3.4 ANALYSIS OF TWO HINGED ARCHES

Two hinged arches

Arches are curved structural members used to support heavy loads on large spans. When the load is imposed on an arch, it remains primarily under compression. Thus, it receives lower bending moments.

As the name indicates, a two hinged arch is an arch hinged on its two supports. Also, it is a statically indeterminate structures. Since the horizontal thrust cannot be calculated by equilibrium equations.

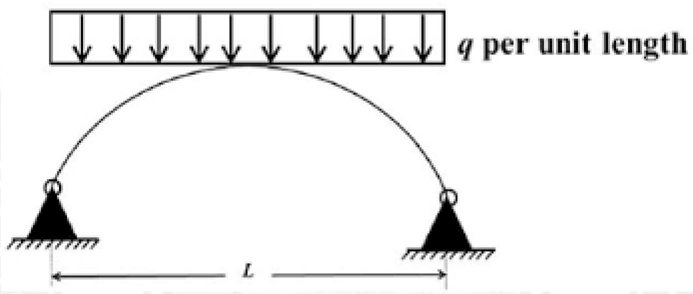

Fig. 3.4.1 Two hinged arches

Reaction locus

Reaction locus is a line that gives the intersection point of two reactions for a particular position of load.

Methods of analysis of two hinged arches.

a. Strain energy principle method or first theorem Castigliono.
b. Consistent deformation method or unit load method

Internal stress resultants induced in arch section

a. Normal thrust
b. Radial shear

Rib-shortening in the case of arches.

In a two hinged arch, the normal thrust which is a compressive force along the axis of the arch will shorten the rib of the arch. This in turn will release part of the horizontal thrust. Normally, this effect is not considered in the analysis (in the case of two hinged arches).

Depending upon the importance of the work we can either take into account or omit the effect of rib shortening. This will be done by considering (or omitting) strain energy due to axial compression along with the strain energy due to bending in evaluating H.

Analysis of two-hinged arch

A typical two-hinged arch is shown in Fig. 33.1a. In the case of two-hinged arch, we have four unknown reactions, but there are only three equations of equilibrium available. Hence, the degree of statically indeterminacy is one for twohinged arch.

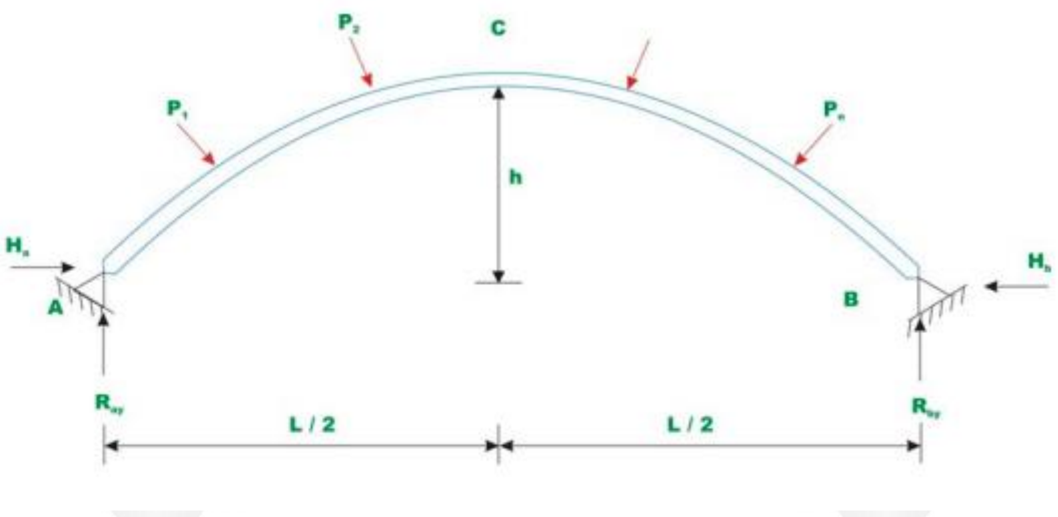

Fig. 3.4.2 Two hinged arches

The fourth equation is written considering deformation of the arch. The unknown redundant reaction is calculated by noting that the horizontal displacement of hinge Hb B is zero. In general the horizontal reaction in the two hinged arch is evaluated by straightforward application of the theorem of least work (see module 1, lesson 4), which states that the partial derivative of the strain energy of a statically indeterminate structure with respect to statically indeterminate action should vanish. Hence to obtain, horizontal reaction, one must develop an expression for strain energy. Typically, any section of the arch (vide Fig 33.1b) is subjected to shear forceV , bending moment M and the axial compression . The strain energy due to bending is calculated from the following expression.

Example:

A parabolic arch hinged at ends has a span of 60m and a rise of 12m.A concentrated load of 8KN act at 15m from the left hinge. The second moment of area varies as the secant of the inclination of arch axis. calculate the horizontal thrust and the reaction at the hinge also calculate the net bending moment of the section.

