UNIT III

PHASE CHANGE HEAT TRANSFER AND HEAT EXCHANGERS

3 Condensation and Boiling

Heat energy is being converted into electrical energy with the help of water as a working fluid. Water is first converted into steam when heated in a heat exchanger and then the exhaust steam coming out of the steam turbine/engine is condensed in a condenser so that the condensate (water) is recycled again for power generation. Therefore, the condensation and boiling processes involve heat transfer with change of phase. When a fluid changes its phase, the magnitude of its properties like density, viscosity, thermal conductivity, specific heat capacity, etc., change appreciably and the processes taking place are greatly influenced by them. Thus, the condensation and boiling processes must be well understood for an effective design of different types of heat exchangers being used in thermal and nuclear power plants, and in process cooling and heating systems.

3.1 Condensation-Filmwise and Dropwise

Condensation is the process of transition from a vapour to the liquid or solid state. The process is accompanied by liberation of heat energy due to the change 10 phase. When a vapour comes 10 contact with a surface maintained at a temperature lower than the saturation temperature of the vapour corresponding to the pressure at which it exists, the vapour condenses on the surface and the heat energy thus released has to be removed. The efficiency of the condensing unit is determined by the mode of condensation that takes place:

Filmwise - the condensing vapour forms a continuous film covering the entire surface,

Dropwise - the vapour condenses into small liquid droplets of various sizes. The dropwise condensation has a much higher rate of heat transfer than filmwise condensation because the condensate in dropwise condensation gets removed at a faster rate leading to better heat transfer between the vapour and the bare surface. .

It is therefore desirable to maintain a condition of dropwise condensation 1D commercial application. Dropwise condensation can only occur either on highly polished surfaces or on surfaces contaminated with certain chemicals. Filmwise condensation is expected to occur in most instances because the formation of dropwise condensation IS greatly influenced by the presence of non-condensable gases, the nature and composition of surfaces and the velocity of vapour past the surface.

5.2.3. Filmwise Condensation Mechanism on a Vertical Plane Surface--Assumption

Let us consider a plane vertical surface at a constant temperature, T_s on which a pure vapour at saturation temperature, T_g ($T_g > T_s$) is condensing. The coordinates are: X-axis along the plane surface wit~ its origin at the top edge and Y-axis is normal to the plane surface as shown in Fig. 11.1. The condensing liquid would wet the solid surface, spread out and form a continuous film over the entire condensing surface. It is further assumed that

(i) the continuous film of liquid will flow downward (positive X-axis) under the action of gravity and its thickness would increase as more and more vapour condenses at the liquid - vapour interface,

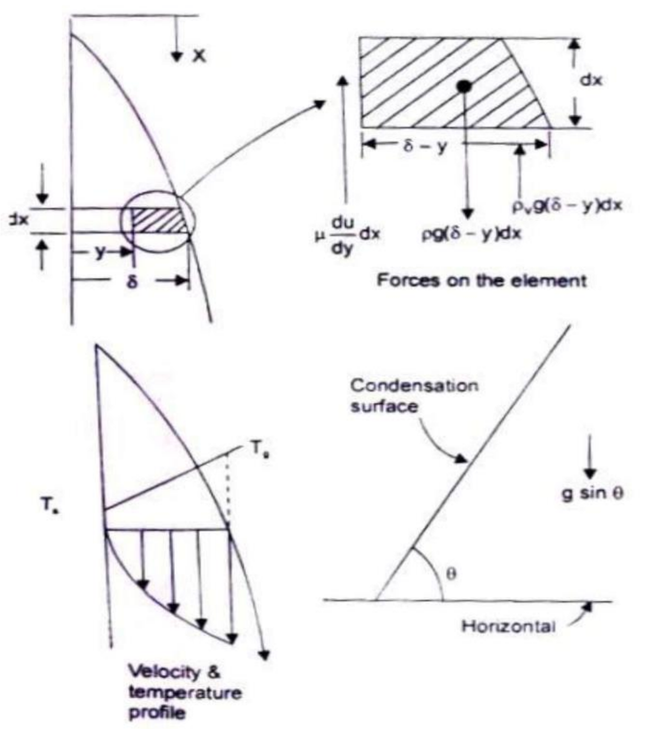


Fig. 5.11 Filmwise condensation on a vertical and Inclined surface

(ii) the continuous film so formed would offer a thermal resistance between the vapour and the surface and would reduce the heat transfer rates,

(iii) the flow in the film would be laminar,

(iv) there would be no shear stress exerted at the liquid vapour interface,

(v) the temperature profile would be linear, and

(vi) the weight of the liquid film would be balanced by the viscous shear in the liquid film and the buoyant force due to the displaced vapour.

5.2.4. An Expression for the Liquid Film Thickness and the Heat Transfer Coefficient in Laminar Filmwise Condensation on a Vertical Plate

We choose a small element, as shown in Fig. 11.1 and by making a force balance, we write

$$\rho g(\delta - y)dx = \mu(du/dy)dx + \rho_v g(\delta - y)dx \quad (5.44)$$

where □ is the density of the liquid, $\square_v$ is the density of vapour, □ is the viscosity of the liquid, Ii is the thickness of the liquid film at any x, and du/dy is the velocity gradient at x.

Since the no-slip condition requires u = 0 at y = 0, by integration we get:

$$u = (\square - \square_v)g(\square_y - y^2/2) / \square \quad (5.45)$$

And the mass flow rate of condensate through any x position of the film would be

$$\dot{m} = \int_0^\delta \rho u \, dy = \int_0^\delta \left[\rho(\rho - \rho_v)(g/\mu)\left(\delta y - y^2/2\right)\right]dy$$

$$= \rho(\rho - \rho_v)\, g\delta^3 / 3\mu \quad (5.46)$$

The rate of heat transfer at the wall in the area dx is, for unit width,

$$\dot{Q} = -kA\,(dt/dy)_{y=0} = k\,(dx \times 1)\left(T_g - T_s\right)/\delta\ ,$$

(temperature distribution is linear)

Since the thickness of the film increases in the positive X-direction, an additional mass of vapour will condense between x and x + dx, i.e.,

$$\frac{d}{dx}\left(\frac{\rho(\rho-\rho_v)g\delta^3}{3\mu}\right)dx = \frac{d}{d\delta}\left(\frac{\rho(\rho-\rho_v)g\delta^3}{3\mu}\right)\frac{d\delta}{dx}dx$$

$$= \frac{\rho(\rho-\rho_v)g\delta^2 d\delta}{\mu}$$

This additional mass of condensing vapour will release heat energy and that has to removed by conduction through the wall, or,

$$\therefore \quad \frac{\rho(\rho-\rho_v)g\delta^2 d\delta}{\mu} \times h_{fg} = k\ dx\left(T_g - T_s\right)/\delta \qquad (5.47)$$

We can, therefore, determine the thickness, δ, of the liquid film by integrating Eq. (11.4) with the boundary condition: at $x = 0$, $\delta = 0$,

$$\text{or,}\quad \delta = \left(\frac{4\mu kx\left(T_g - T_s\right)}{gh_{fg}\rho(\rho-\rho_v)}\right)^{0.25} \qquad (5.48)$$

The rate of heat transfer is also related by the relation,

$$h\,dx\left(T_g - T_s\right) = k\ dx\left(T_g - T_s\right)/\delta;\ \text{or, } h = k/\delta$$

which can be expressed In dimensionless form in terms of Nusselt number,

$$Nu = hx/k = \left[\frac{\rho(\rho-\rho_v)gh_{fg}x^3}{4\mu k\left(T_g - T_s\right)}\right]^{0.25} \qquad (5.49)$$

The average value of the heat transfer coefficient is obtained by integrating over the length of the plate:

$$\bar{h} = (1/L)\int_0^L h_x\ dx = (4/3)h_x\ = L$$

$$Nu_L = 0.943\left[\frac{\rho(\rho-\rho_v)gh_{fg}L^3}{k\mu\left(T_g - T_s\right)}\right]^{0.25} \qquad (5.50)$$

The properties of the liquid in Eq. (5.50) and Eq. (5.49) should be evaluated at the mean temperature, T = $(T_g + T_s)/2$.

The above analysis is also applicable to a plane surface inclined at angle with the horizontal, If g is everywhere replaced by g . sin.

Thus:

Local $$Nu_x = 0.707\left[\frac{\rho(\rho-\rho_v)h_{fg}x^3 g\sin\theta}{\mu k(T_g - T_s)}\right]^{0.25}$$

and the average $$Nu_L = 0.943\left[\frac{\rho(\rho-\rho_v)h_{fg}L^3 g\sin\theta}{\mu k(T_g - T_s)}\right] \qquad (5.51)$$

These relations should be used with caution for small values of because some of the assumptions made in deriving these relations become invalid; for example, when is equal to zero, (a horizontal surface) we would get an absurd result. But these equations are valid for condensation on the outside surface of vertical tubes as long as the curvature of the tube surface is not too great.

Solution: (a) Tube Homontal: The mean film temperature is (50 + 76) = 63°C, and the properties are:

$$\rho = 980\ kg/m^3,\ \mu = 0.432 \times 10^{-3} Pa - s,\ k = 0.66\ W/mK$$

$$h_{fg} = 2320\ kJ/kg,\ \rho >> \rho_v$$

$$h = 0.725\left[\left(\rho^2 h_{fg} k^3 g\right)/\ \mu D\left(T_g - T_s\right)\right]^{0.25}$$

$$= 0.725\left[(980)^2 \times 2320 \times 10^3 \times (0.66)^3 \times 9.81/\left(0.432 \times 10^{-3} \times 0.015 \times 26\right)\right]^{0.25}$$

$= 10\ kW/m^2K$

(b) Tube Vertical: Eq (5.50) should be used if the film thickness is very small in comparison with the tube diameter.

The film thickness, $$\delta = \left[\left\{\left(4\mu k L\left(T_g - T_s\right)\right)\right\}/\left\{g h_{fg}\ \rho\left(\rho - \rho_v\right)\right\}\right]^{0.25}$$

$$= \left[\frac{(980)^2 \times 9.81 \times 2320 \times 10^3 \times (0.66)^3}{0.432 \times 10^{-3} \times 1.5 \times 26} \right]^{0.25}$$

= = 0.212 mm << 15.0 mm, the tube diameter.

Therefore, the average heat transfer coefficient would be

$$h_v = h_h / \left[0.768 (L/D)^{0.25} \right] = 10/2.429 = 4.11 \text{ kW/m}^2\text{K}$$

(Thus, the performance of horizontal tubes for filmwise laminar condensation is much better than vertical tubes and as such horizontal tubes are preferred.)

Example 3.1 A square array of four hundred tubes, 1.5 cm outer diameter is used to condense steam at atmospheric pressure. The tube walls are maintained at 88°C by a coolant flowing inside the tubes. Calculate the amount of steam condensed per hour per unit length of the tubes.

Solution: The properties at the mean temperature (88 + 100)/2 = 94°C are:

□ = 963 kg/mJ, □ = 3.06 × 10^{-4} Pa-s, k = 0.678 W/mK,

h_{fg} = 2255 × 10^3 J/kg

A square array of 400 tubes will have N = 20. From Eq (5.57),

$$h = 0.725 \left[\left(g \rho^2 k^3 h_{fg} \right) \right] / \left[N \mu D \left(T_g - T_s \right) \right]^{0.25}$$

$$= 0.725 \left(\frac{9.81 \times (963)^2 \times (0.678)^3 \times 2255 \times 10^3}{20 \times 0.000306 \times 0.015 \times 12} \right) = 6.328 \text{ kW/m}^2\text{K}$$

Surface area for 400 tubes = 400 × 3.142 × 0.015 × 1 (let L = 1)

= 18.852 m^2 per metre length of the tube

$\dot{Q}$ = hA (□T) = 6.328 × 18.852 × 12 = 1431.56 kW

$\dot{m} = \dot{Q} / h_{fg}$ = 1431.56 × 3600/2255 = 2285.4 kg/hr per metre length.

3 Condensation inside Tubes-Empirical Relation

The condensation of vapours flowing inside a cylindrical tube is of importance in chemical and petro-chemical industries. The average heat transfer coefficient for vapours condensing inside either horizontal or vertical tubes can be determined, within 20 percent accuracy, by the relations:

For $Re_g < 5 \times 10^4$, $Nu_d = 5.03 (Re_g)^{1/3} (Pr)^{1/3}$

For $Re_g > 5 \times 10^4$, $Nu_d = 0.0265 (Re_g)^{0.8} (Pr)^{1/3}$ (5.53)

where Re_g is the Reynolds number defined in terms of the mass velocity, or, Re_g = DG/□, G being the mass rate of flow per unit cross-sectional area.

4 Dropwise Condensation-Merits and Demerits

In dropwise condensation, the condensation is found to appear in t he form 0 f individual drops. These drops increase in size and combine with another drop until their size is great enough that their weight causes them to run off the surface and the condensing surface is exposed for the formation of a new drop. This phenomenon has been observed to occur either on highly polished surfaces or on surface coated/contaminated with certain fatty acids. The heat transfer coefficient in dropwise condensation is five to ten times higher than the filmwise condensation under similar conditions. It is therefore, desirable that conditions should be maintained for dropwise condensation in commercial applications. The presence of non-condensable gases, the nature and composition of the surface, the vapour velocity past the surface have great influence on the formation of drops on coated/contaminated surfaces and It is rather difficult to achieve dr0pwlse condensation.

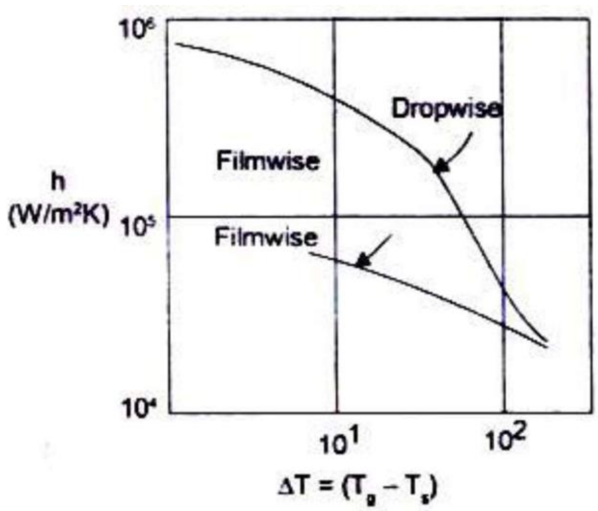


Fig. 5.12 Comparison of h for filmwise and dropwise condensation

Several theories have been proposed for the analysis of dropwise condensation. They do give explanations of the process but do not provide a relation to determine the heat transfer coefficient under various conditions. Fig 5.12 shows the comparison of heat transfer coefficient for filmwise and dropwise condensation.

5. Regimes of Boiling

Let us consider a heating surface (a wire or a flat plate) submerged In a pool of water which is at its saturation temperature. If the temperature of the heated surface exceeds the temperature of the liquid, heat energy will be transferred from the solid surface to the liquid. From Newton's law of cooling, we have

$$\dot{Q} / A = \dot{q} = h\left(T_w - T_s\right)$$

where $\dot{Q}/A$ is the heat flux, T_w is the temperature of the heated surface and T_s, is the temperature of the liquid, and the boiling process will start.

(i) Pool Boiling - Pool boiling occurs only when the temperature of the heated surface exceeds the saturation temperature of the liquid. The liquid above the hot surface I s quiescent and its motion n ear the surface is due to free convection.

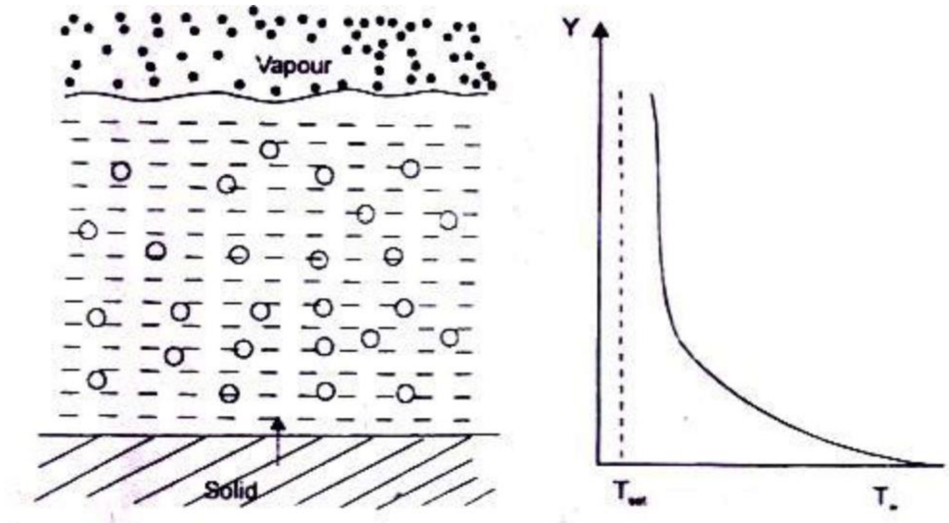


Fig. 5.13 Temperature distribution in pool boiling at liquid-vapour interface

Bubbles grow at the heated surface, get detached and move upward toward the free surface due to buoyancy effect. If the temperature of the liquid is lower than the saturation temperature, the process is called 'subcooled or local boiling'. If the temperature of the liquid is equal to the saturation temperature, the process is known as 'saturated or bulk boiling'. The temperature distribution in saturated pool boiling is shown in Fig5.13. When T_w exceeds T_s by a few degrees, the convection currents circulate in the superheated liquid and the evaporation takes place at the free surface of the liquid.

