2.1 ELECTRIC POTENTIAL

ELECTRIC FILED OR ELECTRIC FIELD INTENSITY:

The electric field or electric field intensity is defined as the electric force per unit charge .It is given by

$$E = \frac{F}{q}$$

According to coulomb's law

$$F = \frac{Qq}{4\pi\varepsilon r^2}$$

Electric Filed

$$E = \frac{F}{q}$$

Substitute F value in above equation

$$E = \frac{\frac{Qq}{4\pi\varepsilon r^2}}{q}$$

$$E = \frac{Qq}{4\pi\varepsilon r^2 q}$$

$$E = \frac{Q}{4\pi\varepsilon r^2}\ V/m$$

The another unit of electric field is $Volts/meter$

ELECTRIC POTENTIAL DUE TO LINE CHARGE:

Considered uniformly charged line of length $\boldsymbol{L}$ whose linear charge density is $\boldsymbol{\rho_l}$ Coulomb/meter. Consider a small element $\boldsymbol{dl}$ at a distance $\boldsymbol{l}$ from one end of the charged line as shown in figure 2.1.1 .Let $\boldsymbol{P}$ be any point at a distance $\boldsymbol{r}$ from the element $\boldsymbol{dl}$.

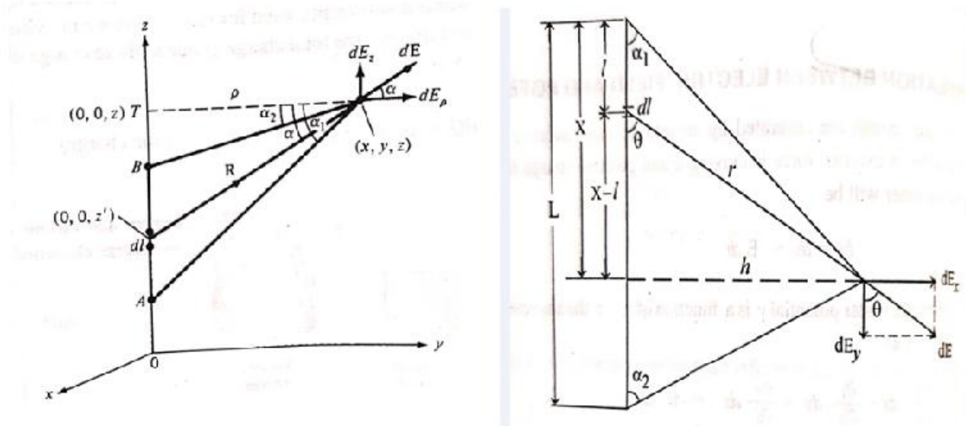


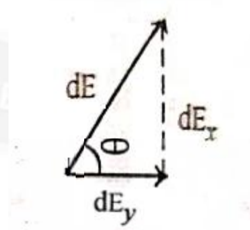
Figure 2.1.1 Evaluation of the electric potential $\boldsymbol{V}$ due to a line charge

[Source: "Elements of Electromagnetics" by Matthew N.O.Sadiku, page-114]

The electric field at a point $\boldsymbol{P}$ due to the charge element $\boldsymbol{\rho dl}$ is given

$$dE = \frac{\rho_l dl}{4\pi\varepsilon r^2}$$

The $\boldsymbol{x}$ and $\boldsymbol{y}$ components of electric field $\boldsymbol{dE}$ are given by



From the above diagram find $\boldsymbol{\sin\theta}$ and $\boldsymbol{\cos\theta}$

$$\sin\theta = \frac{dE_x}{dE}$$

$$dE_x = dE\sin\theta$$

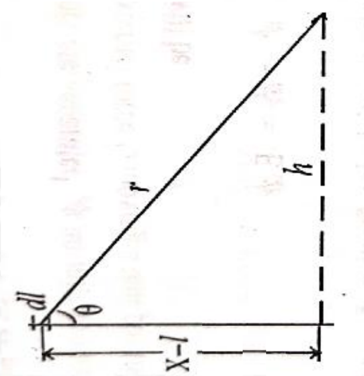
$$\cos\theta = \frac{dE_y}{dE}$$

$$dE_y = dE\cos\theta$$

Substitute $\boldsymbol{dE}$ expression in $\boldsymbol{dE_x}$

$$dE_x = \frac{\rho_l dl \sin\theta}{4\pi\varepsilon r^2}$$

$$dE_y = \frac{\rho_l dl \cos\theta}{4\pi\varepsilon r^2}$$



From the above diagram find $\boldsymbol{\tan\theta}$

$$\tan\theta = \frac{h}{x-l}$$

$$x - l = \frac{h}{\tan\theta}$$

$$x - l = h\cot\theta$$

Differentiate above equation on both sides

$$0 - dl = h(-\operatorname{cosec}^2\theta)$$

$$-dl = -h(\operatorname{cosec}^2\theta)$$

$$dl = h(\operatorname{cosec}^2\theta).d\theta$$

From the above diagram find $\boldsymbol{\sin\theta}$

$$\sin\theta = \frac{h}{r}$$

$$r = \frac{h}{\sin\theta}$$

$$r = h\operatorname{cosec}\theta$$

Substitute $\boldsymbol{dl}$ and $\boldsymbol{r}$ value in $\boldsymbol{dE_x}$

$$dE_x = \frac{\rho_l dl \sin\theta}{4\pi\varepsilon r^2}$$

$$dE_x = \frac{\rho_l h(\operatorname{cosec}^2\theta)\, d\theta \sin\theta}{4\pi\varepsilon\,(h \operatorname{cosec}\theta)^2}$$

$$dE_x = \frac{\rho_l h(\operatorname{cosec}^2\theta)\, d\theta \sin\theta}{4\pi\varepsilon h^2 \operatorname{cosec}^2\theta}$$

$$dE_x = \frac{\rho_l \sin\theta d\theta}{4\pi\varepsilon h}$$

Integrate the above equation dE_x considered the limit as α_1 *to* $\pi - \alpha_2$