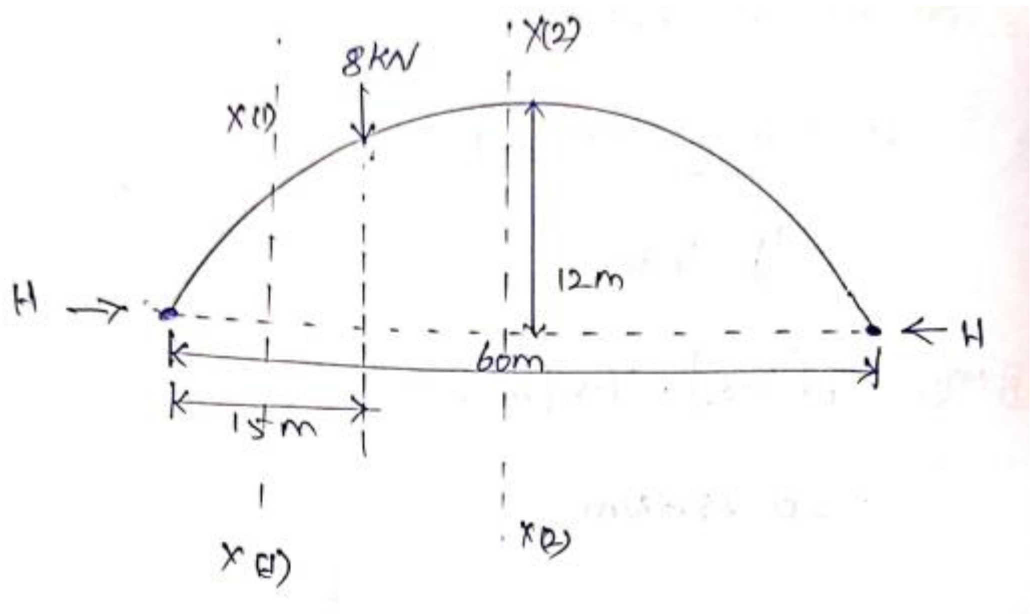

Fig. 3.4.3 Two hinged arches

Solution

Vertical reaction VA and VB

Taking moment about A

VB×60 - 8×15 =0

VB =2KN

Total load =VA+VB

VA = total load -VB

=8 - 2

=6 KN

Horizontal thrust

$H = {}_0\int^{l} \mu\, y\, dx \,/\, {}_0\int^{l} y^2\, dx$

${}_0\int^{l} \mu\, y\, dx = {}_0\int^{l5} \mu_1\, y\, dx + {}_{15}\int^{60} \mu_2\, y dx$

$y = 4\, r / l^2\, x\, (\, l - x\,)$

${}_0\int^{l} y^2\, dx = {}_0\int^{60} [4\, r / l^2\, x\, (\, l - x\,)]^2\, dx$

$= {}_0\int^{60} [4\times12 / 60^2\, x\, (60\text{-}x)]^2\, dx$

$= {}_0\int^{60} [(0.0133x\;\; x\, (60\text{-}x)]^2\, dx$

$= {}_0\int^{60} [(0.8x - 0.0133x^2)^2\;\; dx$

$= {}_0\int^{60} [0.64x^2 - 0.0213x^3 + (1.76\times10^{-4})x^4\,]\, dx$

$= {}_0\int^{60} [0.64x^2 - 0.0213x^3 + (1.76\times10^{-4})x^4\,]dx$

$= {}_0[\, (0.64x^3 / 3) - (0.0213x^4 /4) + (1.76\times10^{-4}x^4 /5)]^{60}$

$= 46.08\times10^3 - 69.012\times10^3 + 27.37\times10^3$

=4439.52

$_0\int^{l} \mu \; y \, dx \quad = {}_0\int^{l5} \mu_1 \, y \, dx + {}_{15}\int^{60} \mu_2 \, ydx$

$\mu_1 \quad = VA \; x1$

$= 6x$

$\mu_1 \quad =VA \; x_2 \; - 8 \; (x_2 \; -15)$

$=6x - 8x + 120$

$=- 2x + 120$

$_0\int^{l5} \mu_1 \, y \, dx$

$= {}_0\int^{l5} 6x(0.8x-0.0133x^2 \,)dx$

$= {}_0\int^{l5} (4.8x^2 \, -0.079x^3 \,)dx$

$= {}_0[\; (4.8x^3 \,/3) – (0.079x^4 \,/4)]^{15}$

$= 5400 - 999.84$

$=4400$

$_{15}\int^{60} \mu_2 \; ydx$

$= {}_{15}\int^{60} (120-2x)(0.8-0.013x^2 \,) \; dx$

$= {}_{15}\int^{60} (96x-1.596x^2 \, -1.6x^2 \, +0.0266x^3 \,) \; dx$

$= {}_{15}\int^{60} (0.0266x^3 \, -3.196x^2+96x)dx$

$= {}_{15}[(0.0266x^4 \,/4) – (3.196x^3/3) + (96x^2/2)]^{60}$

$=[(86184 – 230112 + 172800) - (336.6-359 +1080)]$

$= 21330.9$

H $= {}_0\int^{l} \mu\, y\, dx \,/\, {}_0\int^{l} y^2\, dx$

$= {}_0\int^{15} \mu_1\, y\, dx + {}_{15}\int^{60} \mu_2\, y dx \;/\; {}_0\int^{60} y^2\, dx$

H $= 4400 + 21330.9 / 4439.52$

$= 5.79$ KN

Reaction

RA $=\sqrt{H^2+VA^2}$

$=\sqrt{6^2+5.79^2}$

$=8.18$KN

RB $=\sqrt{H^2+VB^2}$

$=\sqrt{2^2+5.79^2}$

$=5.91$KN

Max Bending moment

Mx $=VA\,(15) - Hy$

y $=4\times12/60^2 \times15(60-15)$

$=9$m

Mx $=6\,(15) - 5.79\,(9)$

$=39.87$KNm