UNIT-I

TESTING THE HYPOTHESIS

(χ^2) Chi-Square Test

1.5 χ^2test of Goodness of Fit

- χ^2test is used to test whether differences between observed and expected frequencies are significant.
- χ^2 is used to test the independence of attributes
- The test statistic $\chi^2 = \sum \left[\frac{(o-E)^2}{E}\right]$
- Where O – Observed Frequency
- E – Expected Frequency
- If the data is given in a series of "n" numbers then degrees of freedom

  $= n - 1.$

Note:

- If the case of Binomial Distribution the degrees of freedom $= n - 1$
- Poisson distribution the degrees of freedom $= n - 2$
- Normal distribution the degrees of freedom $= n - 3$

1.The following table gives the number of aircraft accident that occurred during the various days of the week. Test whether the accidents are uniformly distributed over the week.

Days	**: Mon**	**Tue**	**Wed**	**Thu**	**Fri**	**Sat**	**Total**
No. of accidents	**: 14**	**18**	**12**	**11**	**15**	**14**	**84**

Solution:

The expected number of accidents on any day $= \frac{84}{6} = 14$

Let H_0: The accidents occur uniformly over the week.

Observed Frequency	**Expected Frequency**	**(O – E)**	$\frac{(o-E)^2}{E}$
14	14	0	0
18	14	4	1.143

12	14	-2	0.286
11	14	-3	0.643
15	14	1	0.071
14	14	0	0

Now $\chi^2 = \sum\left[\frac{(o-E)^2}{E}\right] = 2.143$

Number of degrees of freedom $V = n - 1 = 7 - 1 = 6$

Critical value: The tabulated value of χ^2 at 5% for 6 d. f is 12.59

Conclusion:

Since $\chi^2 = 2.143 < 12.59$, then the null hypothesis H_0 is accepted.

i.e., we conclude that the accidents are uniformly distributed over the week

1. **4 coins were tossed 160 times and the following results were obtained.**

No. of heads	**:**	**0**	**1**	**2**	**3**	**4**
Frequency	**:**	**19**	**50**	**52**	**30**	**9**

Test the goodness of fit with the help of χ^2 on the assumption that the coins are unbiased

Solution:

Set the null hypothesis: H_0: The coins are unbiased.

The probability if getting the success of heads is $p = \frac{1}{2}$

- And $q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$
- When 4 coins are tossed, the probability of getting "r" heads is given by $P(X = r) = nC_r p^r q^{n-r}$, r = 0, 1, 2, . . .
- The expected frequency of getting 0, 1, 2, 3, 4 heads are given by
- $P(X = 0) = 160 \times 4C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{4-0} = 10$
- $P(X = 1) = 160 \times 4C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{4-1} = 40$
- $P(X = 2) = 160 \times 4C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{4-2} = 60$

- $P(X = 3) = 160 \times 4C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{4-3} = 40$
- $P(X = 4) = 160 \times 4C_0 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^{4-4} = 10$

Observed Frequency	**Expected Frequency**	**(O – E)**	$\frac{(o-E)^2}{E}$
19	10	-9	8.1
50	40	10	2.5
52	60	-8	1.067
30	40	-10	2.5
9	10	-1	0.1

Now $\chi^2 = \sum \left[\frac{(o-E)^2}{E}\right] = 14.267$

Number of degrees of freedom $V = n - 1 = 5 - 1 = 4$

Critical value: The tabulated value of χ^2 at 5% for 4 d. f is 9.488

Conclusion:

Since $\chi^2 = 14.267 > 9.488$, then the null hypothesis H_0 is rejected.

i.e., The coin are biased