ENGINEERING PHYSICS

UNIT II

WAVES AND FIBRE OPTICS

2.4.Damped Oscillations

2.4.1 Differential Equation And Its Solution

2.4.Damped Oscillations:

When a body is in vibration ,if the amplitude of vibration goes on decreasing and finally the oscillation die. This type of oscillation is said to be a damped oscillation. In this oscillation, the body vibrates with natural frequency.

Examples:

When a pendulum is displaced from its equilibrium position, it oscillates with decreasing amplitude and finally it come to rest.

2.4.1 Differential Equation And Its Solution

Let us consider a mass system . Let 'm' is the mass suspended over the spring. Due to the applied mass (load), the system exhibits two types of forces on it, namely Restoring force and Friction force.

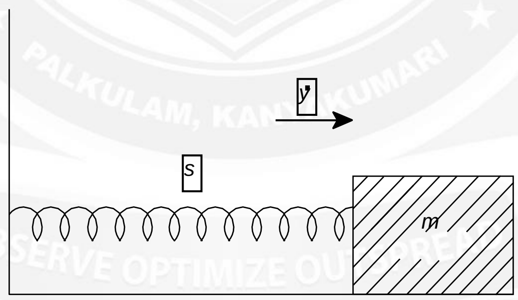

Fig 2.4.1 Damped Oscillations

(source: "The Physics of vibration and vibration" by H.J.Pain Page-38)

Restoring force:

A restoring force is the force which is opposite to the direction of displacement(y).

$$F_1 \, \alpha - y$$

$$F1 = - ky \text{----------------(i)}$$

Where, K is the force constant and y is the displacement. Here the negative sign indicates that the restoring force acts in the opposite direction to the displacement.

Friction force:

Friction force or damping force is due to presence of air resistance, which is opposite to the direction of velocity

$$F_2 = -r\,\frac{dy}{dx}\text{------------------}(2)$$

Total force

$$F = F_1+F_2\text{------------------}(3)$$

Sub (i) & (2) in (3)

$$F=- ky -r\,\frac{dy}{dt}\text{-----------------}(4)$$

But according to Newton's law

$$F = ma$$

$$\text{Here } F= m\,\frac{d^2y}{dt^2}\text{---------------}(5)$$

Where $\frac{d^2y}{dt^2}$ *is the acceleration*

From (4) & (5)

$$m\,\frac{d^2y}{dt^2} =- ky -r\,\frac{dy}{dt}$$

Divide by m

$$\frac{d^2y}{dt^2} = -\frac{k}{m}y - \frac{r}{m}\frac{dy}{dt}$$

$$\frac{d^2y}{dt^2} + \frac{r}{m}\frac{dy}{dt} + \frac{k}{m}y = 0$$

Put r/m = 2b & k/m = ω^2

$$\frac{d^2y}{dt^2} + 2b\,\frac{dy}{dt} + \omega^2 y = 0\text{----------------}\mathbf{(6)}$$

The solution for this equation is

$$y = Ae^{\alpha t}\text{------------------}\mathbf{(7)}$$

Where, A and $\boldsymbol{\alpha}$ are the arbitrary constants

On differentiating (7)

$$\frac{dy}{dt} = Ae^{\alpha t}\,\alpha\text{----------------}(8)$$

$$\frac{d^2y}{dt^2} = Ae^{\alpha t}\,\alpha^2\text{---------------}(9)$$

Sub equations 7,8 & 9 in 6 we get

$$A\alpha^2 e^{\alpha t} + 2b\, Ae^{\alpha t}\, \alpha + \omega^2 Ae^{\alpha t} = 0$$

$$Ae^{\alpha t}\, (\alpha^2 + 2b\, \alpha + \omega^2) = 0$$

$Ae^{\alpha t}$ is not equal to zeo

$$\alpha^2 + 2b\, \alpha + \omega^2 = 0$$

On solving the above equation we get

$$\alpha = -b \pm \sqrt{b^2 - \omega^2}$$

Then the general solution for the damped equation is

$$y = Ae^{(-b \pm \sqrt{b^2-\omega^2})\, t}$$

$$y = A_1 e^{(-b+\sqrt{b^2-\omega^2})\, t} + A_2 e^{(-b-\sqrt{b^2-\omega^2})\, t}$$

A_1 & A_2 are arbitrary constant.

Change of amplitude with respect to displacement is shown in figure 2.4.2

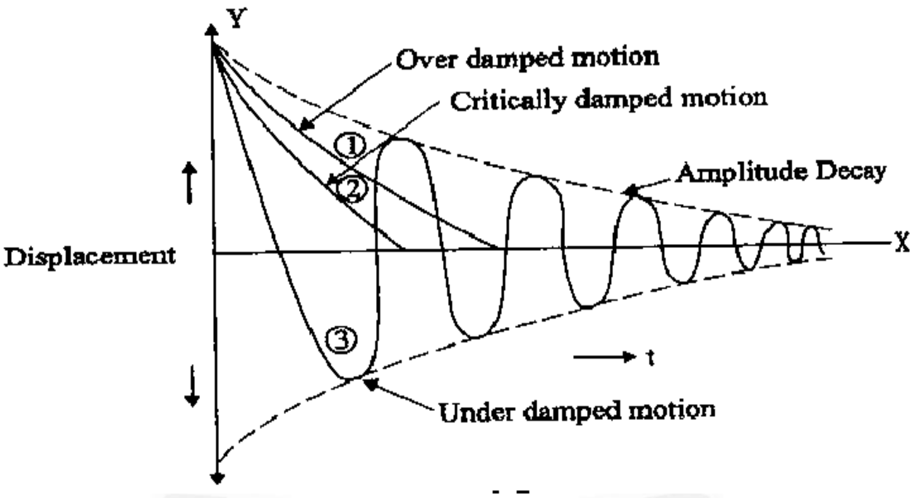

Fig 2.4.2.Damping