The electric field E_x due to the entire length of line charge is given by

$$\int dE_x = \int_{\alpha_1}^{\pi-\alpha_2} \frac{\rho_l \sin\theta d\theta}{4\pi\varepsilon h}$$

$$E_x = \int_{\alpha_1}^{\pi-\alpha_2} \frac{\rho_l \sin\theta d\theta}{4\pi\varepsilon h}$$

$$E_x = \frac{\rho_l}{4\pi\varepsilon h} \int_{\alpha_1}^{\pi-\alpha_2} \sin\theta d\theta$$

$$E_x = \frac{\rho_l}{4\pi\varepsilon h} [-\cos\theta]_{\alpha_1}^{\pi-\alpha_2}$$

$$E_x = \frac{\rho_l}{4\pi\varepsilon h} [-\cos(\pi - \alpha_2) - (\cos\ \alpha_1)]$$

$$E_x = \frac{\rho_l}{4\pi\varepsilon h} [(\cos\alpha_2) + (\cos\ \alpha_1)]$$

$$E_x = \frac{\rho_l}{4\pi\varepsilon h} [(\cos\alpha_1) + (\cos\ \alpha_2)]$$

Substitute dl and r value in dE_x

$$dE_y = \frac{\rho_l h(\operatorname{cosec}^2\theta) d\theta \cos\theta}{4\pi\varepsilon\,(h \operatorname{cosec}\theta)^2}$$

$$dE_y = \frac{\rho_l h(\operatorname{cosec}^2\theta)\, d\theta \cos\theta}{4\pi\varepsilon h^2 \operatorname{cosec}^2\theta}$$

$$dE_y = \frac{\rho_l h(\mathrm{cosec}^2\,\theta)\, d\theta \cos\theta}{4\pi\varepsilon h^2\, \mathrm{cosec}^2\,\theta}$$

$$dE_y = \frac{\rho_l\, d\theta \cos\theta}{4\pi\varepsilon h}$$

$$dE_y = \frac{\rho_l \cos\theta\, d\theta}{4\pi\varepsilon h}$$

Similarly for $\boldsymbol{y}$ component of E

Integrate the above equation $\boldsymbol{dE_y}$ consider the limit as $\boldsymbol{\alpha_1}$ *to* $\boldsymbol{\pi - \alpha_2}$

The electric field $\boldsymbol{E_y}$ due to the entire length of line charge is given by

$$\int dE_y = \int_{\alpha_1}^{\pi-\alpha_2} \frac{\rho_l \cos\theta\, d\theta}{4\pi\varepsilon h}$$

$$E_y = \int_{\alpha_1}^{\pi-\alpha_2} \frac{\rho_l \cos\theta\, d\theta}{4\pi\varepsilon h}$$

$$E_y = \frac{\rho_l}{4\pi\varepsilon h} \int_{\alpha_1}^{\pi-\alpha_2} \cos\theta\, d\theta$$

$$E_y = \frac{\rho_l}{4\pi\varepsilon h} [\sin\theta]_{\alpha_1}^{\pi-\alpha_2}$$

$$E_y = \frac{\rho_l}{4\pi\varepsilon h} [\sin(\pi - \alpha_2) - (\sin\ \alpha_1)]$$

$$E_y = \frac{\rho_l}{4\pi\varepsilon h} [(\sin\alpha_2) - (\sin\ \alpha_1)]$$

Case (i): If the point $\boldsymbol{P}$ is at bisector of a line, then $\boldsymbol{\alpha_1 = \alpha_2 = \alpha}$

$\boldsymbol{E_y = 0}$ $\quad$ $\boldsymbol{E}$ becomes $\boldsymbol{E_x}$

$$E_x = \frac{\rho_l}{4\pi\varepsilon h} [(\cos\alpha_1) + (\cos\ \alpha_2)]$$

$$E_x = \frac{\rho_l}{4\pi\varepsilon h} [(\cos\alpha) + (\cos\alpha)]$$

$$E_x = \frac{\rho_l}{4\pi\varepsilon h}(2\cos\alpha)$$

$$E_x = \frac{\rho_l}{2\pi\varepsilon h}(\cos\alpha)$$

$$E_y = \frac{\rho_l}{4\pi\varepsilon h}[(\sin\alpha_2) - (\sin\ \alpha_1)]$$

Substitute $\alpha_1 = \alpha_2 = \alpha$

$$E_y = \frac{\rho_l}{4\pi\varepsilon h}[(\sin\alpha) - (\sin\alpha)]$$

$$E_y = \frac{\rho_l}{4\pi\varepsilon h}[0]$$

$$E_y = 0$$

E becomes E_x

$$E = E_x$$

$$E = E_x = \frac{\rho_l}{2\pi\varepsilon h}(\cos\alpha)$$

$$E = \frac{\rho_l}{2\pi\varepsilon h}(\cos\alpha)$$

Case (ii): If the line is infinitely long then $\alpha_1 = \alpha_2 = \alpha = 0$