(ii) Nucleate Boiling - Fig. I 1.5 illustrates the different regimes of boiling where the heat flux $\left(\dot{Q}/A\right)$ is plotted against the temperature difference $\left(T_w - T_s\right)$. When the temperature T_w increases a little more, vapour bubbles are formed at a number of favoured spots on the heating surface. The vapour bubbles are initially small and condense before they reach the free surface. When the temperature is raised further, their number increases and they grow bigger and finally rise to the free surface. This phenomenon is called 'nucleate boiling'. It can be seen from the figure (5.14) that in nucleate boiling regime, the heat flux increases rapidly with increasing surface temperature. In the latter part of the nucleate boiling, (regime 3), heat transfer by evaporation is more important and predominating. The point A on the curve represents 'critical heat flux'.

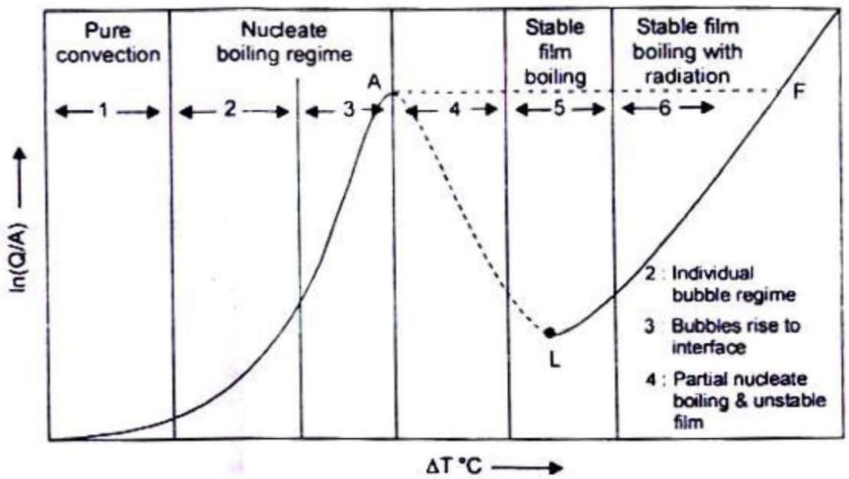


Fig. 5.14 Heat Flux - Temperature difference curve for boiling water heated by a wire (Nukiyama's boiling curve for saturated water at atmospheric pressure) (L is the Laidenfrost Point)

(iii) Film Bolling - When the excess temperature, □T = (T_W–T_S) increases beyond the point A, a vapour film forms and covers the entire heating surface. The heat transfer takes place through the vapour which is a poor conductor and this increased thermal resistance causes a drop in the heat flux. This phase is film boiling'. The transition from the nucleate boiling regime to the film boiling regime is not a sharp one and the vapour film under the action of circulating currents collapses and rapidly reforms. In regime 5, the film is stable and the heat flow rate is the lowest.

(iv) Critical Heat Flux and Burnout Point - For □T beyond 550°C (regime 6) the temperature of the heating metallic surface is very high and the heat transfer occurs predominantly by radiation, thereby, increasing the heat flux. And finally, a point is reached at which the heating surface melts - point F in Fig. 11.5. It can be observed from the boiling curve that the whole boiling process remains in the unstable state between A and F. Any increase in the heat flux beyond point A will cause a departure from the boiling curve and there would be a large increase in surface temperature.

6. Boiling Curve - Operating Constraints

The boiling curve, shown in Fig. 11.5, is based on the assumption that the temperature of the heated surface can be maintained at the desired value. In that case, it would be possible to operate the vapour producing system at the point of maximum flux with nucleate boiling. If the heat flux instead of the surface temperature, is the independent variable and it IS desired to operate the system at the point of maximum flux, it is just possible that a slight increase in the heat flux will increase the surface temperature substantially. And, the equilibrium will be established at point F. If the material of the heating element has its melting point temperature lower than the temperature at the equilibrium point F, the heating element will melt.

7 Factors Affecting Nucleate Boiling

Since high heat transfer rates and convection coefficients are associated with small values of the excess temperature, it is desirable that many engineering devices operate in the nucleate boiling regime. It is possible to get heat transfer coefficients in excess of 10^4 W/m^2 in nucleate boiling regime and these values are substantially larger than those normally obtained in convection processes with no phase change. The factors which affect the nucleate boiling are:

(a) Pressure - Pressure controls the rate of bubble growth and therefore affects the temperature difference causing the heat energy to flow. The maximum allowable heat flux for a boiling liquid first increases with pressure until critical pressure is reached and then decreases.

(b) Heating Surface Characteristics - The material of the heating element has a significant effect on the boiling heat transfer coefficient. Copper has a higher value than chromium, steel and zinc. Further, a rough surface gives a better heat transfer rate than a smooth or coated surface, because a rough surface gets wet more easily than a smooth one.

(c) Thermo-mechanical Properties of Liquids - A higher thermal conductivity of the liquid will cause higher heat transfer rates and the viscosity and surface tension will have a marked effect on the bubble size and their rate of formation which affects the rate of heat transfer.

(d) Mechanical Agitation - The rate of heat transfer will increase with the increasing degree of mechanical agitation. Forced convection increases mixing of bubbles and the rate of heat transfer.

HEAT EXCHANGERS

3.1 Heat Exchangers: Regenerators and Recuperators

A heat exchanger is an equipment where heat energy is transferred from a hot fluid to a colder fluid. The transfer of heat energy between the two fluids could be carried out (i) either by direct mixing of the two fluids and the mixed fluids leave at an intermediate temperature determined from the principles of conservation of energy, (ii) or by transmission through a wall separating the two fluids. The former types are called direct contact heat exchangers such as water cooling towers and jet condensers. The latter types are called regenerators, recuperator surface exchangers.

In a regenerator, hot and cold fluids alternately flow over a surface which provides alternately a sink and source for heat flow. Fig. 10.1 (a) shows a cylinder containing a matrix that rotates in such a way that it passes alternately through cold and hot gas streams which are sealed from each other. Fig. 10.1 (b) shows a stationary matrix regenerator ill which hot and cold gases flow through them alternately.

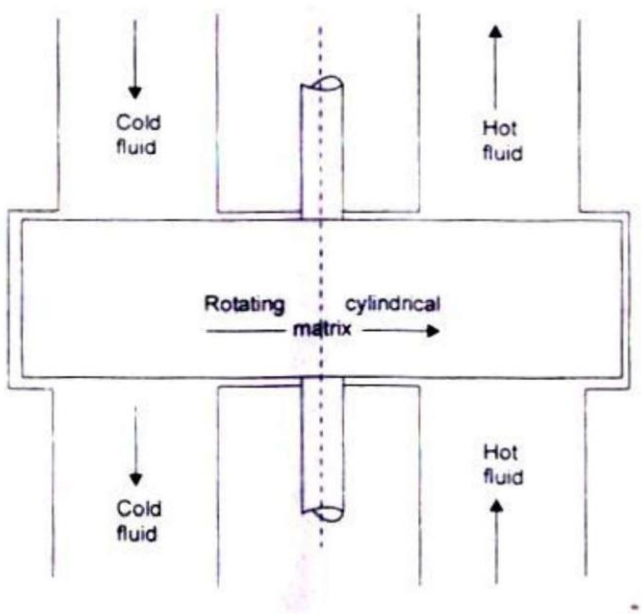


Fig. 3.1 (a) Rotating matrix regenerator

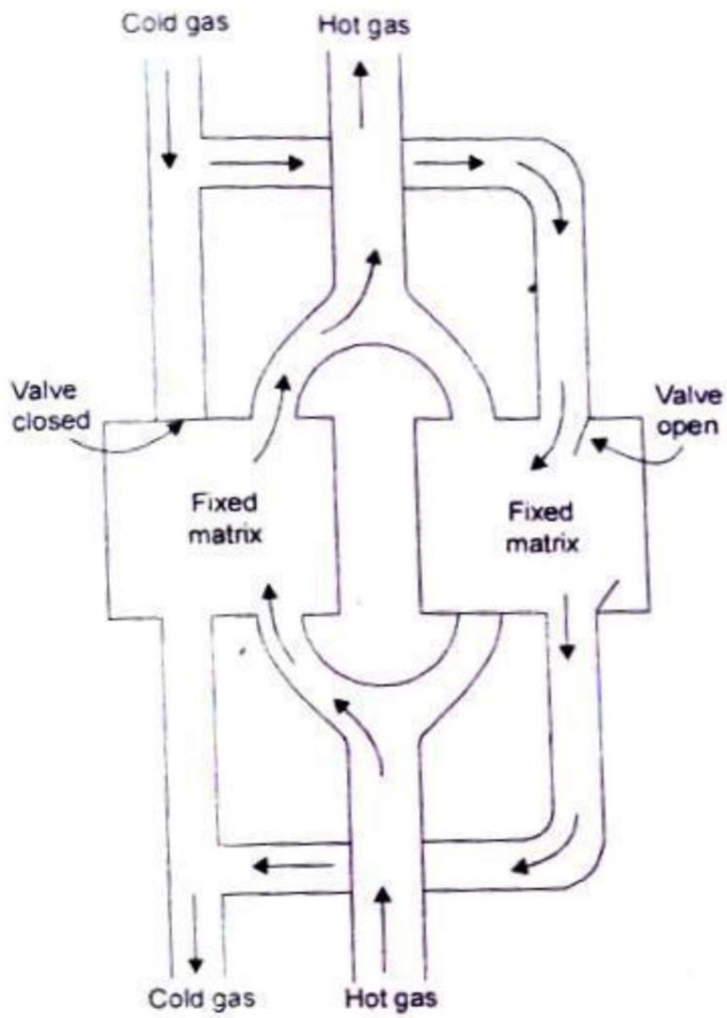


Fig. 3.1 (b) Stationary matrix regenerator

In a recuperator, hot and cold fluids flow continuously following he same path. The heat transfer process consists of convection between the fluid and the separating wall, conduction through the wall and convection between the wall and the other fluid. Most common heat exchangers are of recuperative type having a Wide variety of geometries:

3.2. Classification of Heat Exchangers

Heat exchangers are generally classified according to the relative directions of hot and cold fluids:

(a) Parallel Flow – the hot and cold fluids flow in the same direction. Fig 3.2 depicts such a heat exchanger where one fluid (say hot) flows through the pipe and the other fluid (cold) flows through the annulus.

(b) Counter Flow –the two fluids flow through the pipe but in opposite directions. A common type of such a heat exchanger is shown in Fig. 3.3. By comparing the temperature distribution of the two types of heat exchanger

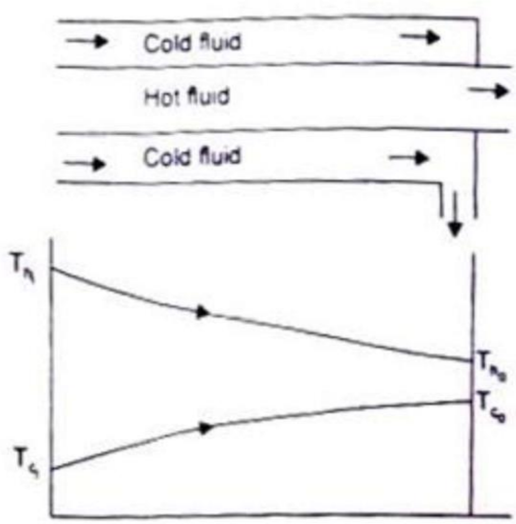


Fig 3.2 Parallel flow heat exchanger with temperature distribution

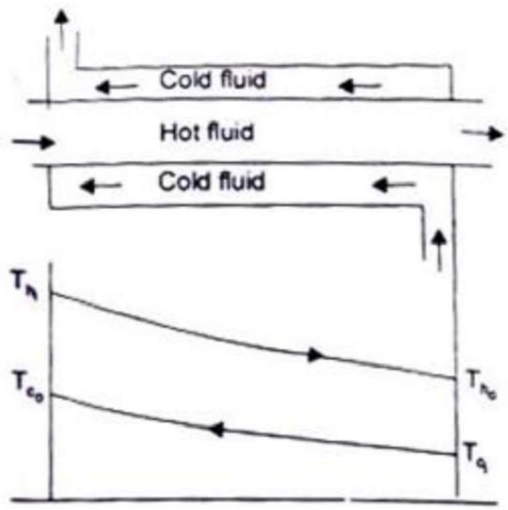


Fig 3.3 Counter-flow heat exchanger with temperature distribution

we find that the temperature difference between the two fluids is more uniform in counter flow than in the parallel flow. Counter flow exchangers give the maximum heat transfer rate and are the most favoured devices for heating or cooling of fluids.

When the two fluids flow through the heat exchanger only once, it is called one-shell-pass and one-tube-pass as shown in Fig. 3.2 and 3.3. If the fluid flowing through the tube makes one pass through half of the tube, reverses its direction of flow, and makes a second pass through the remaining half of the tube, it is called 'one-shell-pass, two-tube-pass' heat exchanger,fig 3.4. Many other possible flow arrangements exist and are being used. Fig. 10.5 depicts a 'two-shell-pass, four-tube-pass' exchanger.

(c) Cross-flow - A cross-flow heat exchanger has the two fluid streams flowing at right angles to each other. Fig. 3.6 illustrates such an arrangement An automobile radiator is a good example of cross-flow exchanger. These exchangers are 'mixed' or 'unmixed' depending upon the mixing or not mixing of either fluid in the direction transverse to the direction of the flow stream and the analysis of this type of heat exchanger is extremely complex because of the variation in the temperature of the fluid in and normal to the direction of flow.

(d) Condenser and Evaporator - In a condenser, the condensing fluid temperature remains almost constant throughout the exchanger and temperature of the colder fluid gradually increases from the inlet to the exit, Fig. 3.7 (a). In an evaporator, the temperature of the hot fluid gradually decreases from the inlet to the outlet whereas the temperature of the colder fluid remains the same during the evaporation process, Fig. 3.7(b). Since the temperature of one of the fluids can be treated as constant, it is immaterial whether the exchanger is parallel flow or counter flow.

(e) Compact Heat Exchangers - these devices have close arrays of finned tubes or plates and are typically used when atleast one of the fluids is a gas. The tubes are either flat or circular as shown in Fig. 10.8 and the fins may be flat or circular. Such heat exchangers are used to a chieve a very large (≥700 m^2/mJ) heat transfer surface area per unit volume. Flow passages are typically small and the flow is usually laminar.

