{2}\right]$

P.I = $\frac{1}{D^3+8}(x^4 + 2x + 1)$

$$= \frac{1}{8\left[1+\frac{D^3}{8}\right]}(x^4 + 2x + 1)$$

$$= \frac{1}{8}\left[1 + \frac{D^3}{8}\right]^{-1}(x^4 + 2x + 1)$$

$$=\frac{1}{8}\left[1-\frac{D^3}{8}+\left(\frac{D^3}{8}\right)^2\ldots\right](x^4+2x+1)$$

$$=\frac{1}{8}\left[1-\frac{D^3}{8}\right](x^4+2x+1)$$

$$=\frac{1}{8}\left[(x^4+2x+1)-\frac{D^3}{8}(x^4+2x+1)\right]$$

$$=\frac{1}{8}\left[x^4+2x+1-\frac{24x}{8}\right]$$

$$=\frac{1}{8}[x^4+2x+1-3x]$$

$$=\frac{1}{8}[x^4-x+1]$$

The general solution is $y=C.F+P.I$

$$y=Ae^{-2x}+B^{\frac{1}{2}x}\left[Bcos\frac{\sqrt{3}}{2}x+Ccos\frac{\sqrt{3}}{2}x\right]+\frac{1}{8}[x^4-x+1]$$

Type V: Problems based on R.H.S $e^{ax}F(x)$

P.I $=\frac{1}{f(D+a)}e^{ax}F(x)$ Replace x by D+a

$$=e^{ax}\frac{1}{f(D+a)}F(x)$$

Example :

Solve $(D^2+1)y=xsinhx$

Solution:

Auxiliary Equation is $m^2+1=0$

$$m^2=-1$$

$$m^2=\pm i$$

C.F $=Acosx+Bsinx$

P.I $=\frac{1}{D^2+1}xsinhx$

$$=\frac{1}{D^2+1}\left[x\left(\frac{e^x-e^{-x}}{2}\right)\right]$$

$$=\frac{1}{2}\left[\frac{1}{D^2+1}xe^x-\frac{1}{D^2+1}xe^{-x}\right]$$

Replace $by\ D+1$; Replace D by $D-1$

$$=\frac{1}{2}\left[e^x\frac{1}{(D+1)^2+1}x-e^x\frac{1}{(D-1)^2+1}\right]$$

$$=\frac{1}{2}\left[e^x\frac{1}{D^2+2D+2}x-e^{-x}\frac{1}{D^2-2D+2}x\right]$$

$$= \frac{1}{2}\left[e^{x}\frac{1}{2\left[1+\left(\frac{D^2+2D}{2}\right)\right]}x - e^{-x}\frac{1}{2\left[1+\left(\frac{D^2-2D}{2}\right)\right]}x\right]$$

$$= \frac{1}{2}\left[\frac{e^{x}}{2}\left[1+\left(\frac{D^2+2D}{2}\right)\right]^{-1}x - \frac{e^{-x}}{2}\left[1+\left(\frac{D^2-2D}{2}\right)\right]^{-1}x\right]$$

$$= \frac{1}{2}\left[\frac{e^{x}}{2}\left(1-\frac{D^2}{2}-\frac{2D}{2}\right)x - \frac{e^{-x}}{2}\left(1-\frac{D^2}{2}+\frac{2D}{2}\right)x\right]$$

$$= \frac{1}{2}\left[\frac{e^{x}}{2}(x-1) - \frac{e^{-x}}{2}(x+1)\right]$$

$$= \frac{1}{2}\left[\frac{e^{x}x}{2} - \frac{e^{x}}{2} - \frac{xe^{-x}}{2} - \frac{e^{-x}}{2}\right]$$

$$= \frac{1}{2}\left[x\left(\frac{e^{x}-e^{-x}}{2}\right) - \left(\frac{e^{x}+e^{-x}}{2}\right)\right]$$

$$= \frac{1}{2}[xsinhx - coshx]$$

The general solution is y = C.F + P.I

$$y = Acosx + Bsinx + \frac{1}{2}(xsinhx - coshx)$$

TYPE VI:

Problems based on f(x) = $x^n \sin ax$ or $x^n \cos ax$ $P.I = \frac{1}{f(x)} x^n \sin ax$ or $x^n \cos ax$

Example

Solve $(D^2+1)y = xsin\,x$

Solution:

Auxiliary Equation is $m^2 + 1 = 0$

$$m^2 + 1 = -1$$

$$m = \pm i$$

C.F $= Acos\,x + Bsin\,x$

P.I $= \frac{1}{D^2+1}\, xsin\,x$

$= \frac{1}{D^2+1}\, x\, I.P\ of\ e^{ix}$ Replace D $by D + i$

$= I.P\ of e^{ix}\ \frac{1}{(D+i)^2+1}\ x$

$= I.P\ of e^{ix}\ \frac{1}{D^2+2Di+i^2+1}\ x$

$= I.P\ of e^{ix}\ \frac{1}{D^2+2Di+i^2+1}\ x$

$$= I.P\ of e^{ix}\ \frac{1}{D^2+2Di}\ x$$

$$= I.P\ of e^{ix}\ \frac{1}{2Di}\left(1+\frac{D}{2i}\right)^{-1} x$$

$$= I.P\ of e^{ix}\ \frac{1}{2Di}\left(x-\frac{D(x)}{2i}\right)$$

$$= I.P\ of (\cos x + i\sin x)\ \left(\frac{x^2}{4i}+\frac{x}{4}\right)$$

$$= I.P\ of (\cos x + i\sin x)\ \left(\frac{-x^2 i}{4}+\frac{x}{4}\right)$$

$$= I.P\ of \left(\frac{-ix^2}{4}\cos x + \frac{x\cos x}{4} - \frac{x^2 sinx}{4} + \frac{ix\ sinx}{4}\right)$$

$$= \frac{-x^2}{4}\cos x + \frac{xsin\ x}{4}$$

The general solution is y = C.F + P.I

$$y = Acosx + Bsinx - \frac{x^2}{4}cosx + \frac{xsin\ x}{4}$$

Example:

Solve $(D^2 - 4D + 4)y = 3x^2e^{2x}sin\ 2x$

Solution:

Auxiliary Equation is $m^2 - 4m + 4 = 0$

$m = 2, 2$

C.F = $(A + Bx)e^{2x}$

P.I $= \frac{1}{D^2-4D+4}\ 3x^2e^{2x}sin\ 2x$ Replace D by $D+2$

$$= 3e^{2x}\frac{1}{(D+2)^2-4(D+2)+4}\ x^2sin2x$$

$$= 3e^{2x}\frac{1}{D^2}\ x^2sin2x$$

$= 3e^{2x}\frac{1}{D^2}\ x^2 I.P\ of e^{i2x}$ Replace D by $D+2i$

$$= 3e^{2x}I.P\ of e^{i2x}\frac{1}{(D+2i)^2}\ x^2$$

$$= 3e^{2x}I.P\ of e^{i2x}\frac{1}{-4\left[1-\frac{D^2+4Di}{4}\right]}\ x^2$$

$$= 3e^{2x}I.P\ of e^{i2x}\frac{1}{-4}\left[1-\frac{D^2+4Di}{4}\right]^{-1} x^2$$

$$= 3e^{2x} I.P\ of e^{i2x} \frac{1}{-4}\left[1 + \left(\frac{D^2+4Di}{4}\right) + \left(\frac{D^2+4Di}{4}\right)^2 + \cdots\right] x^2$$

$$= 3e^{2x} I.P\ of e^{i2x} \frac{1}{-4}\left[1 + \frac{D^2}{4} + Di + D^2 i\right](x^2)$$

$$= \frac{3e^{2x}}{-4} I.P\ of (\cos 2x + i\sin 2x)\left(x^2 + \frac{1}{2} + i2x - 2\right)$$

$$= \frac{-3}{4} e^{2x} I.P\ of (\cos 2x + i\sin 2x)\left(x^2 + 2xi - \frac{3}{2}\right)$$

$$= \frac{-3}{4} e^{2x} I.P\ of (x^2 \cos 2x + i2x\cos 2x - \frac{3}{2}\cos 2x + i\,x^2 \sin 2x - 2x\sin 2x - i\frac{3}{2}\sin 2x\ )$$

$$= \frac{-3e^{2x}}{4}\left[2\,x\cos 2x + x^2 \sin 2x - \frac{3}{2}\sin 2x\right]$$

The general solution y = C.F + P.I

$$y = (A + Bx)e^{2x} - \frac{3}{4} e^{2x}\ \left[2x\cos 2x + x^2 \sin 2x - \frac{3}{2}\sin 2x\right]$$