$E_y = 0$ E becomes E_x

$$E_x = \frac{\rho_l}{4\pi\varepsilon h}[(\cos\alpha_1) + (\cos\ \alpha_2)]$$

$$E_x = \frac{\rho_l}{4\pi\varepsilon h}[(\cos 0) + (\cos 0)]$$

$$E_x = \frac{\rho_l}{4\pi\varepsilon h}[(1) + (1)]$$

$$E_x = \frac{\rho_l}{4\pi\varepsilon h}[2]$$

$$E_x = \frac{\rho_l}{2\pi\varepsilon h}$$

$$E_y = \frac{\rho_l}{4\pi\varepsilon h}[(\sin\alpha_2) - (\sin\ \alpha_1)]$$

Substitute $\alpha_1 = \alpha_2 = \alpha = 0$

$$E_y = \frac{\rho_l}{4\pi\varepsilon h}[(\sin 0) - (\sin 0)]$$

$$E_y = \frac{\rho_l}{4\pi\varepsilon h}[(0) - (0)]$$

$$E_y = \frac{\rho_l}{4\pi\varepsilon h}[0]$$

$$E_y = 0$$

E becomes E_x

$$E = E_x$$

$$E = E_x = \frac{\rho_l}{2\pi\varepsilon h}$$

$$E = \frac{\rho_l}{2\pi\varepsilon h}$$

Work done

$$W = -\int_{r_1}^{r_2} q\, E\, dh$$

Substitute E equation in W

$$W = -\int_{r_1}^{r_2} q\, E\, dh$$

$$W = -\int_{r_1}^{r_2} q\, \frac{\rho_l}{2\pi\varepsilon h}\, dh$$

$$W = -\frac{q\rho_l}{2\pi\varepsilon}\int_{r_1}^{r_2} \frac{1}{h}\, dh$$

$$W = -\frac{q\rho_l}{2\pi\varepsilon}[\ln h]_{r_1}^{r_2}$$

$$W = -\frac{q\rho_l}{2\pi\varepsilon}[(\ln r_2) - (\ln r_1)]$$

Multiply the common minus term with inside terms

$$W = \frac{q\rho_l}{2\pi\varepsilon}[-(\ln r_2) - (-\ln r_1)]$$

$$W = \frac{q\rho_l}{2\pi\varepsilon}[-(\ln r_2) + (\ln r_1)]$$

$$W = \frac{q\rho_l}{2\pi\varepsilon}[(\ln r_1) - (\ln r_2)]$$

$$W = \frac{q\rho_l}{2\pi\varepsilon}\left[\frac{r_1}{r_2}\right]$$

Electric Potential Difference

$$V = \frac{W}{q}$$

Substitute W value in above equation

$$V = \frac{W}{q}$$

$$V = \frac{\frac{q\rho_l}{2\pi\varepsilon}\left[\frac{r_1}{r_2}\right]}{q}$$

$$V = \frac{q\rho_l}{2\pi\varepsilon q}\left[\frac{r_1}{r_2}\right]$$

$$V = \frac{\rho_l}{2\pi\varepsilon}\left[\frac{r_1}{r_2}\right]$$

ELECTRIC POTENTIAL DUE TO CIRCULAR DISC:

Consider a circular disc of radius $\boldsymbol{R}$ is charged uniformly with a charge density of $\rho_s\ coulomb/n^2$ .Let $\boldsymbol{P}$ be any point on the axis of the disc at a distance from the centre. Consider an annular ring of radius $\boldsymbol{r}$ and of radial thickness $\boldsymbol{dr}$ as shown in figure 2.1.2.The area of the annular ring is $\boldsymbol{ds} = \boldsymbol{2\pi r dr}$.

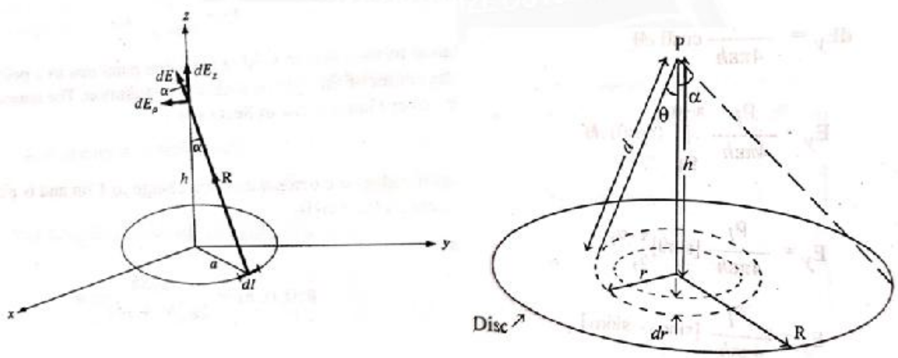


Figure 2.1.2 Evaluation of the *E* field due to a charged ring

[Source: "Elements of Electromagnetics" by Matthew N.O.Sadiku, page-120]

The field intensity at point $\boldsymbol{P}$ due to the charged annular ring is given by

$$dE = \frac{\rho_S ds}{4\pi\varepsilon d^2}$$

Since the horizontal component of electric field intensity is zero, The horizontal components and vertical components are dE_x and dE_y

The horizontal components of angular ring is zero

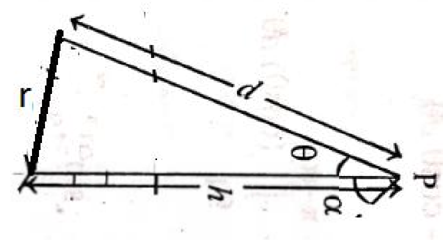
$$dE_x = 0$$

$$E_x = 0$$

The horizontal components of angular ring E_y have to find for circular ring.