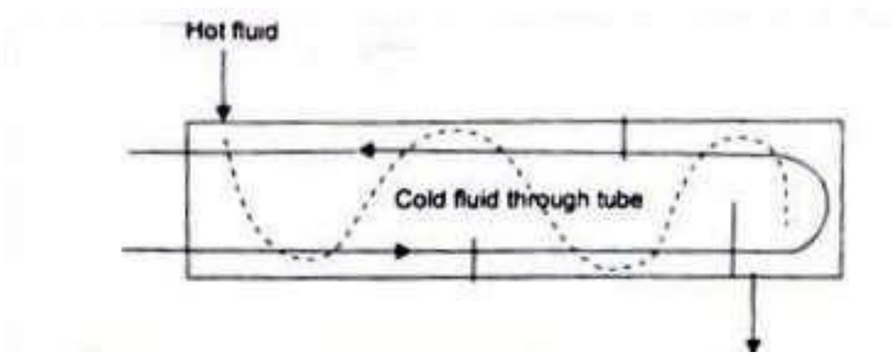


Fig 3.4: multi pass exchanger one shell pass, two shell pass

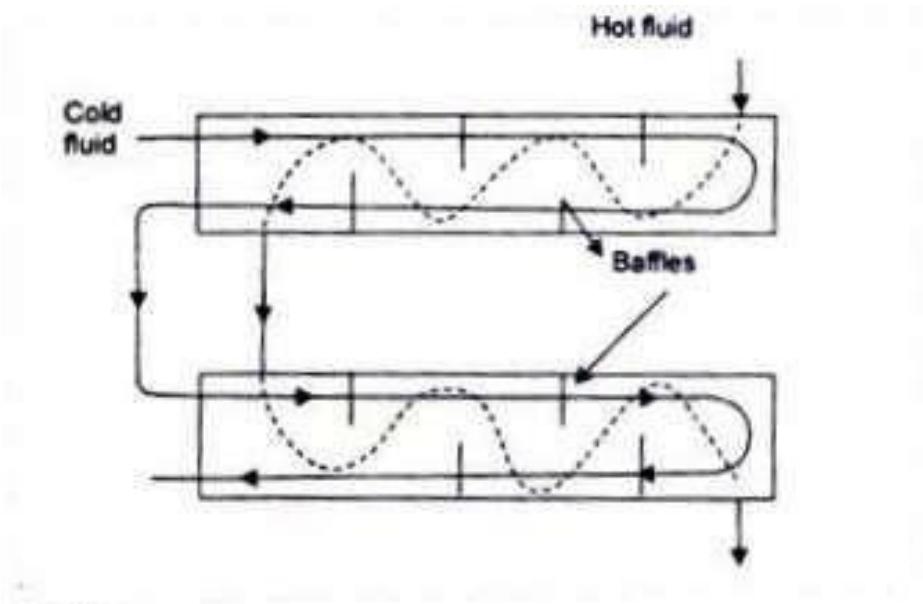


Fig 3.5: Two shell passes, four-tube passes heat exchanger (baffles increases the convection coefficient of the shell side fluid by inducing turbulance and a cross flow velocity component)

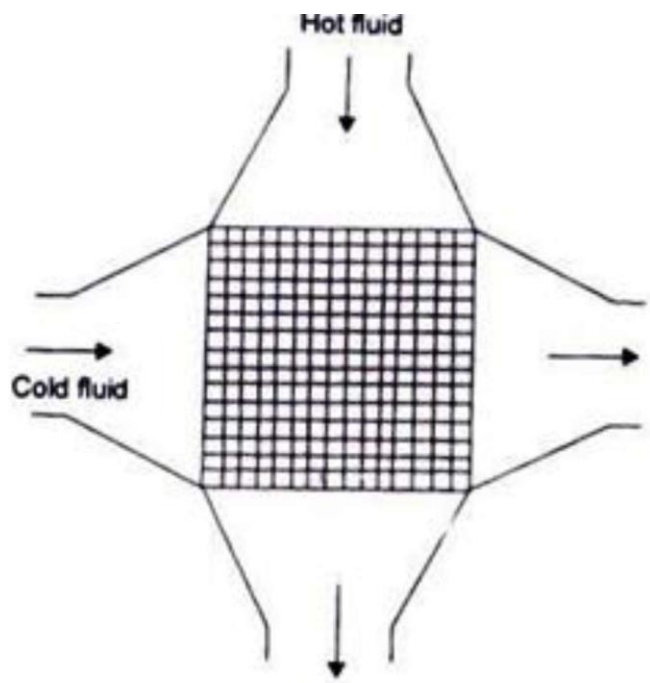


Fig 3.6: A cross-flow exchanger

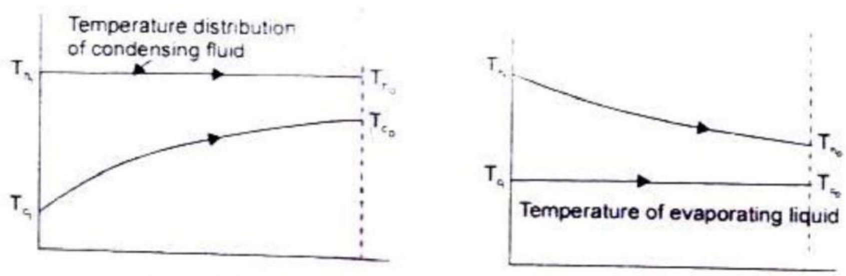


Fig. 10.7 (a) A condenser

Fig. 10.7 (b) An evaporator

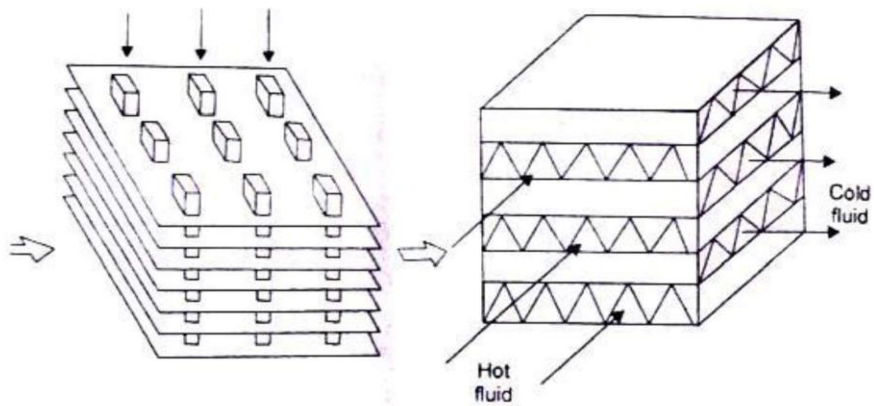


Fig. 3.8 Compact heat exchangers: (a) flat tubes, continuous plate fins, (b) plate fin (single pass)

3.3. Expression for Log Mean Temperature Difference - Its Characteristics

Fig. 10.9 represents a typical temperature distribution which is obtained in heat exchangers. The rate of heat transfer through any short section of heat exchanger tube of surface area dA is: $dQ = U\ dA(T_h - T_c) = U\ dA\ \Delta T$. For a parallel flow heat exchanger, the hot fluid cools and the cold fluid is heated in the direction of increasing area. therefore, we may write

$d\dot{Q} = -\dot{m}_h\ c_h\ dT_h = \dot{m}_c\ c_c\ dT_c$ and $d\dot{Q} = -\dot{C}_h\ dT_h = \dot{C}_c dT_c$ where $\dot{C} = \dot{m} \times c$, and is called the 'heat capacity rate.'

$$\text{Thus,}\quad d(\Delta T) = d(T_h - T_c) = dT_h - dT_c = -(1/C_h + 1/C_c)\ d\dot{Q} \qquad (3.1)$$

For a counter flow heat exchanger, the temperature of both hot and cold fluid decreases in the direction of increasing area, hence

$d\dot{Q} = -\dot{m}_h\ c_h\ dT_h = -\dot{m}_c\ c_c\ dT_c$, and $d\dot{Q} = -C_h\ dT_h = -C_c dT_c$

or, $d(\Delta T) = dT_h - dT_c = (1/C_h - 1/C_c)d\dot{Q}$ (3.2)

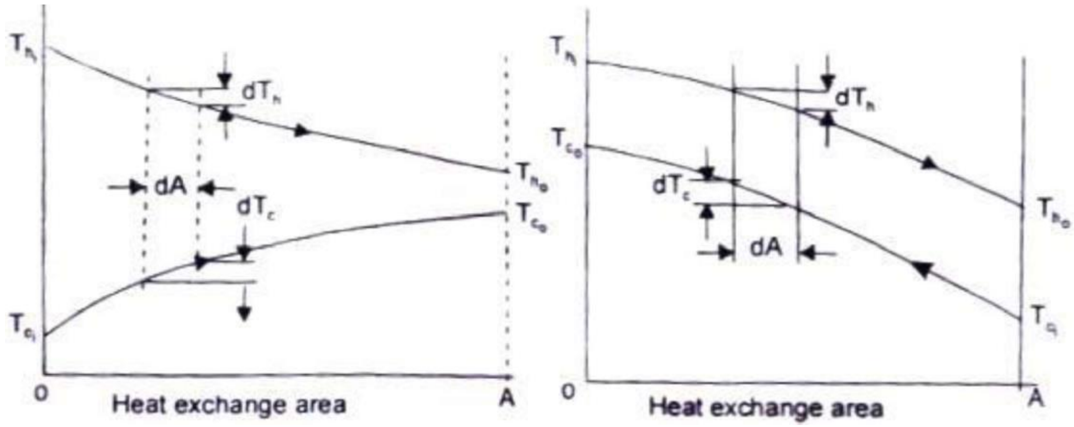


Fig. 3.9 Parallel flow and Counter flow heat exchangers and the temperature distribution with length

Integrating equations (3.1) and (3.2) between the inlet and outlet. and assuming that the specific heats are constant, we get

$$-(1/C_h \pm 1/C_c)\dot{Q} = \Delta T_o - \Delta T_i \quad (3.3)$$

The positive sign refers to parallel flow exchanger, and the negative sign to the counter flow type. Also, substituting for dQ in equations (10.1) and (10.2) we get

$$-(1/C_h \pm 1/C_c)UdA = d(\Delta T)/\Delta T \quad (3.3a)$$

Upon integration between inlet i and outlet 0 and assuming U as a constant,

We have $-(1/C_h \pm 1/C_c)U\ A = \ln(\Delta T_0/\Delta T_i)$

By dividing (10.3) by (10.4), we get

$$\dot{Q} = UA\left[(\Delta T_o - \Delta T_i)/\ln(\Delta T_o/\Delta T_i)\right] \quad (3.5)$$

Thus the mean temperature difference is written as

Log Mean Temperature Difference,

$$\text{LMTD} = (\Delta T_0 - \Delta T_i)/\ln(\Delta T_o/\Delta T_i) \quad (3.6)$$

(The assumption that U is constant along the heat exchanger is never strictly true but it may be a good approximation if at least one of the fluids is a gas. For a gas, the physical properties do not vary appreciably over moderate range of temperature and the resistance of the gas film is considerably higher than that of the metal wall or the liquid film, and the value of the gas film resistance effectively determines the value of the overall heat transfer coefficient U.)

It is evident from Fig.1 0.9 that for parallel flow exchangers, the final temperature of fluids lies between the initial values of each fluid whereas m counter flow exchanger, the temperature of the colder fluid at exit is higher than the temperature of the hot fluid at exit. Therefore, a counter flow exchanger provides a greater temperature range, and the LMTD for a counter flow exchanger will be higher than for a given rate of mass flow of the two fluids and for given temperature changes, a counter flow exchanger will require less surface area.

3.4. Special Operating Conditions for Heat Exchangers

(i) Fig. 3.7a shows temperature distributions for a heat exchanger (condenser) where the hot fluid has a much larger heat capacity rate, $\dot{C}_h = \dot{m}_h c_h$ than that of cold fluid, $\dot{C}_c - \dot{m}_c c_c$ and therefore, the temperature of the hot fluid remains almost constant throughout the exchanger and the temperature of the cold fluid increases. The LMTD, in this case is not affected by whether the exchanger is a parallel flow or counter flow.

(ii) Fig. 3.7b shows the temperature distribution for an evaporator. Here the cold fluid expenses a change in phase and remains at a nearly uniform temperature $\left(\dot{C}_c \to \infty\right)$. The same effect would be achieved without phase change if $\dot{C}_c >> \dot{C}_h$, and the LMTD will remain the same for both parallel flow and counter flow exchangers.

(iii) In a counter flow exchanger, when the heat capacity rate of uoth the fluids are equal, $\dot{C}_c = \dot{C}_h$, the temperature difference is the same all along the length of the tube. And in that case, LMTD should be replaced by $\Delta T_a = \Delta T_b$, and the temperature profiles of the two fluids along Its length would be parallel straight lines.

(Since $d\dot{Q} = -\dot{C}_c dT_c = -\dot{C}_h dT_h$; $dT_c = -d\dot{Q}/\dot{C}_c$, and $dT_h = -d\dot{Q}/\dot{C}_h$

and, $dT_c - dT_h = d\theta = -dQ\left(1/\dot{C}_c - 1/\dot{C}_h\right) = 0$ (because $\dot{C}_c = \dot{C}_h$)

Or, d□ = 0, gives □ = constant and the temperature profiles of the two fluids

along Its length would be parallel straight lines.)

3.5. LMTD for Cross-flow Heat Exchangers

LMTD given by Eq (10.6) is strictly applicable to either parallel flow or counter flow exchangers. When we have multipass parallel flow or counter flow or cross flow exchangers, LMTD is first calculated for single pass counter flow exchanger and the mean temperature difference is obtained by multiplying the LMTD with a correction factor F which takes care of the actual flow arrangement of the exchanger. Or,

$$\dot{Q} = U\,A\,F\,(LMTD) \tag{3.7}$$

The correction factor F for different flow arrangements are obtained from charts given in Fig. 3.10 (a, b, c, d).

3.6. Fouling Factors in Heat Exchangers

Heat exchanger walls are usually made of single materials. Sometimes the walls are bimettalic (steel with aluminium cladding) or coated with a plastic as a protection against corrosion, because, during normal operation surfaces are subjected to fouling by fluid impurities, rust formation, or other reactions between the fluid and the wall material. The deposition of a film or scale on the surface greatly increases the resistance to heat transfer between the hot and cold fluids. And, a scale coefficient of heat transfer h, is defined as:

$$R_s = 1/h_s A, \ {}^oC/W \text{ or } K/W$$

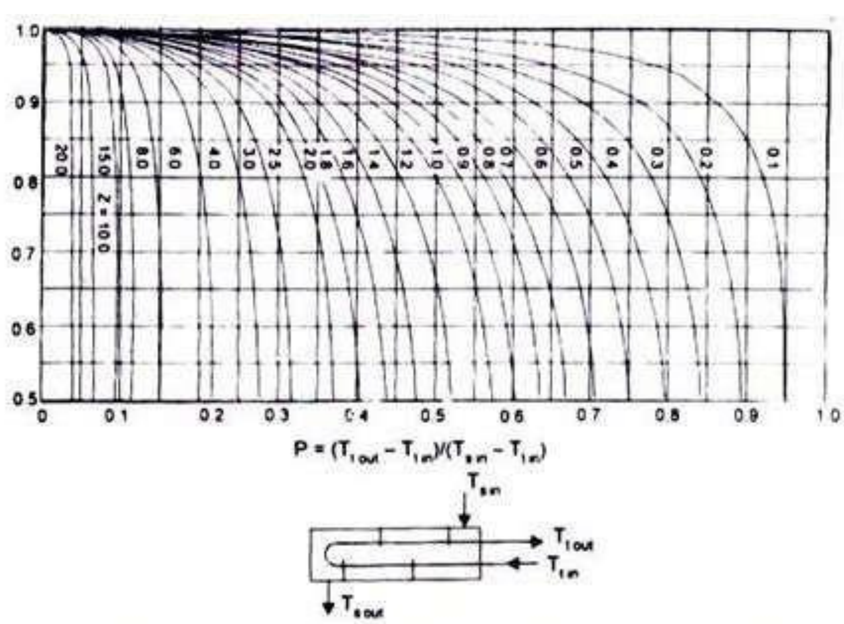


Fig 3.10(a) correctio factor to counter flow LMTD for heat exchanger with one shell pass andtwo, or a muliple of two,tube passes

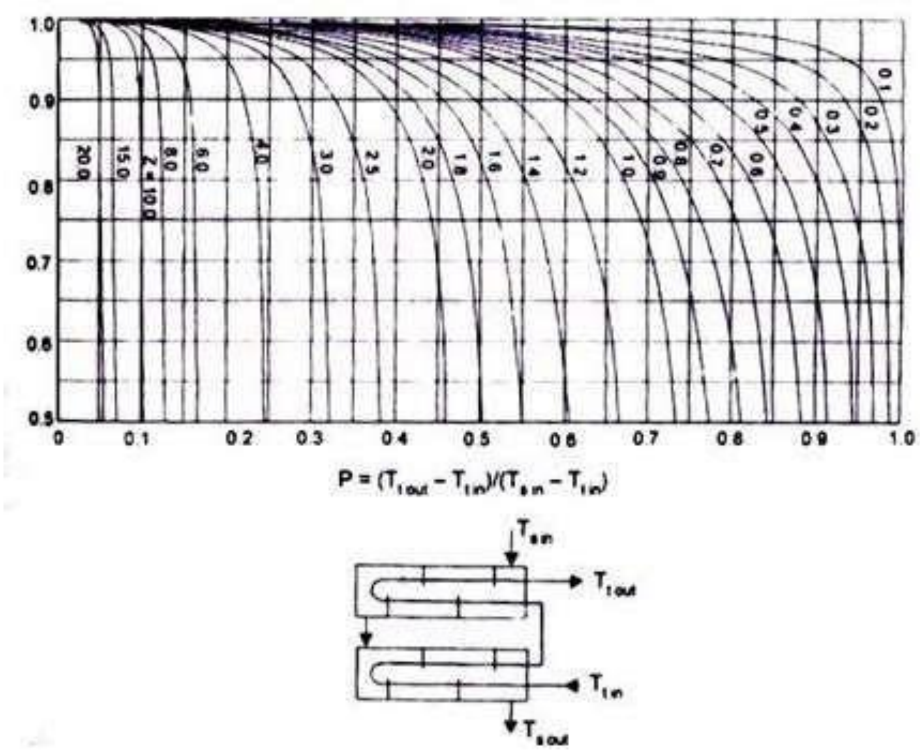


Fig 3.10 (b) Correction factor to counter flow LMTD for heat exchanger with two shell passes and a multiple of two tube passes

where A is the area of the surface before scaling began and l/h_s, is called ‘Fouling Factor'. Its value depends upon the operating temperature, fluid velocity, and length of service of the heat exchanger. Table 10.1 gives the magnitude of l/h, recommended for inclusion in the overall heat transfer coefficient for calculating the required surface area of the exchanger

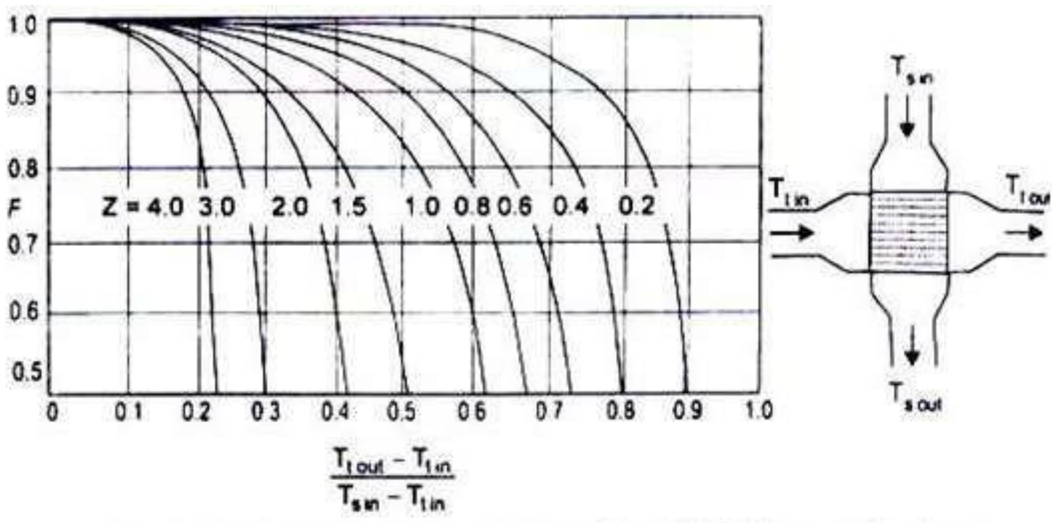


Fig3.10(c) Correction factor to counter flow LMTD for cross flow heat exchangers, fluid on shell side mixed, other fluid unmixed one tube pass..

