2.1 DYNAMIC EQUATIONS OF GRADUALLY VARIED FLOWS

VARIED FLOW

Flow properties, such as depth of flow area of cross section and velocity of flow vary with respect to distance is called Non-uniform flow.

It is, otherwise, called as varied flow. The varied flow is broadly classified into two types:

1) Rapidly varied flow (R.V.F)

2) Gradually varied flow (G.V.F)

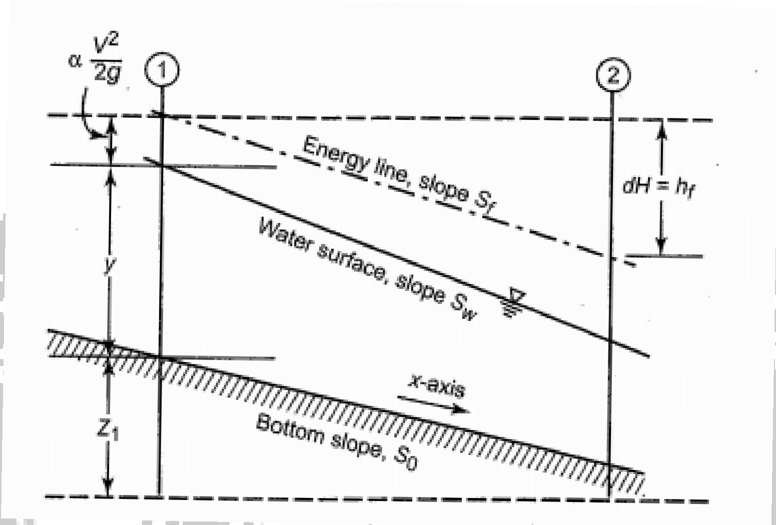
If the depth of flow changes quickly over a small length of the channel, the flow is said to be gradually varied flow (GVF). Example: Back water in a dam.

The following assumptions are made for analyzing the gradually varied flow:

1. The flow is steady
2. The pressure distribution over the channel section is hydrostatic, i.e., streamlines are practically straight and parallel.
3. The head loss is same as for uniform flow.
4. The channel slope is small, so that the depth measured vertically is the same as depth measured normal to the channel bottom.
5. A channel is prismatic.
6. Kinetic energy correction factor is very close to unity.
7. Roughness coefficient is constant along the channel length

8. The formulae, such as Chezy's formula, Manning's formula which are applicable, to the uniform flow are also applicable for the gradually varied flow for determining slope of energy line.

DYNAMIC EQUATION OF GVF



let

Z	=	height of bottom of channel above datum
h	=	depth of flow
V	=	Mean velocity of flow
i b	=	slope of the channel bed.
i e	=	slope of the energy line
b	=	width of channel
Q	=	discharge through the channel

The energy equation at any section is given by Bernoulli's equation.

$$E = Z + h + \frac{v^2}{2g}$$

Differentiating this equation with respect to x is where x is measured along the bottom of the channel in the direction of flow, we get.

Equation of Non-uniform flow (Slope of free water surface)

$$\frac{dE}{dx} = \frac{dZ}{dx} + \frac{dh}{dx} + \frac{d}{dx} \times \frac{v^2}{2g}$$

$$\frac{d}{dx} \times \frac{v^2}{2g} = \frac{d}{dx} \times \frac{Q^2}{2gA^2}$$

$$\frac{\mathrm{d}}{\mathrm{dx}} \times \frac{\mathrm{Q}^2}{2\mathrm{gh}^2\mathrm{b}^2} = \frac{\mathrm{Q}^2}{2\mathrm{gb}^2} \frac{\mathrm{d}}{\mathrm{dx}} \times \frac{1}{\mathrm{h}^2}$$

$$= \frac{\mathrm{Q}^2}{2\mathrm{gb}^2} \frac{\mathrm{d}}{\mathrm{dh}} \times \frac{1}{\mathrm{h}^2} \times \frac{\mathrm{dh}}{\mathrm{dx}}$$

$$= \frac{\mathrm{Q}^2}{2\mathrm{gb}^2} \times \frac{-2}{\mathrm{h}^3} \times \frac{\mathrm{dh}}{\mathrm{dx}}$$

$$= \frac{\mathrm{Q}^2}{2\mathrm{gb}^2} \times \frac{-2}{\mathrm{h}^3} \times \frac{\mathrm{dh}}{\mathrm{dx}}$$

$$\frac{d}{dx} \times \frac{v^2}{2g} = \frac{Q^2}{2gb^2} \times \frac{-2}{h^2} \times \frac{1}{h} \times \frac{dh}{dx} = -\frac{v^2}{gh} \frac{dh}{dx}$$

Substitute the value of $\frac{d}{dx} \times \frac{v^2}{2g}$

$$\frac{dE}{dx} = \frac{dZ}{dx} + \frac{dh}{dx} - \frac{v^2}{2gh} \frac{dh}{dx}$$

$$\frac{dE}{dx} = \frac{dZ}{dx} + \frac{dh}{dx}\left[1 - \frac{v^2}{2gh}\right]$$

$\frac{dE}{dx}$ =Slope of the energy line = -ie.

$\frac{dZ}{dx}$ =Slope of the bed of the channel = - ib.

-Ve sign with ie & ib us taken with the increase of x, the value of E and Z decreases.

Substituting the value of $\frac{dE}{dx}$ and $\frac{dZ}{dx}$

$$-i_b = -i_e + \frac{dh}{dx}\left[1 - \frac{v^2}{2gh}\right]$$

$$i_b - i_e = \frac{dh}{dx}\left[1 - \frac{v^2}{2gh}\right]$$

$$\frac{dh}{dx} = \frac{(i_b - i_e)}{\left[1 - \frac{v^2}{2gh}\right]}$$

$$\frac{dh}{dx} = \frac{(i_b - i_e)}{[1 - f^2]}$$

As h is the depth of flow and x is the distance measured along the bottom of the channel

$\frac{dx}{dh}$ represents the variation of the water depth along the bottom of the channel.

This is also called the slope of the free water surface.

1. If Sf = So, then dy/dx = 0, the surface profile of flow is parallel to the bottom of the channel.
2. If Sf < So, then dy/dx is positive which means the flow profile is rising gradually.
3. If Sf > So, then dy/dx is negative which means the flow profile is lowering gradually.

problem 1

In a rectangular channel 12 m wide, depth 3.6 m with a velocity of 12m/s. The bed slope of channel is 1 in 4000. If the flow of water through the channel is regulated in such away the energy line having a slope of 0.00004. Find the rate of change of depth of water in the channel.

Given

b = 12m,

h = 3.6m

V = 1.2 m/s,

i_b= 1/4000

i_e = 0.00004

To find

1. the rate of change of depth of water in the channel.

Solution

$$\frac{dh}{dx} = \frac{(i_b - i_e)}{\left[1 - \frac{v^2}{2gh}\right]}$$

$$\frac{dh}{dx} = \frac{(i_b - i_e)}{[1 - f^2]}$$

=2.189 ×10^{-4}