the vertical component is given by

$$dE_y = \frac{\rho_S ds \cos\theta}{4\pi\varepsilon d^2}$$



From the above diagram find $\tan\theta$ *and* $\sin\theta$

$$\tan\theta = \frac{r}{h}$$

$$r = h\tan\theta$$

$$\sin\theta = \frac{r}{d}$$

$$d = \frac{r}{sin\,\theta}$$

Assume $$ds = 2\pi r dr$$

$$dE_y = \frac{\rho_S ds\cos\theta}{4\pi\varepsilon d^2}$$

Substitute ds in dE_y

$$dE_y = \frac{\rho_S 2\pi r dr\cos\theta}{4\pi\varepsilon d^2}$$

$$r = h\tan\theta$$

Differentiate above equation

$$dr = h\sec^2\theta d\theta$$

Substitute dr and d in dE_y

$$dE_y = \frac{\rho_S(2\pi r)h\sec^2\theta d\theta\cos\theta}{4\pi\varepsilon d^2}$$

$$dE_y = \frac{\rho_S(2\pi r)h\sec^2\theta d\theta\cos\theta}{4\pi\varepsilon\left(\frac{r}{\sin\theta}\right)^2}$$

$$dE_y = \frac{\rho_S(2\pi r)(h\sec^2\theta)d\theta\cos\theta\ \sin^2\theta}{4\pi\varepsilon r^2}$$

$$dE_y = \frac{\rho_S(2\pi r)(h\sec^2\theta)\sin^2\theta\cos\theta\,d\theta}{4\pi\varepsilon r^2}$$

$$dE_y = \frac{\rho_S(2\pi r)(h)\sin^2\theta\cos\theta\,d\theta}{4\pi\varepsilon r^2\ \cos^2\theta}$$

$$dE_y = \frac{\rho_S(2\pi r)(h)\sin^2\theta d\theta}{4\pi\varepsilon r^2\ \cos\theta}$$

$$dE_y = \frac{\rho_S(2\pi r)(h)\tan\theta\sin\theta\,d\theta}{4\pi\varepsilon r^2}$$

$$dE_y = \frac{\rho_S(2\pi r)(h)\tan\theta\sin\theta\,d\theta}{4\pi\varepsilon r^2}$$

$$dE_y = \frac{\rho_S(h)\tan\theta\sin\theta\,d\theta}{2\varepsilon r}$$

Substitute $\boldsymbol{r}$ in $\boldsymbol{dE_y}$

$$dE_y = \frac{\rho_S(h)\tan\theta\sin\theta\,d\theta}{2\varepsilon r}$$

$$dE_y = \frac{\rho_S(h)\tan\theta\sin\theta\,d\theta}{2\varepsilon\,h\tan\theta}$$

$$dE_y = \frac{\rho_S\sin\theta\,d\theta}{2\varepsilon}$$

Integrate the above equation $\boldsymbol{dE_y}$ considered the limit as $\mathbf{0}\ to\ \boldsymbol{\alpha}$

$$\int dE_y = \int_0^{\alpha}\frac{\rho_S\,sin\,\theta\,d\theta}{2\varepsilon}$$

$$\int dE_y = \frac{\rho_S}{2\varepsilon}\int_0^{\alpha} sin\,\theta\,d\theta$$

$$E_y = \frac{\rho_S}{2\varepsilon}\left[-\cos\theta\right]_0^{\alpha}$$

$$E_y = \frac{\rho_S}{2\varepsilon} [(-\cos\alpha) - (-\cos 0)]$$

$$E_y = \frac{\rho_S}{2\varepsilon} [(-\cos\alpha) + (1)]$$

$$E_y = \frac{\rho_S}{2\varepsilon} [(1) + (-\cos\alpha)]$$

$$E_y = \frac{\rho_S}{2\varepsilon} [1 - \cos\alpha]$$

The total electric field $E = E_x + E_y$

$$E = E_x + E_y$$

$$E_x = 0$$

$$E_y = \frac{\rho_S}{2\varepsilon} [1 - \cos\alpha]$$

$$E = 0 + \frac{\rho_S}{2\varepsilon} [1 - \cos\alpha]$$

$$E = \frac{\rho_S}{2\varepsilon} [1 - \cos\alpha]$$

The electric potential V at any point P due to charge disc

$$V = -\int_d^0 E dx$$

Substitute E value in above equation

$$V = -\int_d^0 \frac{\rho_S}{2\varepsilon} [1 - \cos\alpha]\, dx$$

Substitute $\alpha = \theta$ in above equation

$$V = -\int_d^0 \frac{\rho_S}{2\varepsilon} [1 - \cos\theta]\, dx$$

Integrate above equation with respect to x

$$V = -\frac{\rho_S}{2\varepsilon}\,[1-\cos\theta]\int_d^0 dx$$

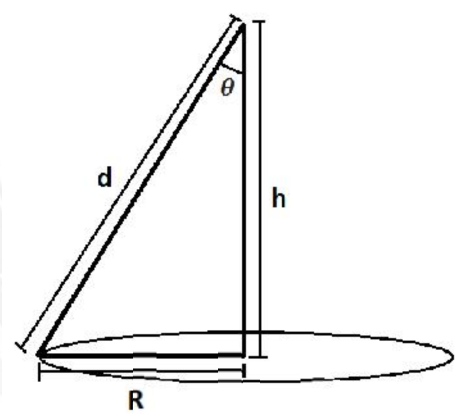
$$V = -\frac{\rho_S}{2\varepsilon}\,[1-\cos\theta][x]_d^0$$