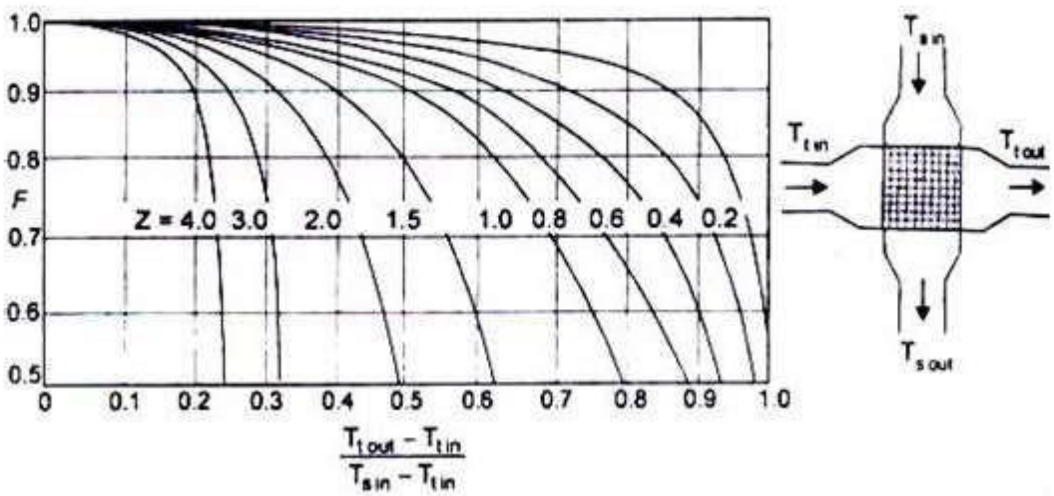


Fig. 3.10 (d) Correction factor to counter flow LMTD for cross flow heat exchangers, both fluids unmixed, one tube pass..

Table 3.1 Representative fouling factors ($1/h_s$)

Type of fluid	Fouling factor	Type of fluid	Fouling Factor
Sea water below 50°C	000009 m'K/W	Refrigerating liquid	0.0002 m'K/W

above 50°C	0.002		
Treated feed water	0.0002	Industrial air	0.0004
Fuel oil	0.0009	Steam, non-oil-bearing	0.00009
Quenching oil	0.0007	Alcohol vapours	0.00009

However, fouling factors must be obtained experimentally by determining the values of U for both clean and dirty conditions in the heat exchanger.

7. The Overall Heat Transfer Coefficient

The determination of the overall heat transfer coefficient is an essential, and often the most uncertain, part of any heat exchanger analysis. We have seen that if the two fluids are separated by a plane composite wall the overall heat transfer coefficient is given by:

$$1/U = (1/h_i) + (L_1/k_1) + (L_2/k_2) + (1/h_o) \qquad (3.8)$$

If the two fluids are separated by a cylindrical tube (inner radius r_i, outer radius r_0), the overall heat transfer coefficient is obtained as:

$$1/U_i = (1 + h_i) + (r_i/k)\ln(r_o/r_i) + (r_i/r_o)(1/h_o) \qquad (3.9)$$

where h_i, and h_o are the convective heat transfer coefficients at the inside and outside surfaces and V, is the overall heat transfer coefficient based on the inside surface area. Similarly, for the outer surface area, we have:

$$1/U_o = (1/h_o) + (r_o/k)\ln(r_o + r_i) + (r_o + r_i)(1/h_i) \qquad (3.10)$$

and U_iA_i will be equal to U_oA_o; or, $U_ir_i = U_or_o$.

The effect of scale formation on the inside and outside surfaces of the tubes of a heat exchanger would be to introduce two additional thermal resistances to the heat flow path. If h_{si} and h_{so} are the two heat transfer coefficients due to scale formation on the inside and outside surface of the inner pipe, the rate of heat transfer is given by

$$Q = (T_i - T_o)/\left[(1/h_iA_i) + 1/h_{si}A_i + \ln(r_o + r_i)/2\pi Lk + 1/h_{so}A_o + (1/h_oA_o)\right] \qquad (3.11)$$

where T_i, and T_o are the temperature of the fluid at the inside and outside of the tube. Thus, the overall heat transfer coeffiCIent based on the inside and outside surface area of the tube would be:

$$1/U_i = 1/h_i + 1/h_{si} + (r_i/k)\ln(r_o/r_i) + (r_i/r_o)(1/h_{so}) + (r_i/r_o)(1/h_o); \qquad (3.12)$$

and

$$1/U_o = (r_o/r_i)(1/h_i) + (r_o/r_i)(1/h_{si}) + \ln(r_o/r_i)(r_0/k) + 1/h_{so} + 1/h_o$$

Example 3.1 In a parallel flow heat exchanger water flows through the inner pipe and is heated from 25°C to 75°C. Oil flowing through the annulus is cooled from 210°C to 110°C. It is desired to cool the oil to a lower temperature by increasing the length of the tube. Estimate the minimum temperature to which the oil can be cooled.

Solution: By making an energy balance, heat received by water must be equal to 4he heat given out by oil.

$$\dot{m}_w c_w (75-25) = \dot{m}_o c_o (210-110); \dot{C}_w/\dot{C}_o = 100/50 = 2.0$$

In a parallel flow heat exchanger, the minimum temperature to which oil can be cooled will be equal to the maximum temperature to which water can be heated,

Fig. 10.2: $(T_{ho} = T_{co})$

therefore, C_w (T –25) = C_o (210 –T);

(T –25)/(210 –T) = 1/2 = 0.5; or, T = 260/3 = 86.67°C.

or the same capacity rates the oil can be cooled to 25°C (equal to the water inlet temperature) in a counter-flow arrangement.

Example 3.2 Water at the rate of 1.5 kg/s IS heated from 30°C to 70°C by an oil (specific heat 1.95 kJ/kg C). Oil enters the exchanger at 120°C and leaves the exchanger at 80°C. If the overall heat transfer coefficient remains constant at 350 W /m^2°C, calculate the heat exchange area for (i) parallel-flow, (ii) counter-flow, and (iii) cross-flow arrangement.

Solution: Energy absorbed by water,

$$\dot{Q} = \dot{m}_w c_w (\Delta T) = 1.5 \times 4.182 \times 40 = 250.92 \text{ kW}$$

(i) Parallel flow: Fig. 10.9; $\Delta T_a = 120 - 30 = 90$; $\Delta T_b = 80 - 70 = 10$

LMTD = (90 – 10)/ln(90/10) = 36.4;

Area = Q/U (LMTD) = 250920 /(350 × 36.4) = 19.69 m^2.

(ii) Counter flow: Fig 10.9; □T_a. = 120 – 70 = 50, □T_b = 80 – 30 = 50

Since □T_a. = □T_b,. LMTD should be replaced by □T = 50

Area A = $\dot{Q}/U(\Delta T) = 250920/(350 \times 50) = 14.33\ m^2$

(iii) Cross flow: assuming both fluids unmixed - Fig. 10.10d

using the nomenclature of the figure and assuming that water flows through the tubes and oil flows through the shell,

$$P = (T_{to} - T_{ti})/(T_{si} - T_{ti}) = (70 - 30)/(120 - 30) = 0.444$$

$$Z = (T_{si} - T_{so})/(T_{to} - T_{ti}) = (120 - 80)/(70 - 30) = 1.0$$

and the correction factor, F = 0.93

$\dot{Q} = UAF(\Delta T)$; or Area A = 250920/(350 × 0.93 × 50) = 15.41 m^2.

Example 3.3 0.5 kg/s of exhaust gases flowing through a heat exchanger are cooled from 400°C to 120°C by water initially at 25°C. The specific heat capacities of exhaust gases and water are 1.15 and 4.19 kJ/kgK respectively, and the overall heat transfer coefficient from gases to water is 150 W/m^2K. If the cooling water flow rate is 0.7 kg/s, calculate the surface area when (i) parallel-flow (ii) cross-flow with exhaust gases flowing through tubes and water is mixed in the shell.

Solution: The heat given out by the exhaust gases is equal to the heat gained by water.

or, 0.5 × 1.15 × (400 – 120) = 0.7 × 4.19 × (T – 25)

Therefore, the temperature of water at exit, T = 79.89°C

For paralle1-flow: $\Delta T_a = 400 - 25 = 375; \quad \Delta T_b = 120 - 79.89 = 40.11$

LMID = (375 – 40.11)/ln(375/40.11) = 149.82

$\dot{Q}$ = 0.5 × 1.15 × 280 = 161000 W;

Therefore Area A = 161000/(150 × 149.82) = 7.164 m^2

For cross-flow: $\dot{Q}$ = U A F (LMTD);

and LMTD is calculated for counter-flow system.

$$\Delta T_a = (400 - 79.89) = 320.11;\ \Delta T_b = 120 - 25 = 95$$

$$\text{LMTD} = (320.11 - 95) / \ln(320.11/95) = 185.3$$

Using the nomenclature of Fig 10.10c,

$$P = (120 - 400)/(25 - 400) = 0.747$$

$$Z = (25 - 79.89)/(120 - 400) = 0.196 \qquad \therefore\ F = 0.92$$

and the area $A = 161000/(150 \times 0.92 \times 185.3) = 6.296\ m^2$

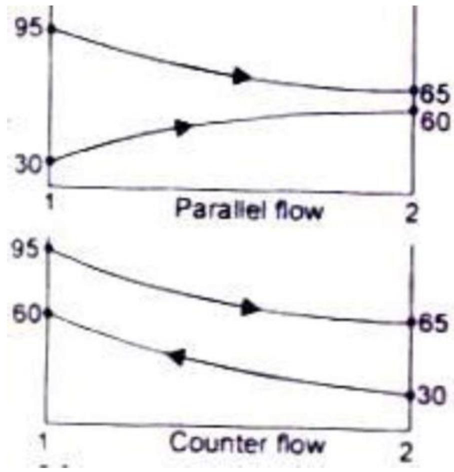
Example 3.4 In a certain double pipe heat exchanger hot water flows at a rate of 5000 kg/h and gets cooled from 95°C to 65°C. At the same time 5000 kg/h of cooling water enters the heat exchanger. The overall heat transfer coefficient is 2270 W/m^2K. Calculate the heat transfer area and the efficiency assuming two streams are in (i) parallel flow (ii) counter flow. Take C_p for water as 4.2 kJ/kgK, cooling water inlet temperature 30°C.

Solution: By making an energy balance:

Heat lost by hot water = 5000 ×4.2 ×(95 –65)

= heat gained by cold water = 5000 ×4.2 ×(T –30) 30

T = 60°C



(i) Parallel now

$$\theta_1 = (95 - 30) = 65$$

$$\theta_2 = (65 - 60) = 5$$

LMTD = $(65 - 5)/\ln(65/5) = 23.4$

Area, $A = \dot{Q}/(U \times LMTD) = \dfrac{500 \times 4.2 \times 10^3 \times 30}{3600 \times 2270 \times 23.4} = 3.295 \text{ m}^2$

(ii) Counter flow: θ_1 = (95 – 60) = 35

θ_2 = (65 – 30) = 35

LMTD = □T = 35

Area A = 500 × 4200 × 30/(3600 × 2270 × 35) = 2.2 m^2

$\in$, Efficiency.= Actual heat transferred/Maximum heat that could be transferred. Therefore, for parallel flow, $\in$ = (95 – 65)/(95 – 60) = 0.857

For counter flow, $\in$ = (95 - 65)/(95 – 30) = 0.461.

Counter now

Example 3.5 The flow rates of hot and cold water streams running through a double pipe heat exchanger (inside and outside diameter of the tube 80 mm and 100 mm) are 2 kg/s and 4 kg/so The hot fluid enters at 75°C and comes out at 45°C. The cold fluid enters at 20°C. If the convective heat transfer at the inside and outside surface of the tube is 150 and 180 W /m^2K, thermal conductivity of the tube material 40 W/mK, calculate the area of the heat exchanger assuming counter flow.

Solution: Let T is the temperature of the cold water at outlet.

By making an energy balance, $\dot{Q} = \dot{m}_h c_h (T_{h1} - T_{h2}) = \dot{m}_c c_c (T_{c2} - T_{c1})$

since $c_h = c_c$, 4.2 kJ / kgK; $2 \times (75 - 45) = 4 \times (T - 20)$; $T = 35^o C$

and $\dot{Q} = 252$ kW

for counter flow: $\theta_1 = (75 - 35) = 40$; $\theta_2 = (45 - 20) = 25$

LMTD = (40 – 25) / ln (40/25) = 31.91

overall heat transfer coefficient based in the inside surface of tube

$1/U = (1/h_i) + (r_i/k)\ln(r_o/r_i) + (r_o r_i)(1/h_o)$

$= 1/150 + (0.04/40)\ln(50/40) + (50/40)(1/180) = 0.0138$

and U = 72.28

area $A = \dot{Q}/(U \times LMTD) = 252 \times 10^3 / (72.28 \times 31.91) = 109.26\ m^2$

Example 3.6 Water flows through a copper tube (k = 350 W/mK, inner and outer diameter 2.0 cm and 2.5 cm respectively) of a double pipe heat exchanger. Oil flows through the annulus between this pipe and steel pipe. The convective heat transfer coefficient on the inside and outside of the copper tube are 5000 and 1500 W /m2K. The fouling factors on the water and oil sides are 0.0022 and 0.00092 K1W. Calculate the overall heat transfer coefficient with and without the fouling factor.

Solution: The scales formed on the inside and outside surface of the copper tube introduces two additional resistances in the heat flow path. Resistance due to inside convective heat transfer coefficient

$1/h_iA_i = 1/5000\ A_i$

Resistance due to scale formation on the inside = $1/h_sA_i$ = 0.0022

Resistance due to conduction through the tube wall = $\ln(r_o/r_i)/2\pi Lk$

$= \ln(2.5/2.0)/2\pi \times L \times 350 = 1.014 \times 10^{-4}/L$

Resistance due to convective heat transfer on the outside

$1/h_oA_o = 1/1500A_o$

Resistance due to scale formation on the outside = $1/h_sA_o = 0.00092$

Since, $Q = \Delta T \sum R = U_iA_1(\Delta T) = \Delta T/(1/U_iA_i)$; we have

(a) With fouling factor:-

Overall heat transfer coefficient based on the inside pipe surface

$$U_i = 1/\left(1/5000 + \pi \times 0.02(0.0022 + 0.00092) + 0.02\pi \times 1.014 \times 10^{-4} + 8.33 \times 10^{-4}\right)$$

= 809.47 W/m^{2}K per metre length of pipe

(b) Without fouling factor

$$U_i = 1/\left(1/5000 + 0.02\pi \times 1.014 \times 10^{-4} + 8.33 \times 10^{-4}\right)$$

= 962.12 W/m^{2}K per m of pipe length.

The heat transfer rate will reduce by (962.12–809.47)1962.12 = 15.9 percent when fouling factor is considered.

Example 3.7 In a surface condenser, dry and saturated steam at 50^{o}C enters at the rate of 1 kg/so The circulating water enters the tube, (25 nun inside diameter, 28 mm outside diameter, k = 300 W/mK) at a velocity of 2 m/s. If the convective heat transfer coefficient on the outside surface of the tube is 5500 W/m^{2}K, the inlet and outlet temperatures of water are 25^{o}C and 35^{o}C respectively, calculate the required surface area.

Solution: For calculating the convective heat transfer coefficient on the inside surface of the tube, we calculate the Reynolds number on the basis of properties of water at the mean temperature of 30^{o}C. The properties are:

μ = 0.001 Pa-s, □ = 1000 kg/m^3, k = 0.6 W/mK, h_{fg} at 50^{o}C = 2375 kJ/kg

Re = □VD/□ = $10^3 \times 2 \times 0.025/0.001$ = 50,000, a turbulent flow. Pr = 7.0.

