

5.1. STIFFNESS METHOD.

5.1.1. INTRODUCTION:

Stiffness method is the more popular younger brother of Flexibility method. Although the two methods are the opposites to each other, they are akin to each other in several respects.

Like in Flexibility method, this also involves generating element matrices, assembling them to get the system matrix and inverting the same to solve for nodal displacements, member displacements and eventually member forces. In the solution of the structure the important result is the member forces. Displacements are generally of lesser importance.

However in stiffness method we get to the displacements first and thence to member forces.

5.1.2. PROPERTIES OF THE STIFFNESS MATRIX:

It is a symmetric matrix and the sum of elements in any columns must be equal to zero. It is an unstable element therefore the determinant is equal to zero.

The given indeterminate structure is first made kinematically determinate by introducing constraints at the nodes. The required number of constraints is equal to degrees of freedom at the nodes that is kinematic indeterminacy k . The kinematically determinate structure comprises of fixed ended members, hence, all nodal displacements are zero. These results in stress resultant discontinuities at these nodes under the action of applied loads or in other words the clamped joints are not in equilibrium.

In order to restore the equilibrium of stress resultants at the nodes the nodes are imparted suitable unknown displacements. The number of simultaneous equations representing joint equilibrium of forces is equal to kinematic indeterminacy k . Solution of these equations gives unknown nodal displacements. Using stiffness properties of members the member end forces are computed and hence the internal forces throughout the structure.

Since nodal displacements are unknowns, the method is also called displacement method. Since equilibrium conditions are applied at the joints the method is also called equilibrium method. Since stiffness properties of members are used the method is also called stiffness method.

5.1.3. ELEMENT AND GLOBAL STIFFNESS MATRICES

Local co ordinates

In the analysis for convenience we fix the element coordinates coincident with the member axis called element (or) local coordinates (coordinates defined along the individual member axis)

Global co ordinates

It is normally necessary to define a coordinate system dealing with the entire structure is called system on global coordinates (Common coordinate system dealing with the entire structure)

Transformation matrix

The connectivity matrix which relates the internal forces Q and the external forces R is known as the force transformation matrix. Writing it in a matrix form,

$$\{Q\} = [b] \{R\}$$

Where; Q = member force matrix/vector,

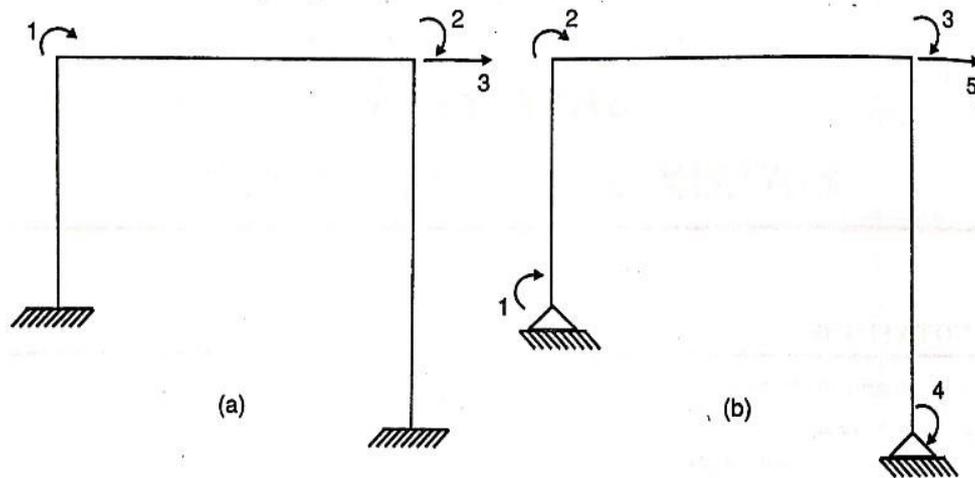
b = force transformation matrix

R = external force/load matrix/ vector

5.1.4.RESTRAINED STRUCTURE

In the Flexibility methods the difficulty of solving a structure increases with the static indeterminacy of a structure.

In stiffness methods the difficulty increases with its kinematic indeterminacy. Thus, structures with more constraints (supports, fixities etc.) are more easily solved than structures with more freedom. Strangely, the structure in fig, (a) is easier to tackle than the structure in fig, (b). Thus we have to get familiar with kinematic indeterminacies or freedoms.



5.1.5. PIN JOINTED FRAMES

In the case of pin jointed plane frames, we have to assign two degrees of freedom to each node.

The elements in trusses are very distinct. Each element shall have one degree of freedom for each end except the ends that are restrained. Normally the questions of forces not at co-ordinates will not arise in trusses.

An introduction to the stiffness method was given in the previous Page. The basic principles involved in the analysis of beams, trusses were discussed. The problems were solved with hand computation by the direct application of the basic principles.

In this session a formal approach has been discussed which may be readily programmed on a computer. In this less on the direct stiffness method as applied to planar truss structure is discussed.

Planetrusses are made up of short thin members inter connected a thin gesto form triangulated patterns. A hinge connection can only transmit forces from one member to another member but not the moment. For analysis purpose, the truss is loaded at the joints. Hence, a truss member is subjected to only axial forces and the forces remain constant along the length of the member. The forces in the member at its two ends must be of the same magnitude but act in the opposite directions for equilibrium.