$$V = -\frac{\rho_S}{2\varepsilon}\,[1-\cos\theta]\,[(0)-(d)]$$

$$V = -\frac{\rho_S}{2\varepsilon}\,[1-\cos\theta]\,[-(d)]$$

$$V = \frac{\rho_S}{2\varepsilon}\,[1-\cos\theta]\,[(d)]$$

$$V = \frac{\rho_S d}{2\varepsilon}\,[1-\cos\theta]$$



$$d^2 = R^2 + h^2$$

$$d = \sqrt{R^2 + h^2}$$

$$\cos\theta = \frac{h}{d}$$

Substitute d value in above equation

$$\cos\theta = \frac{h}{\sqrt{R^2 + h^2}}$$

Substitute $\cos\theta$ equation in V

$$V = \frac{\rho_S d}{2\varepsilon}\,[1 - \cos\theta]$$

$$V = \frac{\rho_S \sqrt{R^2 + h^2}}{2\varepsilon}\left[1 - \frac{h}{\sqrt{R^2 + h^2}}\right]$$

$$V = \frac{\rho_S \sqrt{R^2 + h^2}}{2\varepsilon} \times \left[\frac{\sqrt{R^2 + h^2} - h}{\sqrt{R^2 + h^2}}\right]$$

$$V = \frac{\rho_S \sqrt{R^2 + h^2}}{2\varepsilon} \times \left(\frac{1}{\sqrt{R^2 + h^2}}\right) \times \left[\sqrt{R^2 + h^2} - h\right]$$

$$V = \frac{\rho_S}{2\varepsilon}\left[\sqrt{R^2 + h^2} - h\right]$$

ELECTRIC POTENTIAL DUE TO INFINITE SHEET OF CHARGE:

Consider an infinite plane sheet which is uniformly charged with a charge density of ρ_s ***Coulom/n*** ^{2} as shown in figure 2.1.3.

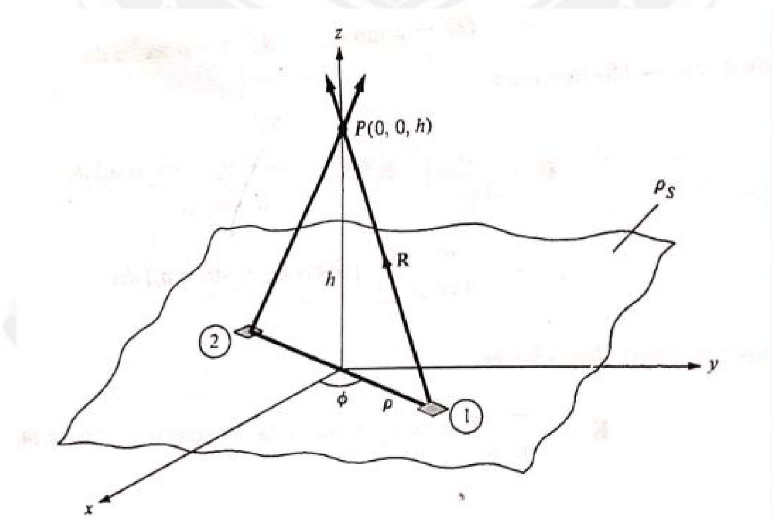


Figure 2.1.3 Evaluation of the *E* field due to an infinite sheet of charge

[Source: "Elements of Electromagnetics" by Matthew N.O.Sadiku, page-116]

The field intensity at any point $\boldsymbol{P}$ due to infinite plane sheet of charge can be evaluated by applying expression of charged circular disc.

The field intensity at point $\boldsymbol{P}$ due to the charged annular ring is given by

$$dE = \frac{\rho_S ds}{4\pi\varepsilon d^2}$$

Since the horizontal component of electric field intensity is zero, The horizontal components and vertical components are dE_x and dE_y

The horizontal components of angular ring is zero

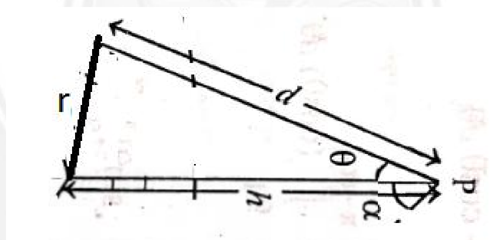
$$dE_x = 0$$

$$E_x = 0$$

The horizontal components of angular ring E_y have to find for circular ring.

the vertical component is given by

$$dE_y = \frac{\rho_S ds \cos\theta}{4\pi\varepsilon d^2}$$



From the above diagram find $\tan\theta$ *and* $\sin\theta$

$$\tan\theta = \frac{r}{h}$$

$$r = h\tan\theta$$

$$\sin\theta = \frac{r}{d}$$

$$d = \frac{r}{\sin\theta}$$

Assume $$ds = 2\pi r dr$$

$$dE_y = \frac{\rho_S ds \cos\theta}{4\pi\varepsilon d^2}$$

Substitute ds in dE_y

$$dE_y = \frac{\rho_S 2\pi r dr \cos\theta}{4\pi\varepsilon d^2}$$

$$r = h\tan\theta$$

Differentiate above equation

$$dr = h\sec^2\theta d\theta$$

Substitute dr and d in dE_y

$$dE_y = \frac{\rho_S(2\pi r)h\sec^2\theta d\theta\cos\theta}{4\pi\varepsilon d^2}$$

$$dE_y = \frac{\rho_S(2\pi r)h\sec^2\theta d\theta\cos\theta}{4\pi\varepsilon\left(\frac{r}{\sin\theta}\right)^2}$$