The heat transfer coefficient at the inside surface can be calculated by:

Nu = 0.023 $Re^{0.8}$ 8 $Pr^{0.3}$ = 0.023 $(50000)^{0.8}$ $(7)^{0.3}$ = 236.828

and h_i = 236.828 × 0.6/0.025 = 5684 W/m^{2}K.

The overall heat transfer coefficient based on the outer diameter,

U = 1/(0.028/(0.025 × 5684) + 1/5500 + 0.014 ln(28/25)/300)

= 2603.14 W/m^{2}K

□T_a. = (50 - 25) = 25; □T_b = (50 - 35) = 15;

□□□□LMTD = (25 -15)/ln(25/15) = 19.576.

Assuming one shell pass and one tube pass, Q = UA (LMTD)

or A = 2375 × 10^3/(2603.14 x 19.576) = 46.6 m^2

Mass of Circulating water = Q/(c_p □T) = 2375/(4.182 × 10) = 56.79 kg/s

also, m_w = □ × area × V × n, where n is the number of tubes.

n = 56.79 × 4/(2 × □ × 0.025 × 0.025 × 1000) = 58 tubes

Surface area, 46.6 = n × □ × d × L

and L = 46.6/(58 × □ × 0.025) = 10.23 m.

Hence more than one pass should be used.

Example 3.8 A heat exchanger is used to heat water from 20°C to 50°C when thin walled water tubes (inner diameter 25 mm, length IS m) are laid beneath a hot spring water pond, temperature 75°C. Water flows through the tubes with a velocity of 1 m/s. Estimate the required overall heat transfer coefficient and the convective heat transfer coefficient at the outer surface of the tube.

Solution: Water flow rate, $\dot{m}$ = □ × V × A = 10^3 × 1 × (□/4) $(0.025)^2$

= 0.49 kg/so

Heat transferred to water, Q = $\dot{m}$c (□T) = 0.49 × 4200 × 30 = 61740 W.

Since the temperature of the water in the hot spring is constant,

$\theta_1 = (75 - 20) = 55; \ \theta_2 = (75 - 50) = 25$;

LMTD = (55 – 25) / ln(55/25) = 38

Overall heat transfer coefficient, U = Q/ (A × LMTD)

= 61740/(38 × □ × 0.025 × 15) = 1378.94 W/m^2K.

The properties of water at the mean temperature (20 + 50)/2 = 35°C are:

$\mu = 0.001$ Pa – s, k = 0.6 W/ mk and Pr = 7.0

Reynolds number, $Re = \rho Vd/\mu = 1000 \times 1.0 \times 0.25/0.001 = 25000$, turbulent flow.

Nu = 0.023 $(Re)^{0.8}$ $(Pr)^{0.33}$ = 0.023 $(25000)^{0.8}$ × $(7)^{0.33}$ = 144.2

and h_i = 144.2 × k/d = 144.2 × 0.6/0.025 = 3460.8 W/m^2K

Neglecting the resistance of the thin tube wall,

$1/U = 1/h_i + 1/h_o; \quad \therefore 1/h_o = 1/1378.94 = 1/3460.8$

or, $h_o = 2292.3$ W / m^2K

Example 3.9 A hot fluid at 200°C enters a heat exchanger at a mass rate of 10000 kg/h. Its specific heat is 2000 J/kg K. It is to be cooled by another fluid entering at 25°C with a mass flow rate 2500 k g/h and specific heat 400 J/kgK. The overall heat transfer coefficient based on outside area of 20 m2 is 250 W/m^2K. Find the exit temperature of the hot fluid when the fluids are in parallel flow.

Solution: From Eq(10.3a), $-U \ dA(1/C_h + 1/C_c) = d(\Delta T)/\Delta T$

Upon integration,

$$-UA(1/C_h + 1/C_c) = \ln(\Delta T)\big|_1^2 = \ln(T_{h_0} - T_{c_0})/(T_{h_i} - T_{c_i})$$

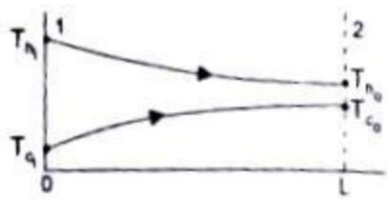
The values are: U = 250 W/m^{2}K

A = 20 m^2

l/C_h = 3600/(10000 × 2000) = 1.8 × 10^{-4}

1/C_c = 3600/(2500 × 400) = 3.6 × 10^{-3}

–UA(1/C_h + l/C_c) = –250 × 20 (1.8 × 10^{-4} + 3.6 × 10^{-3}) = -18.9



; or, $T_{h_0} = T_{c_0}$

By making an energy balance,

$$10000 \times 2000 \left(200 - T_{h_0}\right) = 2500 \times 400 \left(T_{c_0} - 25\right)$$

$$= 2500 \times 400 \left(T_{h_0} - 25\right) \text{ and } 21\, T_{h_0} = 20 \times 200 + 25$$

or, $T_{h_0} = 191.67^o\, C$

Example 3.10 Cold water at the rate of 4 kg/s is heated from 30°C to 50°C in a shell and tube heat exchanger with hot water entering at 95°C at a rate of 2 kg/s. The hot water flows through the shell. The cold water flows through tubes 2 cm inner diameter, velocity of flow 0.38 m/s. Calculate the number 0 f tube passes, the number 0 f tubes per pass if the maximum length of the tube is limited to 2.0 m and the overall heat transfer coefficient is 1420 W/m^{2}K.

Solution: Let T be the temperature of the hot water at exit. By making an energy balance: $4c\left(50-30\right) = 2c\left(95 - T\right); \quad \therefore T = 55^o\, C$

For a counter-flow arrangement:

$$\Delta T_a = \left(95 - 50\right) = 45, \quad \Delta T_b = \left(55 - 30\right) = 25,$$

$\therefore$ LMTD $= (45-25)/\ln(45/25) = 34$; $Q = mC(\Delta T) = 4 \times 4.182 \times 20 = 334.56$ kW

Since the cold water is flowing through the tubes, the number of tubes, n is given by

$\dot{m}$ $\times \rho \times$ Area × velocity; the cross-sectional area 3.142 × 10^{-2} m^2

4 = n × 1000 × 3.142 × 10^{-4} × 0.38; □ n = 33.5, or 34 (say)

Assuming one shell and two tube pass, we use Fig. 10.9(a).

$P(50-30)/(95-30) = 0.3$; $Z = (95-55)/(50-30) = 2.0$

Therefore, the correction factor, F = 0.88

Q = UAF LMTD; 34560 = 1420 × A × 0.88 × 34; or A = 7.875 m^2.

For 2 tube pass, the surface area of 34 tubes per pass = 2 L□d 34

L = 1.843 m

Thus we will have 1 shell pass, 2 tube; 34 tubes of 1.843 m in length.

Example 3.11 A double pipe heat exchanger is used to cool compressed air (pressure A bar, volume flow rate 5 mJ/mm at I bar and 15°C) from 160°C to 35°C. Air flows with a velocity of 5 m/s through thin walled tubes, 2 cm inner diameter. Cooling water flows through the annulus and its temperature rises from 25°C to 40°C. The convective heat transfer coefficient at the inside and outside tube surfaces are 125 W/m^2K and 2000 W/m^2K respectively. Calculate (i) mass of water flowing through the exchanger, and (ii) number of tubes and length of each tube.

Solution: Air is cooled from 1600C to 35°C while water is heated from 25°C to 400C and therefore this must be a counter flow arrangement.

Temperature difference at section 1 : $(T_{h_i} - T_{c_0}) = (160-40) = 120$

Temperature difference at section 2 : $(T_{h_0} - T_{c_i}) = (35-25) = 10$

LMTD = (120 –10)/ln 120/10) = 44.27

Mass of air flowing, $\dot{m}$ = □ × Volume = $(10^5 / 287 \times 288)(5/60) = 0.1$ kg/s

Heat given out by air = Heat taken in by water,

$\therefore\ 0.1 \times 1.005 \times (160 - 35) = \dot{m}_w \times 4.182 \times (40 - 25)$; Or $\dot{m}_w = 0.20$ kg/s

Density of air flowing through the tube, □ = p/RT. The mean temperature of air flowing through the tube is (160 + 35)/2 = 97.5 °C = 370.5K

□□□□□□ = 4 × 10^5/(287 × 370.5) = 3.76 kg/m^3. If n is the number of tubes, from the conservation of mass, $\dot{m} = \rho AV$; 0.1 = 3.76 × (□/4) $(0.02)^2$ × 5 × n

□□□n = 16.9 ≡ 17 tubes; $\dot{Q}$ = UA (LMTD)

U = 1/(1/2000 + 1/125) = 117.65, Area for heat transfer A = □DLn

Q = UA(LMTD); 0.1 × 1005 × 125 = 117.65 × 3.142 × 0.02 × L × 17 × 44.27 and L = 2.26 m.

Example 3.12 A refrigerant (mass rate of flow 0.5 kg/s, S = 907 J/kgK k = 0.07 W/mK, □ = 3.45 × 10^{-4} Pa-s) at 20°C flows through the annulus (inside diameter 3 c m) of a double pipe counter flow heat exchanger used to cool water (mass flow rate 0.05 kg/so k = 0.68 W/mK, □ = 2.83 × 10^{-4} Pa-s) at 98°C flowing through a thin walled copper tube of 2 cm inner diameter. If the length of the tube is 3m, estimate (i) the overall heat transfer coefficient, and (ii) the temperature of the fluid streams at exit.

Solution: Mass rate of flow, $\dot{m} = \rho AV = \rho(\pi/4) D^2 V$;

$\rho VD = 4\dot{m}/\pi D$ and, Reynolds number, $Re = \rho VD/\mu = 4\dot{m}/\pi D\mu$

Water is flowing through the tube of diameter 2 cm,

$\therefore\ Re = 4 \times 0.05/\left(3.142 \times 0.02 \times 2.83 \times 10^{-4}\right) = 1.12 \times 10^4$, turbulent flow.

$$Nu = 0.023\, Re^{0.8} (Pr)^{0.33} = 0.023\left(1.12 \times 10^4\right)^{0.8} (1.8)^{0.33}$$

= 48.45; and $h_i = Nu \times k/D = 48.45 \times 0.68/0.02 = 1647.3\ W/m^2K$

Refrigerant is flowing through the annulus. The hydraulic diameter is

D_o - D_i, and the Reynolds number would be, $Re = 4m / \mu\pi(D_0 + D_i)$

$$Re = 4 \times 0.5 / \left(3.45 \times 10^{-4} \times 3.142 \times (0.02 + 0.03)\right)$$ = 3.69 × 10^4, a turbulent flow.

$$Nu = 0.023(Re)^{0.8}(Pr)^{0.33},$$

where $Pr = \mu\ c / k = 3.45 \times 10^{-4} \times 907 / 0.07 = 4.47$

$$= 0.023\left(3.69 \times 10^4\right)^{0.8}(4.47)^{0.33} = 169.8$$

$$\therefore\ h_o = nu \times k / (D_o - D_i) = 169.8 \times 0.07 / 0.01 = 1188.6\ W / m^2K$$

and, the overall heat transfer coefficient, U = 1/(1/1647.3 + 1/1188.6)

= 690.43 W/m^2K

For a counter flow heat exchanger, from Eq. (10.4), we have,

$$(1/C_c - 1/C_h)UA = \ln(\Delta T_0 / \Delta T_i) = \ln\left[\left(T_{h_0} - T_{c_i}\right)/\left(T_{h_i} - T_{c_0}\right)\right]$$

$$C_c = 0.5 \times 907 = 453.5;\ C_h = 0.05 \times 4182 = 209.1$$

$$1/C_c - 1/C_h UA = (1/453.5 - 1/209.1) \times 690.43 \times 3.142 \times 0.02 \times 3 = -0.335$$

$$\therefore\ \left(T_{h_0} - T_{c_i}\right)/\left(T_{h_i} - T_{c_0}\right) = \exp(-0.335) = 0.715$$

or, $\left(T_{h_0} + 20\right)/\left(98 - T_{c_0}\right) = 0.715$; By making an energy balance,

$$453.5\left(T_{c_0} + 20\right) = 209.1\left(98 - T_{h_0}\right)$$

which gives $T_{c_0} = 3.12^o\,C;\ T_{h_0} = 47.8^o\,C$

3.8. Heat Exchangers Effectiveness - Useful Parameters

In the design of heat exchangers, the efficiency of the heat transfer process is very important. The method suggested by Nusselt and developed by Kays and London is now being extensively used. The effectiveness of a heat exchanger is defined as the ratio of the actual heat transferred to the maximum possible heat transfer.

Let $\dot{m}_h$ and $\dot{m}_c$ be the mass flow rates of the hot and cold fluids, c_h and c_c be the respective specific heat capacities and the terminal temperatures be Th and T h for the hot fluid at inlet and outlet, T_{h_i} and T_{h_0} for the cold fluid at inlet and outlet. By making an energy balance and assuming that there is no loss of energy to the surroundings, we write

$$\dot{Q} = \dot{m}_h\ c_h\left(T_{h_i} - T_{h_0}\right) = \dot{C}_h\left(T_{h_i} - T_{c_0}\right), \text{ and}$$

$$= \dot{m}_c c_c\left(T_{c_0} - T_{c_i}\right) = \dot{C}_c\left(T_{c_0} - T_{c_i}\right) \qquad (3.13)$$

From Eq. (10.13), it can be seen that the fluid with smaller thermal capacity, C, has the greater temperature change. Further, the maximum temperature change of any fluid would be $\left(T_{h_i} - T_{c_i}\right)$ and this Ideal temperature change can be obtained with the fluid which has the minimum heat capacity rate. Thus,

$$\text{Effectiveness, } \in = \dot{Q}/C_{min}\left(T_{h_i} - T_{c_i}\right) \qquad (3.14)$$

Or, the effectiveness compares the actual heat transfer rate to the maximum heat transfer rate whose only limit is the second law of thermodynamics. An useful parameter which also measures the efficiency of the heat exchanger is the 'Number of Transfer Units', NTU, defined as

NTU = Temperature change of one fluid/LMTD.

Thus, for the hot fluid: NTU = $\left(T_{h_i} - T_{h_0}\right)/\text{ LMTD}$, and

for the cold fluid: $\text{NTU} = \left(T_{c_0} - T_{c_i}\right)/\text{ LMTD}$

Since $\dot{Q} = UA\left(\text{LMTD}\right) = C_h\left(T_{h_i} - T_{h_0}\right) = \dot{C}_c\left(T_{c_0} - T_{c_i}\right)$

we have $\text{NTU}_h = UA/C_h$ and $\text{NTU}_c = UA/C_c$

The heat exchanger would be more effective when the NTU is greater, and therefore,

$$NTU = AU/C_{min} \quad (3.15)$$

An another useful parameter in the design of heat exchangers is the ratio the minimum to the maximum thermal capacity, i.e., $R = C_{min}/C_{max}$,

where R may vary between I (when both fluids have the same thermal capacity) and 0 (one of the fluids has infinite thermal capacity, e.g., a condensing vapour or a boiling liquid).

3.9. Effectiveness - NTU Relations

For any heat exchanger, we can write: $\in = f\left(NTU,\ C_{min}/C_{max}\right)$. In order to determine a specific form of the effectiveness-NTU relation, let us consider a parallel flow heat exchanger for which $C_{min} = C_h$. From the definition of effectiveness (equation 10.14), we get

$$\varepsilon = \left(T_{h_i} - T_{h_0}\right)/\left(T_{h_i} - T_{c_i}\right)$$

and, $C_{min}/C_{max} = C_h/C_c = \left(T_{c_0} - T_{c_i}\right)/\left(T_{h_i} - T_{h_0}\right)$ for a parallel flow heat exchanger, from Equation 10.4,

$$\ln\left(T_{h_0} - T_{c_0}\right)/\left(T_{h_i} - T_{c_i}\right) = -UA\left(1/C_h + 1/C_c\right) = \frac{-UA}{C_{min}}\left(1 + C_{min}/C_{max}\right)$$

or, $\left(T_{h_0} - T_{c_0}\right)/\left(T_{h_i} - T_{c_i}\right) = \exp\left[-NTU\left(1 + C_{min}/C_{max}\right)\right]$

But, $\left(T_{h_0} - T_{c_0}\right)/\left(T_{h_i} - T_{c_i}\right) = \left(T_{h_0} - T_{h_i} + T_{h_i} - T_{c_0}\right)/\left(T_{h_i} - T_{c_i}\right)$

$$= \left[\left(T_{h_0} - T_{h_i}\right) + \left(T_{h_i} - T_{c_i}\right) - \left\{R\left(T_{h_i} - T_{h_0}\right)\right\}\right]/\left(T_{h_i} - T_{c_i}\right)$$

$$= \in + 1 - R \in = 1 - \in\left(1 + R\right)$$

Therefore, $\in = \left[1 - \exp\left\{-NTU(1+R\right\}\right]/\left(1 + R\right)$

$$NTU = -\ln\left[1 - \in\left(1 + R\right)\right]/\left(1 + R\right)$$

Similarly, for a counter flow exchanger, $\varepsilon = \dfrac{\left[1 - \exp\left\{-NTU(1-R)\right\}\right]}{\left[1 - R\exp\left\{-NTU(1-R)\right\}\right]}$;

and, $NTU = \left[1/\left(R-1\right)\right] \ln\left[\left(\in -1\right)/\left(\in R-1\right)\right]$

Heat Exchanger Effectiveness Relation

Flow arrangement	relationship
Concentric tube	
Parallel flow	$\in = \dfrac{1-\exp\left[-N\left(1+R\right)\right]}{\left(1+R\right)}$; $R = C_{min} / C_{max}$
Counter flow	$\in = \dfrac{1-\exp\left[-N\left(1-R\right)\right]}{1-R\exp\left[-N\left(1-R\right)\right]}$; $R < 1$
	$\in = N/\left(1+N\right)$ for $R = 1$
Cross flow (single pass)	
Both fluids unmixed	$\in = 1-\exp\left[\left(1/R\right)\left(N\right)^{0.22}\left\{\exp\left(-R\left(N\right)^{0.78}\right)-1\right\}\right]$
C_{max} mixed , C_{min} unmixed	$\in = \left(1/R\right)\left[1-\exp\left\{-R\left(1-\exp\left(-N\right)\right)\right\}\right]$
C_{min} mixed, C_{max} unmixed	$\in = 1-\exp\left[-R^{-1}\left\{1-\exp\left(-RN\right)\right\}\right]$
All exchangers (R = 0)	$\in = 1-\exp\left(-N\right)$

Kays and London have presented graphs of effectiveness against NTU for Various values of R applicable to different heat exchanger arrangements, Fig. 3.11 to Fig. (3.15).

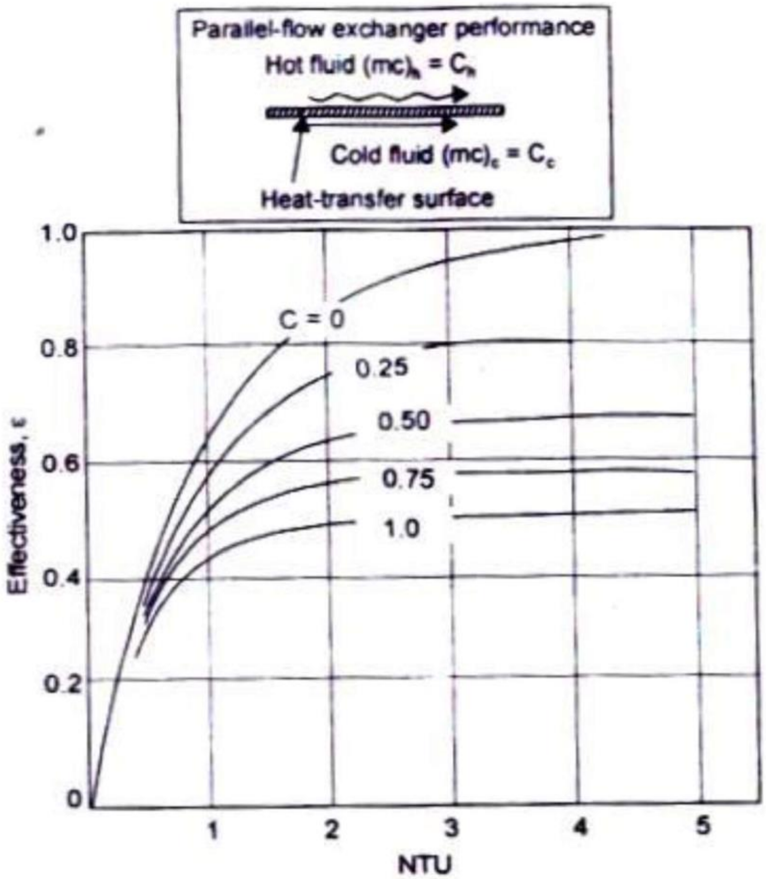


Fig 3.11 Heat exchanger effectiveness for parallel flow

Example 3.13 A single pass shell and tube counter flow heat exchanger uses exhaust gases on the shell side to heat a liquid flowing through the tubes (inside diameter 10 mm, outside diameter 12.5 mm, length of the tube 4 m). Specific heat capacity of gas 1.05 kJ/kgK, specific heat capacity of liquid 1.5 kJ/kgK, density of liquid 600 kg/m^3, heat transfer coefficient on the shell side and on the tube sides are: 260 and 590 W/m^{2}K respectively. The gases enter the exchanger at 675 K at a mass flow rate of 40 kg/s and the liquid enters at 375 K at a mass flow rate of 3 kg/s. If the velocity of liquid is not to exceed 1 m/s, calculate (i) the required number of tubes, (ii) the effectiveness of the heat exchanger, and (iii) the exit temperature of the liquid. Neglect the thermal resistance of the tube wall.

Solution: Volume flow rate of the liquid = 3/600 = 0.005 m/s. For a velocity of 1 mls through the tube, the cross-sectional area of the tubes will be 0.005 m^2. Therefore, the number of tubes would be

$$n(0.005\times 4)/(3.142\times 0.01)^2 = 63.65 = 64 \text{ tubes}$$

The overall heat transfer coefficient based on the outside surface area of the tubes, after neglecting the thermal resistance of the tube wall, is

$$U = 1/\left(1/h_0 + r_o/r_i h_i\right) = 1/\left[1/260 + 12.5/(10\times 590)\right] = 167.65 \text{ W/m}^2\text{K}$$

$$C_{max} = 40\times 1.05 = 42;\ C_{min} = 3\times 1.5 = 4.5;\ R = 4.5/42 = 0.107$$

$$NTU = AU/C_{min} = 3.142\times 0.0125\times 4\times 64\times 167.65/(4.5\times 1000) = 0.374$$

From Fig. 10.12, for R = 0.107, and NTU = 0.3 74, E = 0.35 approximately Therefore, $0.35 = \left(T_{c_0} - 375\right)/(675-375)$ or $T_{c_0} = 207^o\,C$

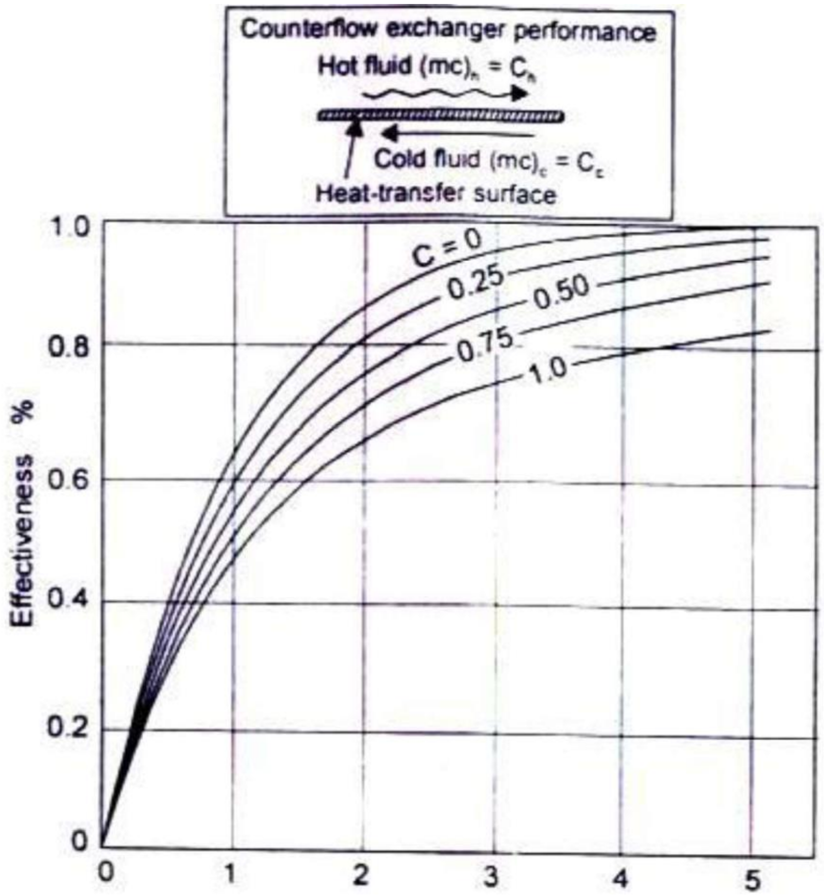


Fig 3.12 Heat exchanger effectiveness for counter flow

Example 3.14 Air at 25°C, mass flow rate 20 kg/min, flows over a cross-flow heat exchanger and cools water from 85°C to 50°C. The water flow rate is 5 kg/mm. If the overall heat transfer coefficient IS 80 W/m^{2}K and air is the mixed fluid, calculate the exchanger effectiveness and the surface area.

Solution: Let the specific heat capacity of air and water be 1.005 and 4.182 kJ/ kgK. By making an energy balance:

$$\dot{m}_c \times c_c \times \left(T_{c_0} - T_{c_i}\right) = \dot{m}_h \times c_h \times \left(T_{h_i} - T_{h_0}\right)$$

or, 5 × 4182 × (85 – 50) = 20 × 1005 × $\left(T_{c_0} - 25\right)$

i.e., the air will come out at 61.4 ^{o}C.

Heat capacity rates for water and air are:

$C_w = 4182 \times 5/60 = 348.5; \quad C_a = 1005 \times 20/60 = 335$

$R = C_{min}/C_{max} = 335/348.5 = 0.96$

The effectiveness on the basis of minium heat capacity rate is

$\in = (61.4 - 25)/(85 - 25) = 0.6$

From Fig. 10.13, for R = 0.96 and $\in$ = 06, NTU = 2.5

Since NTU = AU/C_{min}; A = 2.5 × 335/80 = 10.47 m^2

Since all the four terminal temperatures are easily obtained, we can also use the LMTD approach. Assuming a simple counter flow heat exchanger,

LMTD = (25 – 23.6)/ ln (25/23.6) = 24.3

The correction factor for using a cross-flow heat exchanger with one fluid mixed and the other unmixed, from Fig. 10.10(d), F = 0.55

$\dot{Q}$ = U A F (LMTD)

Therefore, A = 348.5 × 35/(80 × 0.55 × 24.3) = 11.4 m^2

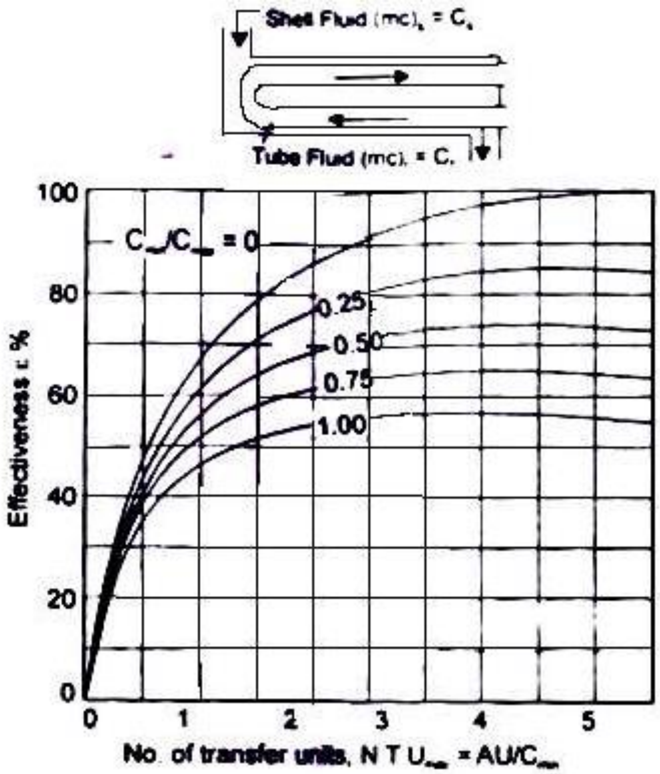
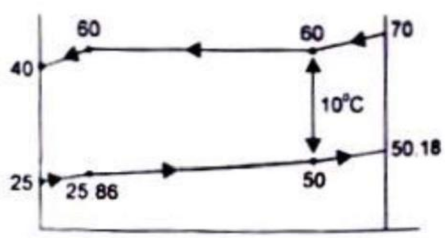


Fig. 3.13 Heat exchanger effectiveness for shell and tube heat exchanger with one shell pass and two, or a multiple of two, tube passes

Example 3.15 Steam at 20 kPa and 70°C enters a counter flow shell and tube exchanger and comes out as subcooled liquid at 40°C. Cooling water enters the condenser at 25°C and the temperature difference at the pinch point is 10°C. Calculate the (i) amount of water to be circulated per kg of steam condensed, and (ii) required surface area if the overall heat transfer coefficient is 5000 W/m^{2}K and is constant.

Solution: The temperature profile of the condensing steam and water is shown in the 40 accompanying sketch.



The saturation temperature corresponding to 20 kPa is 60°C and as such the temperature of the cooling water at the pinch pint is 50°C. The condensing unit may be considered as a combination of three sections:

(i) desuperheater - the superheated steam is condensed to saturated steam from 70°C to 60°C.

(ii) the condenser - saturated steam is condensed into saturated liquid.

(iii) subcooler - saturated liquid at 60°C is cooled to 40°C.

Assuming that the specific heat capacity of superheated steam is 1.8 kJ/kgK, heat given out in the desuperheater section is 1.8 × (70 - 60) = 18000 Jlkg. Heat given out in the condenser section = 2358600 J/kg (= hfg)

Heat given out in the subcooler = 4182 × (60 - 40) = 83640 J/kg

By making an energy balance, for subcooler and condenser section, we have

$$\dot{m}_w \times 4182 \times (50 - 25) = (83640 + 2358600) \; ;$$

∴. Mass of water circulated, $\dot{m}_w$ = 23.36 kg/kg steam condensed.

The temperature of water at exit

= 25 + (83640 + 2358600 + 18000)/ (23.36 × 4182) = 50.18 °C

LMTD for desuperheater section

= [(70 - 50.18) - (60 - 50))/ln(20.18/1 0) = 14.5

LMTD for condenser section = [(60 - 50) - (60 - 25.86))/ln(1 0/34.14)

= 19.66

LMTD for subcooler section = [(34.14 - 15)/1n (34.14/15) = 23.27

Since U is constant through out,

Surface area for subcooler section = 83640/(5000 × 23.27) = 0.7188 m^2

Surface area for condenser section = 2358600/(5000 × 19.66) = 23.9939 m^2

Surface area for desuperheater section = 18000/(5000 × 14.5) = 0.2483 m^2

$\therefore$. Total surface area = 24.96 m^2 and average temperature difference = 19.71°C.

Example 3.16 In an economiser (a cross flow heat exchanger, both fluids unmixed) water, mass flow rate 10 kg/s, enters at 175°C. The flue gas mass flow rate 8 kg/s, specific heat 1.1 kJ/kgK, enters at 350°C. Estimate the temperature of the flue gas and water at exit, if U = 500 W/m^2K, and the surface area 20 m^2 What would be the exit temperature if the mass flow rate of flue gas is (i) doubled, and (ii) halved.

Solution: The heat capacity rate of water = 4182 × 10 = 41820 W /K

The heat capacity rate of flue gas = 1100 × 8 = 8800 W/K

C_{min}/C_{max} = 8800/41820 = 0.21

$$NTU = AU/C_{min} = 500 \times 20/8800 = 1.136$$

From Fig. 10.14. for NTU = 1.136 and $C_{min}/C_{max} = 0.21$, $\in = 0.62$

Therefore, 0.62 = (350 - T)/(350 - 175) and T = 241.5°C

The temperature of water at exit, T_w = 175 + 8800 × (350 - 241.5)/41820

= 197.83°C

When the mass flow rate of the flue gas is doubled. C_{gas} = 17600 W/K

$C_{min}/C_{max} = 0.42$, $NTU = AU/C_{min} = 0.568$

$\in$ = 0.39 = (350 - T)/(350 - 175);

T = 281. 75°C, an increase of 40°C

and T_w = 175 + 28.72 = 203.72°C, an increase of about 6°C.

When the mass flow rate of the flue gas is halved, C_{min} = 4400 W/K

$C_{min}/C_{max} = 0.105$, NTU = 2.272, and from the figure, $\in$ = 0.83, an increase and T_g = 204.75 and T_w = 190.3°C

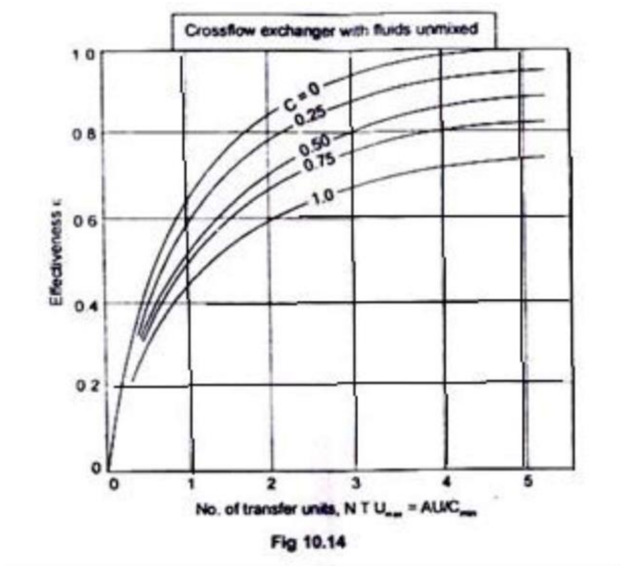


Fig 3.14

Example 3.17 In a tubular condenser, steam at 30 kPa and 0.95 dry condenses on the external surfaces of tubes. Cooling water flowing through the tubes has mass flow rate 5 kg/s, inlet temperature 25°C, exit temperature 40°C. Assuming no subcooling of the condensate, estimate the rate of condensation of steam, the effectiveness of the condenser and the NTU.