$$dE_y = \frac{\rho_S(2\pi r)(h\sec^2\theta)d\theta\cos\theta\ \sin^2\theta}{4\pi\varepsilon r^2}$$

$$dE_y = \frac{\rho_S(2\pi r)(h\sec^2\theta)\sin^2\theta\cos\theta\, d\theta}{4\pi\varepsilon r^2}$$

$$dE_y = \frac{\rho_S(2\pi r)(h)\sin^2\theta\cos\theta\, d\theta}{4\pi\varepsilon r^2\ \cos^2\theta}$$

$$dE_y = \frac{\rho_S(2\pi r)(h)\sin^2\theta d\theta}{4\pi\varepsilon r^2\ \cos\theta}$$

$$dE_y = \frac{\rho_S(2\pi r)(h)\tan\theta\sin\theta\, d\theta}{4\pi\varepsilon r^2}$$

$$dE_y = \frac{\rho_S(2\pi r)(h)\tan\theta\sin\theta\, d\theta}{4\pi\varepsilon r^2}$$

$$dE_y = \frac{\rho_S(h)\tan\theta\sin\theta\, d\theta}{2\varepsilon r}$$

Substitute r in dE_y

$$dE_y = \frac{\rho_S(h)\tan\theta\sin\theta\, d\theta}{2\varepsilon r}$$

$$dE_y = \frac{\rho_S(h)\tan\theta\sin\theta\, d\theta}{2\varepsilon\, h\tan\theta}$$

$$dE_y = \frac{\rho_S \sin\theta \, d\theta}{2\varepsilon}$$

Integrate the above equation dE_y consider the limit as $0 \ to \ \alpha$

$$\int dE_y = \int_0^{\alpha} \frac{\rho_S \sin\theta \, d\theta}{2\varepsilon}$$

$$\int dE_y = \frac{\rho_S}{2\varepsilon} \int_0^{\alpha} \sin\theta \, d\theta$$

$$E_y = \frac{\rho_S}{2\varepsilon} [-\cos\theta]_0^{\alpha}$$

$$E_y = \frac{\rho_S}{2\varepsilon} [(-\cos\alpha) - (-\cos 0)]$$

$$E_y = \frac{\rho_S}{2\varepsilon} [(-\cos\alpha) + (1)]$$

$$E_y = \frac{\rho_S}{2\varepsilon} [(1) + (-\cos\alpha)]$$

$$E_y = \frac{\rho_S}{2\varepsilon} [1 - \cos\alpha]$$

The total electric field $E = E_x + E_y$

$$E = E_x + E_y$$

$$E_x = 0$$

$$E_y = \frac{\rho_S}{2\varepsilon} [1 - \cos\alpha]$$

$$E = 0 + \frac{\rho_S}{2\varepsilon} [1 - \cos\alpha]$$

$$E = \frac{\rho_S}{2\varepsilon} [1 - \cos\alpha]$$

The electric field due to infinite uniformly charge sheet $\alpha = 90°$

$$E = \frac{\rho_S}{2\varepsilon}\,[1 - \cos\alpha]$$

$$E = \frac{\rho_S}{2\varepsilon}\,[1 - \cos 90°]$$

$$E = \frac{\rho_S}{2\varepsilon}\,[1 - 0]$$

$$E = \frac{\rho_S}{2\varepsilon}\,[1]$$

$$E = \frac{\rho_S}{2\varepsilon}$$

The electric potential *V* at any point *P* is given by

$$V = -\int_d^0 E dx$$

Substitute E value in above equation

$$V = -\int_d^0 \frac{\rho_S}{2\varepsilon}\, dx$$

Substitute $\alpha = \theta$ in above equation

$$V = -\int_d^0 \frac{\rho_S}{2\varepsilon}\, dx$$

Integrate above equation with respect to x

$$V = -\frac{\rho_S}{2\varepsilon}\int_d^0 dx$$

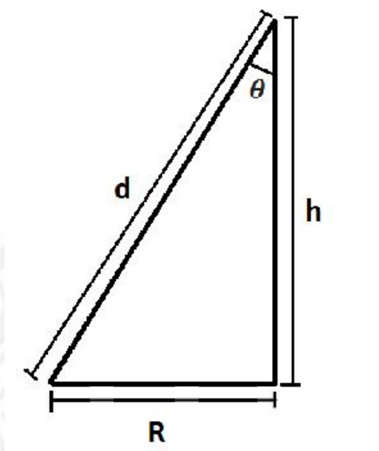
$$V = -\frac{\rho_S}{2\varepsilon}\,[x]_d^0$$

$$V = -\frac{\rho_S}{2\varepsilon}\,[(0) - (d)]$$

$$V = -\frac{\rho_S}{2\varepsilon}\,[-(d)]$$

$$V = \frac{\rho_S}{2\varepsilon}\ [(d)]$$

$$V = \frac{\rho_S d}{2\varepsilon}$$



$$d^2 = R^2 + h^2$$

$$d = \sqrt{R^2 + h^2}$$

Substitute $\boldsymbol{d}$ equation in $\boldsymbol{V}$

$$V = \frac{\rho_S d}{2\varepsilon}$$

$$V = \frac{\rho_S \sqrt{R^2 + h^2}}{2\varepsilon}$$

$$V = \frac{\rho_S}{2\varepsilon}\left[\sqrt{R^2 + h^2}\right] \quad volts$$

COAXIAL CYLINDER

Consider the two coaxial cylindrical conductors forming a coaxial cable. The radius of the inner cylinder is $\boldsymbol{a}$ while the radius of the outer cylinder is $\boldsymbol{b}$.The coaxial cable is shown in figure 2.1.4.The length of cable is $\boldsymbol{L}$.

The line charge density of inner cylinder is $\boldsymbol{\rho_l}$. The line charge density of inner cylinder is $-\boldsymbol{\rho_l}$.

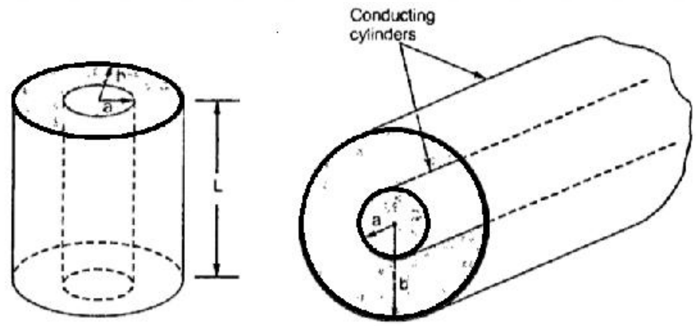


Figure 2.1.4 Coaxial Cable

[Source: "Electromagnetic Theory" by U.A.Bakshi, page-3.19]

In outer side the integral of electric flux density over a space is equal to charge.

$$\int Dd_s = Q$$

The line charge density

$$\rho_l = \frac{Q}{l} \quad Coulomb/meter(c/n)$$

$$\rho_l = \frac{Q}{l}$$

$$Q = \rho_l l$$

Substitute Q`in $\int Dd_s$

$$\int Dd_s = Q$$

$$\int Dd_s = \rho_l l$$

Substitute D`in $\int Dd_s$ equation

$$D = \varepsilon E$$

$$\int \varepsilon E\, d_s = \rho_l l$$

$$\varepsilon E \int d_s = \rho_l l$$

$$\int d_s = S = A$$

Substitute $\int d_s$`value in above equation

$$\varepsilon E A = \rho_l l$$

$$E = \frac{\rho_l l}{\varepsilon A}$$

Area of Cylinder $A = 2\pi r l$

$$E = \frac{\rho_l l}{\varepsilon 2\pi r l}$$

$$E = \frac{\rho_l}{\varepsilon 2\pi r}$$

$$E = \frac{\rho_l}{2\pi\varepsilon r}$$

The potential difference between the two cylinders

$$V = -\int_b^a E\,dr$$

Substitute $\boldsymbol{E}$ in $\boldsymbol{V}$

$$V = -\int_b^a \frac{\rho_l}{2\pi\varepsilon r}\,dr$$

$$V = -\frac{\rho_l}{2\pi\varepsilon}\int_b^a \frac{1}{r}\,dr$$

$$V = -\frac{\rho_l}{2\pi\varepsilon}[\ln r]_b^a$$

$$V = -\frac{\rho_l}{2\pi\varepsilon}[(\ln a) - (\ln b)]$$

$$V = \frac{\rho_l}{2\pi\varepsilon}[-(\ln a) - (-\ln b)]$$

$$V = \frac{\rho_l}{2\pi\varepsilon}[(\ln b) - (\ln a)]$$

$$V = \frac{\rho_l}{2\pi\varepsilon}\left[\ln\left(\frac{b}{a}\right)\right]$$

The electric fields can b written as in terms of potential

$$V = \frac{\rho_l}{2\pi\varepsilon}\left[\ln\left(\frac{b}{a}\right)\right]$$

Substitute $\boldsymbol{E}$ expression in above equation

$$V = \frac{\rho_l}{2\pi\varepsilon}\left[\ln\left(\frac{b}{a}\right)\right]$$

$$\frac{\rho_l}{2\pi\varepsilon} = Er$$

$$V = -\frac{\rho_l}{2\pi\varepsilon}\left[\ln\left(\frac{b}{a}\right)\right]$$

$$V = Er\left[\ln\left(\frac{b}{a}\right)\right]$$

$$E = \frac{V}{r\left[\ln\left(\frac{b}{a}\right)\right]}$$

ELECTRIC POTENTIAL DUE TO SHELL OF CHARGE

Electric Potential Single Shell of Charge:

A positive charge $\boldsymbol{Q}$ is uniformly distributed over a spherical surface of radius as shown in figure.

By applying Gauss's law inside the shell the integral of flux density $\boldsymbol{D}$ over a spherical surface is zero as no charge is enclosed by the surface.

$$\oint D\,ds = 0 \qquad r < a$$

$$D = \varepsilon E$$

Substitute $\boldsymbol{D}$ expression in $\boldsymbol{D\,ds}$

$$\oint D\,ds = 0$$

$$\oint \varepsilon E\, ds = 0$$

$$\varepsilon \oint E\, ds = 0$$

$$E = 0 \qquad\qquad r < a$$

Electric field is zero inside the shell.