Solution: Since there is no subcooling of the condensate, the steam will lose its latent heat of condensation = 0.95 × h_{fg} = 0.95 × 2336100 = 2.22 × 10^6 J/kg. At pressure, 30kPa, saturation temperature is 69.124^{o}C

Steam condensation rate × 2.22 × 10^6 = Heat gained by water

= 5 × 4182 × (40 – 25) = 313650 J

Therefore, m, = 313650/2.22 × 106 = 847.7 kg/hour.

When the temperature of the evaporating or condensing fluid remains constant, the value of LMTD is the same whether the system is having a parallel flow or counter flow arrangement, therefore,

LMTD = [(69.124 - 25) - (69.124-40)]/ln(44.124/29.124) = 36.1

Q = UA(LMTD)

Therefore, UA = 5 × 4182 × (40 - 25)/36.1 = 8688.36 W/K

NTU = UA/C_{min} = 8688.36/(5 × 4182) = 0.4155

Effectiveness= Actual temp. difference; Maximum possible temp. difference

= (40 - 25)/(69.124 - 25) = 34%.

Example 3.18 A single shell 2 tube pass steam condenser IS used to cool steam entering at 50°C and releasing 2000 MW of heat energy. The cooling water, mass flow rate 3 × 104 kg/s, enters the condenser at 25°C. The condenser has 30,000 thin walled tube of 30 mm diameter. If the overall heat transfer coefficient is 4000 Wim^2K, estimate the (I) rise m temperature of the cooling water, and (II) length of the tube per pass.

Solution: By making an energy balance:

Heat released by steam = heat taken in by cooling water,

or, $2000 \times 10^6 = 3 \times 10^4 \times 4182 \times (\Delta T)$; □□□□T = 15.94°C.

Since in a condenser, heat capacity rate of condensing steam is usually very large m comparison with the heat capacity rate of cooling water, the effectiveness

$$\in = \left(T_{c_0} - T_{c_i}\right) / \left(T_{h_i} - T_{c_i}\right) = 15.94 / (50 - 25) = 0.6376$$

And, for $C_{min} / C_{max} = 0, \in = 1 - \exp(-NTU)$

$$\therefore \exp(-NTU) = 1.0 - 0.6376 = 0.3624$$

And, $NTU = 1.015 = AU / C_{min} = (2 \times 3.142 \times 0.03 \times L \times 30000) \times 4000 / (1.25 \times 108)$

L = 5.546 m

3.10. Heat Exchanger Design-Important Factors

A comprehensive design of a heat exchanger involves the consideration of the thermal, mechanical and manufacturing aspect. The choice of a particular design for a given duty depends on either the selection of an existing design or the development of a new design. Before selecting an existing design, the analysis of his performance must be made to see whether the required performance would be obtained within acceptable limits.

In the development of a new design, the following factors are important:

(a) Fluid Temperature - the temperature of the two fluid streams are either specified for a given inlet temperature, or the designer has to fix the outlet temperature based on flow rates and heat transfer considerations. Once the terminal temperatures are defined, the effectiveness of the heat exchanger would give an indication of the type of flow path-parallel or counter or cross-flow.

(b) Flow Rates - The maximum velocity (without causing excessive pressure drops, erosion, noise and vibration, etc.) in the case of liquids is restricted to 8 m1s and m case of gases below 30 m/s. With this restriction, the flow rates of the two fluid streams lead to the selection of flow passage cross-sectional area required for each of the two fluid steams.

(c) Tube Sizes and Layout - Tube sizes, thickness, lengths and pitches have strong influence on heat transfer calculations and therefore, these are chosen with great care. The sizes of tubes vary from 1/4" O.D. to 2" O.D.; the more commonly used sizes are: 5/8", 3/4" and 1" O.D. The sizes have to be decided after making a compromise between higher heat transfer from smaller tube sizes and the easy clean ability of larger tubes. The tube thickness will depend pressure, corrosion and cost. Tube pitches are to be decided on the basis of heat transfer calculations and difficulty in cleaning. Fig. 3.16 shows several arrangements for tubes in bundles. The two standard types of pitches are the square and the triangle. the usual number 0 f tube passes in a given shell ranges from one to eight. In multipass designs, even numbers of passes are generally used because they are Simpler to design.

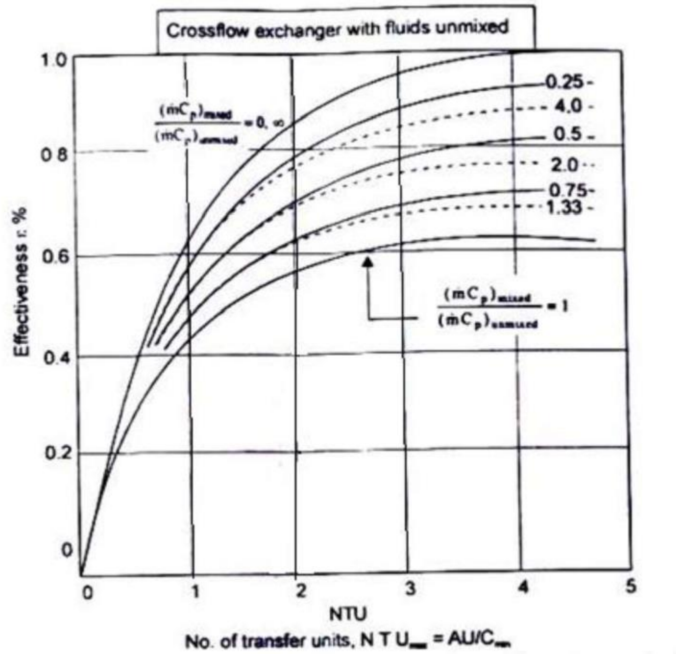


Fig 3.15: Heat exchanger effectiveness for crossflowwith one fluid mixed and the other unmixed

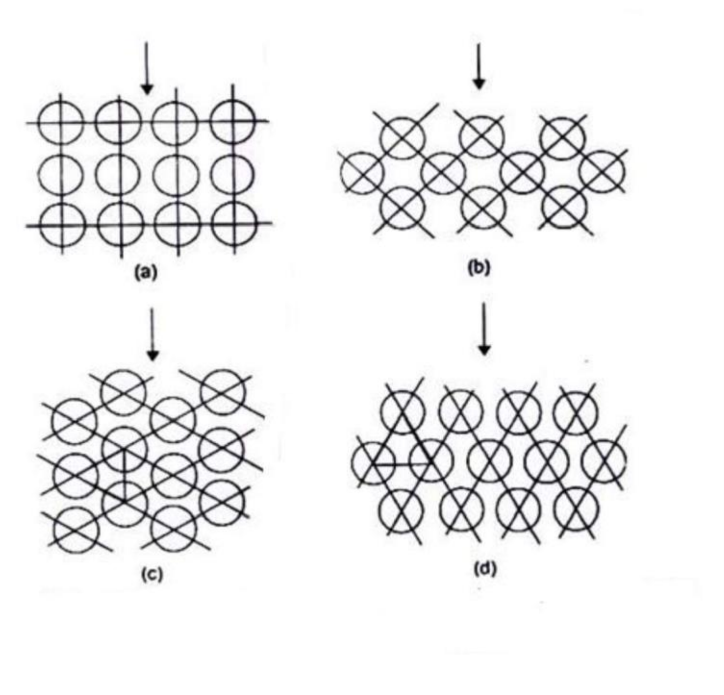


Fig 3.16 Several arrangements of tubes in bundles : (a) I line arrangement with square pitch, (b) staggered arrangement with triangular pitches (c) and(d) stagged arrangement with triangular pitches

Fig 3.17 shows three types of transverse baffles used to increase velocity on the shell side. The choice of baffle spacing and baffle cut is a variable and the optimum ratio of baffle cuts and spacing cannot be specified because of many uncertainties and insufficient data.

(d)Dirt F actor and Fouling - the accumulation of dirt or deposits affects significantly the r ate of heat transfer and the pressure drop. P roper allowance for the fouling factor and dirt factor should receive the greatest attention design because they cannot be avoided. A heat exchanger requires frequent cleaning. Mechanical cleaning will require removal of the tube bundle for cleaning. Chemical cleaning will require the use of non-corrosive materials for the tubes.

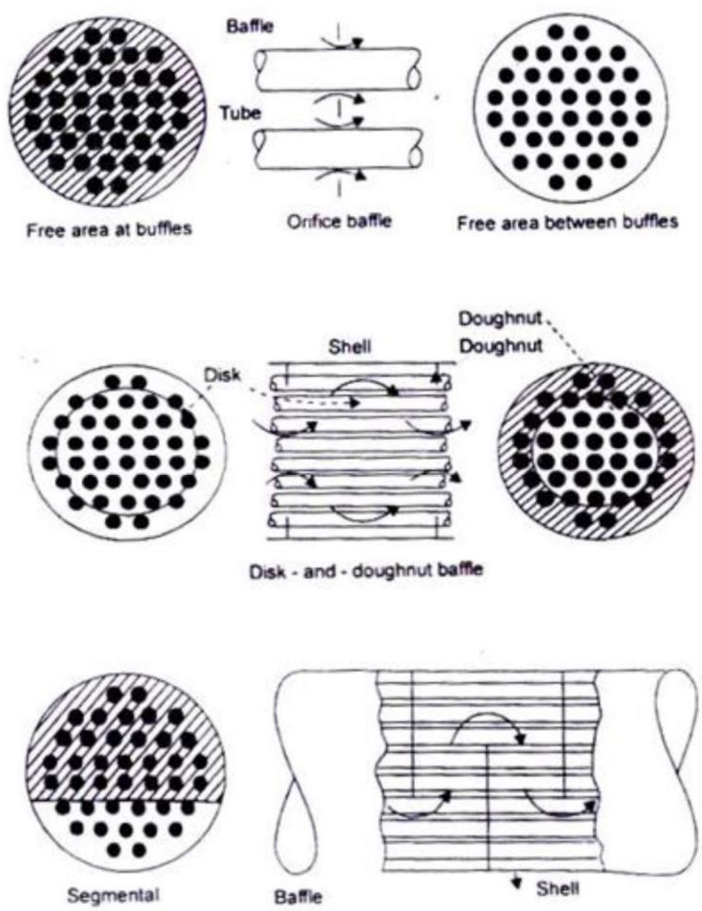


Fig. 3.17 Three types of transverse baffles

(e) Size and Installation - In designing a heat exchanger, It is necessary that the constraints on length, height, width, volume and weight is known at the outset. Safety regulations should also be kept in mind when handling fluids under pressure or toxic and explosive fluids.

(f) Mechanical Design Consideration - While designing, operating temperatures, pressures, the differential thermal expansion and the accompanying thermal stresses require attention.

And, above all, the cost of materials, manufacture and maintenance cannot be Ignored.

Example 3.19 In a counter flow concentric tube heat exchanger cooling water, mass flow rate 0.2 kg/s, enters at 30°C through a tube inner diameter 25rnnl. The oil flowing through the annulus, mass flow rate 0.1 kg/s, diameter 45 rum, has temperature at inlet 100°C. Calculate the length of the tube if the oil comes out at 60°C. The properties of oil and water are:

Oil: C_p – 2131 J/kgK, □ = 3.25×10^{-2} Pa-s, k = 0.138 W/mK,

Water; C_p = 4178 J/kg K, □ = 725×10^{-6} Pa-s,

k = 0.625 W/mK, Pr = 4.85

Solution: By making an energy balance: Heat given out by oil = heat taken in by water.

$$0.1 \times 2131 \times (100 - 60) = 0.2 \times 4187 \times \left(T_{c_0} - 30\right)$$

$$T_{c_0} = 40.2^o \text{ C}$$

$$\text{LMTD} = \left[\left(T_{h_i} - T_{c_0}\right) - \left(T_{h_0} - T_{c_i}\right)\right] / \ln\left[\left(T_{h_i} - T_{c_0}\right) / \left(T_{h_0} - T_{c_0}\right)\right]$$

$$= \left[(100 - 40.2) - (60 - 30)\right] / \ln(59.8/30) = 43.2^o \text{ C}$$

Since water is flowing through the tube,

$$\text{Re} = 4\dot{m} / \pi D \mu = \frac{4 \times 0.2}{3.142 \times 0.025 \times 725 \times 10^{-6}}$$ = 14050, a turbulent flow.

□□□□Nu = 0.023 $\text{Re}^{0.8}$ $\text{Pr}^{0.4}$, fluid being heated.

= 0.023 $(14050)^{0.8}$ $(4.85)^{0.4}$ = 90; $\therefore$ h_i = 90 × 0.625/0.025 = 2250 W/m^2K

The oil is flowing through the annulus for which the hydraulic diameter is:

(0.045 - 0.025) = 0.02 m

$$Re = 4\dot{m}/\pi(D_o + D_i)\mu = 4 \times 0.1/\left(3.142 \times 0.07 \times 3.25 \times 10^{-2}\right) = 56.0$$

laminar flow.

Assuming Uniform temperature along the Inner surface of the annulus and a perfectly insulated outer surface.

Nu = 5.6, by interpolation (chapter 6)

$h_o = 5.6 \times 0.138/0.02 = 38.6\ W/m^2K$.

The overall heat transfer coefficient after neglecting the tube wall resistance,

$U = 1/(1/2250 + 1/38.6) = 38\ W/m^2K$

$\dot{Q} = UA(LMTD)$, where where $A = \pi D_i \times L$

$L = (0.1 \times 2131 \times 40)/(38 \times 3.142 \times 0.025 \times 43.2) = 66.1$ m requires more than one pass.

Example 3.20 A double pipe heat exchanger has an effectiveness of 0.5 for the counter flow arrangement and the thermal capacity of one fluid is twice that of the other fluid. Calculate the effectiveness of the heat exchanger if the direction of flow of one of the fluids is reversed with the same mass flow rates as before.

Solution: For a counter flow arrangement and R = 0.5, $\in = 0.5$

$$NTU = \left[1/(R-1)\right]\ln(\in R - 1) = -2.0\ \ln(0.5/0.75) = 0.811$$

For parallel flow, $\in\ = \left[1 - \exp\{-NTU(1+R)\}\right]/(1+R)$

$$= \left[1 - \exp(-0.811 \times 1.5)\right]/1.5 = 0.469$$

Example 3.21 Oil is cooled in a cooler from 65°C to 54°C by circulating water through the cooler. The cooling load is 200 kW and water enters the cooler at 27°C. If the overall heat transfer coefficient, based on the outer surface area of the tube is 740 W/m^2K and the temperature rise of cooling water is 11°C, calculate the mass flow rate of water, the effectiveness and the heat transfer area required for a single pass In a parallel flow and in a counter flow arrangement.

Solution: Cooling load = 200 kW = mass of water × sp. heat × temp. rise Mass of water = 200/(4.2 × 11) = 4.329 kg/s

(i) Parallel flow:

From the temperature profile:

LMTD = (38 – 16)/ln(38/16) = 25.434 Q = U A (LMTD);

Area A = 200 × 10^3/(740 × 25.434) = 10.626 m^2

Effectiveness, $\in$ = (38 - 27)/(54 - 27) = 0.407.

(ii) Counter flow:

From the temperature profile:

LMTD = mean temperature difference = 27°C

Area A = 200 × 10^3/(740 × 27) = 10 m^2

Effectiveness, E = (38 - 27)/(65 - 27) = 0.289.

Example 3.22 Oil (mass flow rate 1.5 kg/s C_p = 2 kJ/kgK) is cooled in a single pass shell and tube heat exchanger from 65 to 42°C. Water (mass flow rate 1 kg/s, C_p = 4.2 kJ/kgK) has an inlet temperature of 28°C. If the overall heat transfer coefficient is 700 W/m^2K, calculate heat transfer area for a counter flow arrangement using $\in$NTU method.