By applying Gauss's law just outside the shell, the integral of flux density D over a spherical surface is the charge of the shell.

$$\oint_s D\, ds = Q$$

$$D = \varepsilon E$$

Substitute D expression in $D\, ds$

$$\oint_s D\, ds = Q$$

$$\oint_s \varepsilon E ds = Q$$

$$\varepsilon E \oint_s ds = Q$$

$$\oint_s ds = S = A$$

$$\oint_s ds = A$$

Area of sphere

$$A = 4\pi r^2$$

Substitute A expression in above equation

$$\oint_s ds = A$$

$$\oint_s ds = 4\pi r^2$$

Substitute $\oint_s ds$ expression in $\oint D\, ds$ equation

$$\varepsilon E \oint_s ds = Q$$

$$\varepsilon E 4\pi r^2 = Q$$

$$E = \frac{Q}{4\pi\varepsilon r^2}$$

This is the electric field just outside the spherical shell

The potential just outside the shell is

$$V = -\int E\, dr$$

Substitute V expression in E equation

$$V = -\int E\, dr$$

$$V = -\int \frac{Q}{4\pi\varepsilon r^2}\, dr$$

$$V = -\frac{Q}{4\pi\varepsilon r^2}\int \frac{1}{r^2}\, dr$$

$$V = -\frac{Q}{4\pi\varepsilon r^2}\int r^{-2}\, dr$$

Integrate the above equation V

$$V = -\frac{Q}{4\pi\varepsilon}\frac{r^{-2+1}}{-2+1}$$

$$V = -\frac{Q}{4\pi\varepsilon}\frac{r^{-1}}{(-1)}$$

$$V = -\frac{Q}{4\pi\varepsilon}\frac{-1}{(r)}$$

$$V = \frac{Q}{4\pi\varepsilon \times r}$$

$$V = \frac{Q}{4\pi\varepsilon r} \qquad r > a$$

At $r = a$ electric potential on the sphere

$$V = \frac{Q}{4\pi\varepsilon r}$$

Substitute $r = a$ in above equation

$$V = \frac{Q}{4\pi\varepsilon a}$$

Since electric field E inside the shell is zero. It requires no work to move a test charge inside the shell and hence the electric potential V inside the shell is constant.

$$V = -\int E\, d r = Constant$$

$$V = \frac{Q}{4\pi\varepsilon a} \qquad r < a$$

ELECTRIC POTENTIAL TWO CONCENTRIC SHELL OF CHARGE:

Electric field intensity between two shells:

Consider two spherical shells of radius a and b.Let $Q_1$and Q_2 be the charges uniformly distributed over the inner shell of radius a and outer shell of radius b respectively.

By applying Gauss's law the line integral of flux density D over a closed surface is zero.

$$\oint D\, ds = 0 \qquad r < a$$

$$D = \varepsilon E$$

Substitute D expression in $D\, ds$

$$\oint D\, ds = 0$$

$$\oint \varepsilon E\, ds = 0$$

$$\varepsilon \oint E\, ds = 0$$

$$E = 0 \qquad r < a$$

Electric field is zero inside the shell.

The electric field intensity between the two shells $(\boldsymbol{a} < r < b)$

$$E = \frac{Q_1}{4\pi\varepsilon r^2} \qquad (\boldsymbol{a} < r < b)$$

The electric field intensity just outside both shells due to Q_1 and Q_2 $(\boldsymbol{r} > b > a)$

$$E = \frac{Q_1 + Q_2}{4\pi\varepsilon r^2} \qquad (\boldsymbol{r} > b > a)$$

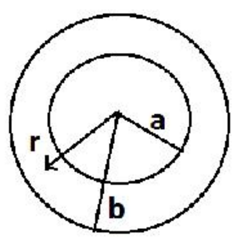
At the inner shell $\boldsymbol{r} = \boldsymbol{a}$ the electric field intensity

$$E = \frac{Q_1}{4\pi\varepsilon r^2}$$

At the outer shell $\boldsymbol{r} = \boldsymbol{b}$ the electric field intensity

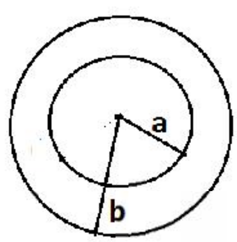
$$E = \frac{Q_1 + Q_2}{4\pi\varepsilon r^2}$$

The variation of electric field intensity is shown in figure



The potential between two concentric shells:

The potential difference between the two shells is given by



$$V = -\int_b^a E\, dr$$

Substitute $\boldsymbol{E}$ expression in above equation

$$E = \frac{Q}{4\pi\varepsilon r^2}$$

$$V = -\int_b^a \frac{Q}{4\pi\varepsilon r^2}\, dr$$

Integrate above equation with respect to r

$$V = -\int_b^a \frac{Q}{4\pi\varepsilon r^2}\, dr$$

$$V = -\frac{Q}{4\pi\varepsilon r^2}\int_b^a \frac{1}{r^2}\, dr$$

$$V = -\frac{Q}{4\pi\varepsilon r^2}\int_b^a r^{-2} dr$$

$$V = -\frac{Q}{4\pi\varepsilon r^2}\left[\frac{r^{-2+1}}{-2+1}\right]_b^a$$

$$V = -\frac{Q}{4\pi\varepsilon r^2}\left[\frac{r^{-1}}{(-1)}\right]_b^a$$

$$V = -\frac{Q}{4\pi\varepsilon r^2}\left[\frac{(-1)}{r}\right]_b^a$$

$$V = \frac{Q}{4\pi\varepsilon r^2}\left[\frac{(1)}{r}\right]_b^a$$

$$V = \frac{Q}{4\pi\varepsilon r^2}\left[\left(\frac{1}{a}\right) - \left(\frac{1}{b}\right)\right]$$

$$V = \frac{Q}{4\pi\varepsilon r^2}\left[\frac{1}{a} - \frac{1}{b}\right]$$

If Q_1 be the charge distributed over inner shell and Q_2 be the charge distributed over outer shell, then potential difference

$$V = \frac{1}{4\pi\varepsilon r^2}\left[\frac{Q_1}{a} - \frac{Q_2}{b}\right]$$