Solution: Heat capacity rate of 011; 1.5 × 2.0 = 3 kW/K

Heat capacity rate of water = 1 × 4.2; 4.2 kW/K

$C_{min} = 3.0 \text{ kW/K}$ and $R = C_{min}/C_{max} = 3/4.2 = 0.714$

For a counter flow arrangement, $NTU = [1/(R-1)] \ln[(\in -1)/(\in R - 1)]$

Effectiveness, $\in = (65-42)/(65-28) = 0.6216$

and NTU = 1.346 = AU/C_{min}; A = 1.346 × 3000 / 700 = 5.77 m^2

By making an energy balance, we can compute the water temperature at outlet.

or 3.0 × (65 - 42); 4.2 × (T - 28), T; 44.428

LMTD for a counter flow arrangement:

LMTD; (20.572 - 14)/ln (20.572/14) = 17.076

Area, A = $\dot{Q}/U \times (LMTD) = 3 \times 10^3 \times (65-42)/(700 \times 17.076) = 5.77 \text{ m}^2$

Example 3.23 A fluid (mass flow rate 1000 kg/min, sp. heat capacity 3.6 kJ/kgK) enters a heat exchanger at 700 C. Another fluid (mass flow rate 1200 kg/mm, sp. heal capacity 4.2 kJ/kgK) enters al 100 C. If the overall heat transfer coefficient is 420 W/m^2K and the surface area is 100m2, calculate the outlet temperatures of both fluids for both counter flow and parallel flow arrangements.

Solution: Heat capacity rate for the hot fluid

$$1000 \times 3.6 \times 10^3 \times 60 = 60 \times 10^3 \text{W/K}$$

Heat capacity rate for the cold fluid = 1200 × 4.2 × 10^3/60 = 84 × 10^3 W/K

R = C_{min}/C_{max} = 60/84; 0.714, NTU = U A/C_{min} = 420 × 100/60000 = 0.7

(i) For counter flow heat exchanger:

$$\in = [1 - \exp\{-N(1-R)\}]/[1 - R\exp\{-N(1-R)\}]$$

$$[1 - \exp\{-0.7(1-0.714)\}]/1[1 - 0.714 \exp\{-0.7(1-0.714)\}] = 0.4367$$

Since heat capacity rate of the hot fluid IS lower,

$$\in = \left(700 - T_{h_0}\right) / \left(700 - 100\right)$$

and $T_{h_0} = 700 - 0.4367 \times 600 = 438^o C$

By making an energy balance, $60 \times 10^3 \left(700 - 438\right) = 84 \times 10^3 \left(T_{c_0} - 100\right)$

or, $T_{c_0} = 60 \times 262 / 84 + 100 = 87.14^o$ C

(ii) For parallel flow heat exchanger

$$\in = \left[1 - \exp\left\{-N\left(1 + R\right)\right\}\right] / \left(1 + R\right) = \left[1 - \exp\left\{0.7\left(1 + 0.714\right)\right\}\right] / \left(1.714\right)$$

$\in = 0.4077$, a lower value

and $\left(T_{h_i} - T_{h_0}\right) / \left(T_{h_i} - T_{c_0}\right) = 0.4077 = \left(700 - T_{h_0}\right) / \left(700 - T_{c_0}\right)$

By making an energy balance: $60 \times 10^3 \times \left(700 - T_{h_0}\right) = 84 \times 10^3 \times \left(T_{c_0} - 100\right)$

or, $\left(700 - T_{c_0}\right) = \left(700 - T_{h_0}\right) / \ 0.4077$

and $84 \times \left(T_{c_0} - 100\right) / 60 = \left(1.4 T_{c_0} - 140\right)$

Therefore, $T_{c_0} = 237.5^o C$

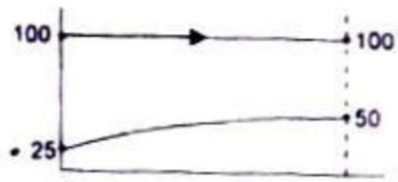
and $T_{h_0} = 511.4^o$ C

Example 3.24 Steam enters the surface condenser at 100°C and water enters at 25°C with a temperature rise of 25°C. Calculate the effectiveness and the NTU for the condenser. If the water temperature at inlet changes to 35 C, estimate the temperature rise for water.

Solution: Effectiveness, $\in$ = 25/(100 25) = 0.33

For R = 0, $\in$ 1 - exp(- N)

or, N = -ln(1 - $\in$) = 0.405



Since other parameters remain the same,

$$25/(100 - 25) = \square T/(100 - 35)$$

and $\square T = 21.66$; or, $T_{c_0} = 35 + 21.66 = 56.66°C$.

3.11. Increasing the Heat Transfer Coefficient

For a heat exchanger, the heat load is equal to Q = UA (LMTD). The effectiveness of the heat exchanger can be increased either by increasing the surface area for heat transfer or by increasing the heat transfer coefficient. Effectiveness versus NTU(AU/C_{min}) curves, Fig. 10.10 - 15, reveal that by increasing the surface area beyond a certain limit (the knee of the curves), there is no appreciable improvement in the performance of the exchangers. Therefore, different methods have been employed to increase the heat transfer coefficient by increasing turbulence, improved mixing, flow swirl or by the use of extended surfaces. The heat transfer enhancement techniques is gaining industrial importance because it is possible to reduce the heat transfer surface area required for a given application and that leads to a reduction in the size of the exchanger and its cost, to increase the heating load on the exchanger and to reduce temperature differences.

The 'different techniques used for increasing the overall conductance U are: (a) Extended Surfaces - these are probably the most common heat transfer enhancement methods. The analysis of extended surfaces has been discussed in Chapter 2. Compact heat exchangers use extended surfaces to give the required heat transfer surface area in a small volume. Extended surfaces are very effective when applied in gas side heat transfer. Extended surfaces find their application in single phase natural and forced convection pool boiling and condensation.

(b) Rough Surfaces - the inner surfaces of a smooth tube is artificially roughened to promote early transition to turbulent flow or to promote mixing between bulk flow and the various sub-layer in fully developed turbulent flow. This method is primarily used in single phase forced convection and condensation.

(c) Swirl Flow Devices - twisted strips are inserted into the flow channel to impart a rotational motion about an axis parallel to the direction of bulk flow. The heat transfer coefficient increases due to increased flow velocity, secondary flows generated by swirl, or increased flow path length in the flow channel. This technique is used in flow boiling and single phase forced flow.

(d) Treated Surfaces - these are used mainly in pool boiling and condensation.

Treated surfaces promote nucleate boiling by providing bubble nucleation sites. The rate of condensation increases by promoting the formation of droplets, instead of a liquid film on the condensing surface. This can be accomplished by coating the surface with a material that makes the surface non-wetting.

All of these techniques lead to an increase in pumping work (increased frictional losses) and any practical application requires the economic benefit of increased overall conductance. That is, a complete analysis should be made to determine the increased first cost because of these techniques, increased heat exchanger heat transfer performance, the effect on operating costs (especially a substantial increase in pumping power) and maintenance costs.

3.12. Fin Efficiency and Fin Effectiveness

Fins or extended surfaces increase the heat transfer area and consequently, the amount of heat transfer is increased. The temperature at the root or base of the fin is the highest and the temperature along the length of the fm goes on decreasing Thus, the fin would dissipate the maximum amount of heat energy if the temperature all along the length remains equal to the temperature at the root. Thus, the fin efficiency is defined as:

η_{fin} = (actual heat transferred) / (heat which would be transferred if the entire fin area were at the root temperature)

In some cases, the performance of the extended surfaces is evaluated by comparing the heat transferred with the fin to the heat transferred without the fin. This ratio is called 'fin effectiveness' E and it should be greater than 1, if the rate of heat transfer has to be increased with the use of fins.

For a very long fin, effectiveness $E = \dot{Q}_{with\ fin} / \dot{Q}_{without\ fin}$

$= (hpkA)^{1/2}\theta_0 \, / hA\,\theta_0 = (kp/hA)^{1/2}$

And $\eta_{fin} = (hpkA)^{1/2}\theta_0 \, (hpL\,\theta_0) = (hpkA)^{1/2} / (hpL)$

$$\frac{E}{\eta_{fin}} = \frac{(kp/hA)^{1/2}}{(hpkA)^{1/2}} \times hpL = \frac{pL}{A} = \frac{\text{Surface area of fin}}{\text{Cross-sectional area of the fin}}$$

i.e., effectiveness increases by increasing the length of the fin but it will decrease the fin efficiency.

Expressions for Fin Efficiency for Fins of Uniform Cross-section

1. Very long fins: $(hpkA)^{1/2}(T_0 - T_\infty)/\left[hpL(T_0 - T_\infty)\right] = 1/\,mL$

2 For fins having insulated tips:

$$\frac{(hpkA)^{1/2}(T_0 - T_\infty)\tan h(mL)}{hpL(T_0 - T_\infty)} = \frac{\tan h\,(mL)}{mL}$$

Example 3.25 The total efficiency for a finned surface may be defined as the ratio of the total heat transfer of the combined area of the surface and fins to the heat which would be transferred if this total area were maintained at the root temperature T_0 . Show that this efficiency can be calculated from

$\eta_t = 1 - A_f / A(1 - \eta_t)$ where η_t = total effltiency, A_f = surface area of all fins, A = total heat transfer area, η_f = fin efficiency

Solution: Fin efficiency,

$$\eta_f = \frac{\text{Actual heat transfered}}{\text{Heat that would be transferred if the entire fin were at the root temperature}}$$

or, $\eta_f = \dfrac{\text{Actual heat transfer}}{hA_f(T_0 - T_\infty)}$

$\therefore$ Actual heat transfer from finned surface = $\eta_f hA_f (T_0 - T_\infty)$

Actual heat transfer from un finned surface which are at the root temperature: $h(A-A_f)$

$(T_0 - T_\infty)$

Actual total heat transfer = $h(A - A_f)(T_0 - T_\infty) + \eta_f h A_f (T_0 - T_\infty)$

By the definition of total efficiency,

$$\eta_t = \left[h(A - A_f)(T_0 - T_\infty) + \eta_f h A_f (T_0 - T_\infty)\right] / \left[hA(T_0 - T_\infty)\right]$$

$$= \frac{(A - A_f) + \eta_f h A_f}{A} = 1 - A_f / A + \eta_f A_f / A$$

$$= 1 - (A_f / A) + (1 - \eta_f).$$

3.13. Extended Surfaces do not always Increase the Heat Transfer Rate

The installation of fins on a heat transferring surface increases the heat transfer area but it is not necessary that the rate of heat transfer would increase. For long fins, the rate of heat loss from the fin is given by $(hpkA)^{1/2}\theta_0$ = $kA(hp/kA)^{1/2}\theta_0$ = $kAm\,\theta_0$. When h/ mk = 1, Q = $hA\,\theta_0$ which is equal to the heat loss from the primary surface with no extended surface. Thus, when h = mk, an extended surface will not increase the heat transfer rate from the primary surface whatever be the length of the extended surface.

For h/mk > 1, Q < $hA\,\theta_0$ and hence adding a secondary surface reduces the heat transfer, and the added surface will act as an insulation. For h/mk < 1, Q > $hA\,\theta_0$, and the extended surface will increase the heat transfer, Fig. 2.3l. Further, $h/mk = (h^2.kA/k^2hp)^{1/2}$ = $(hA/kP)^{1/2}$, i.e. when h/mk < 1, the heat transfer would be more effective when h/k is low for a given geometry.

3.14. An Expression for Temperature Distribution for an Annular Fin of Uniform Thickness

In order to increase the rate of heat transfer from cylinders of air-cooled engines and in certain type of heat exchangers, annular fins of uniform cross-section are employed. Fig. 2.32 shows such a fin with its nomenclature.

In the analysis of such fins, it is assumed that:

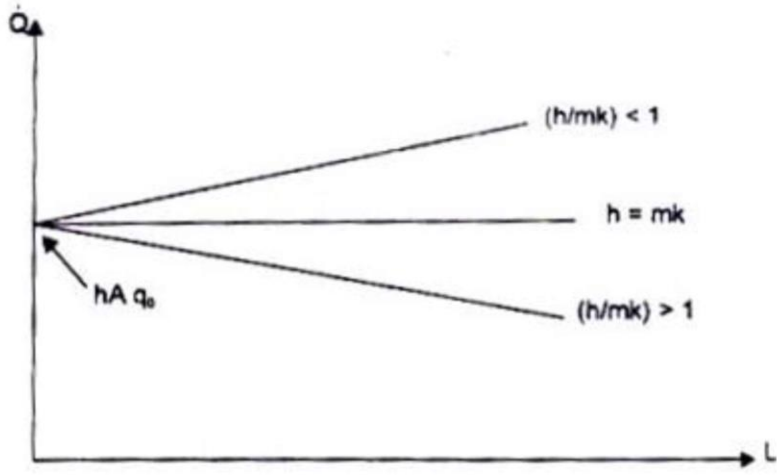
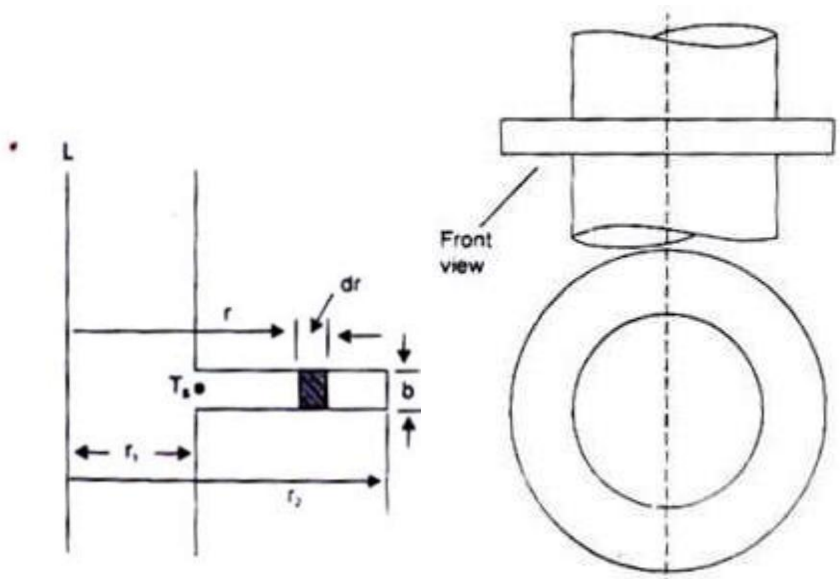


Fig 3.18

(For increasing the heat transfer rate by fins, we should have (i) higher value of thermal conductivity, (ii) a lower value of h, fins are therefore generally placed on the gas side, (iii) perimeter/cross-sectional area should be high and this requires thin fins.)

(i) the thickness b is much smaller than the radial length (r_2 - r_1) so that one-dimensional radial conduction of heat is valid;

(ii) steady state condition prevails.



Annular fin of uniform thickness   Top view of annular fin

Fig 3.19

We choose an annular element of radius r and radial thickness dr. The cross-sectional area for radial heat conduction at radius r is $2\pi rb$ and at radius r + dr is $2\pi(r + dr)b$. The surface area for convective heat transfer for the annulus is $2(2\pi r.dr)$. Thus, by making an energy balance,

$$-k2\pi rb\frac{dT}{dr} = -k2\pi(r+dr)b\left(\frac{dT}{dr}+\frac{d^2T}{dr^2}dr\right)+h\times 4\pi r.dr(T-T_\infty)$$

or, $d^2T / dr^2 + (1/r)dT/dr - 2h/kb(T - T_\infty) = 0$

Let, $\theta = (T - T_\infty)$ the above equation reduces to

$d^2\theta / dr^2 + (l/r)\, d\theta/dr - (2h\ kb)\,\theta = 0$

The equation is recognised as Bessel's equation of zero order and the solution is $\theta = C_1 I_0(nr) + C_2 K_0(nr)$, where $n=(2h/kb)^{1/2}$, I_0 is the modified Bessel function, 1st kind and K_0 is the modified Bessel function, 2nd kind, zero order, The constants C_1 and C_2 are evaluated by applying the two boundary conditions:

at $r = r_l$, $T = T_s$ and $\theta = T_s - T_\infty$

at $r = r_2$, $dT/dr = 0$ because $b << (r_2 - r_l)$

By applying the boundary conditions, the temperature distribution is given by

$$\frac{\theta}{\theta_0} = \frac{I_0(nr)K_1(nr_2) + K_0(nr)I_1(nr_2)}{I_0(nr_1)K_1(nr_2) + K_0(nr_1)I_1(nr_2)} \qquad (3.16)$$

$I_1(nr)$ and $K_1(nr)$ are Bessel functions or order one.

And the rate of heat transfer is given by:

$$Q = 2\pi knb\,\theta_0 r_1 \frac{K_1(nr_1)I_1(nr_2) - I_1(nr_1)K_1(nr_2)}{K_0(nr_1)I_1(nr_2) + I_0(nr_1)K_1(nr_2)} \qquad (3.17)$$

Table 2.1 gives selected values of the Modified Bessel Functions of the First and Second kinds, order Zero and One. (The details of solution can be obtained from: C.R. Wylie, Jr: Advanced Engineering Mathematics, McGraw-Hill Book Company, New York.)

The efficiency of circumferential fins is also obtained from curves for efficiencies

$(\text{along Y-axis}) \square \left(r_2 + \frac{b}{2} - r_1\right)^{\frac{3}{2}} \left(\frac{2h}{Kb}(r_2 - r_1)\right)^{\frac{1}{2}}$ for different values of $\left(r_2 + \frac{b}{2}\right)/r_1$.