Transient Stability-Equal Area Criterion:-

The transient stability studies involve the determination of whether or not synchronism is maintained after the machine has been subjected to severe disturbance. This may be sudden application of load, loss of generation, loss of large load, or a fault on the system.

A method known as the equal area criterion can be used for a quick prediction of stability. This method is based on the graphical interpretation of the energy stored in the rotating mass as an aid to determine if the machine maintains its stability after a disturbance. This method is only applicable to a one-machine system connected to an infinite bus or a two-machine system. Because it provides physical insight to the dynamic behavior of the machine.

Now consider the swing equation (18),

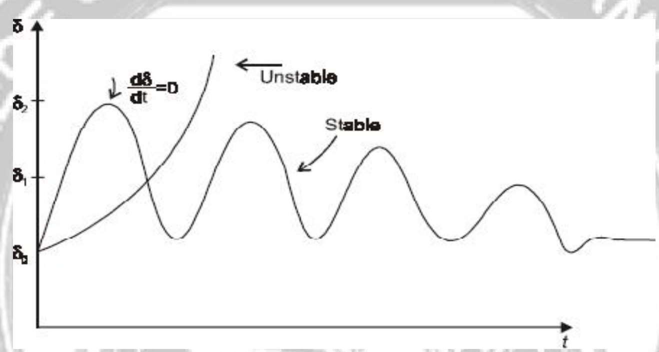
$$M_{(pu)} \cdot \frac{d^2\delta}{dt^2} = (P_i - P_e)$$

or

$$M_{(pu)} \cdot \frac{d^2\delta}{dt^2} = P_a$$

or $$\frac{d^2\delta}{dt^2} = \frac{P_a}{M} \qquad (52)$$

As shown in Fig.5, in an unstable system, δ increases indefinitely with time and machine looses synchronism. In a stable system, δ undergoes oscillations, which eventually die out due to damping. From Fig.4, it is clear that, for a system to be stable, it must be that $\frac{d\delta}{dt} = 0$ at some instant. This criterion ($\frac{d\delta}{dt} = 0$) can simply be obtained from equation (52).



(Fig. 5 A plot of δ (t))

Multiplying equation (52) by $\frac{2d\delta}{dt}$, we have

$$\frac{2d\delta}{dt}\frac{d^2\delta}{dt^2} = \frac{2P_a}{M}\cdot\frac{d\delta}{dt} \qquad (53)$$

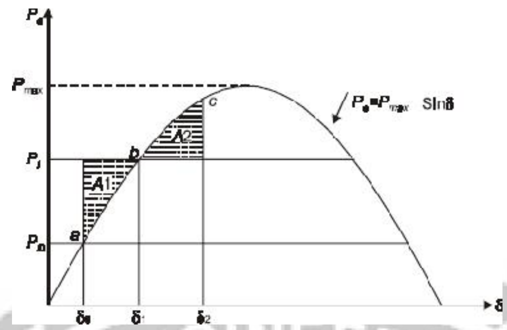
This upon integration with respect to time gives

$$\left(\frac{d\delta}{dt}\right)^2 = \frac{2}{M}\int_{\delta_0}^{\delta} P_a d\delta \qquad (54)$$

Where $P_a = P_i - P_e$ = accelerating power and δ_0 is the initial power angle before the rotor begins to swing because of a disturbance. The stability ($\frac{d\delta}{dt} = 0$) criterion implies that

$$\int_{\delta_0}^{\delta} P_a d\delta = 0 \qquad (55)$$

For stability, the area under the graph of accelerating power P_a versus δ must be zero for some value of δ; i.e., the positive (accelerating) area under the graph must be equal to the negative (decelerating) area. This criterion is therefore know as the equal area criterion for stability and is shown in Fig. 6.



(Fig.6 Power angle characteristic)

Application to sudden change in power input:-

In Fig. 6 point 'a' corresponding to the δ_0 is the initial steady-state operating point. At this point, the input power to the machine, $P_{i0} = P_{e0}$, where P_{e0} is the developed power. When a sudden increase in shaft input power occurs to P_i, the accelerating power P_a, becomes positive and the rotor moves toward point 'b'

We have assumed that the machine is connected to a large power system so that $|V_t|$ does not change and also x_d does not change and that a constant field current maintains $|E_g|$. Consequently, the rotor accelerates and power angle begins to increase. At point $P_i = P_e$ and $\delta = \delta_1$. But $\frac{d}{dt}$ is still positive and δ overshoots 'b', the final steady-state operating point. Now P_a is negative and δ ultimately reaches a maximum value δ_2 or point 'c' and swing back towards point 'b'. Therefore the rotor settles back to point 'b', which is ultimate steady-state operating point.

In accordance with equation (55) for stability, equal area criterion requires

$$\text{Area } A_1 = \text{Area } A_2$$

or $$\int_{\delta_0}^{\delta_1}(P_i - P_{max}\sin\delta)\,d\delta = \int_{\delta_1}^{\delta_2}(P_{max}\sin\delta - P_i)\,d\delta \text{ .... (56)}$$

or $$P_i(\delta_1 - \delta_0) + P_{max}(\cos\delta_1 - \cos\delta_0) =$$
$$P_i(\delta_1 - \delta_2) + P_{max}(\cos\delta_1 - \cos\delta_2) \text{ .... (57)}$$

But $$P_i = P_{max}\sin\delta$$

Which when substituted in equation (57), we get

$$P_{max}(\delta_1 - \delta_0)\sin\delta + P_{max}(\cos\delta_1 - \cos\delta_0) = P_{max}(\delta_1 - \delta_2)\sin\delta +$$
$$P_{max}(\cos\delta_1 - \cos\delta_2) \text{ .... (58)}$$

On simplification equation (58) becomes

$$(\delta_2 - \delta_0)\sin\delta_1 + \cos\delta_2 - \cos\delta_0 = 0 \quad \text{.................................(59)}$$

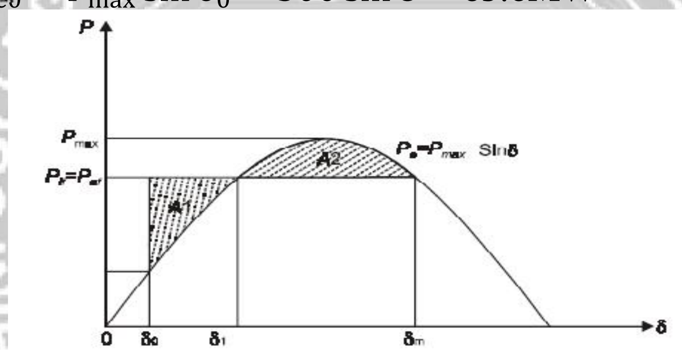
<u>Example 3</u>:-

A synchronous generator, capable of developing 500MW power per phase, operates at a power angle of 8^o. By how much can the input shaft power be increased suddenly without loss of stability? Assume that P_{max} will remain constant.

<u>Solution</u>:-

Initially, $\delta_0 = 8^o$

$$P_{e0} = P_{max}\sin\delta_0 = 500\sin 8^0 = 69.6\text{MW}$$



(Fig. 7 Power angle characteristics)

Let δ_m be the power angle to which the rotor can swing before losing synchronism. If this angle is exceeded, P_i will again become greater than P_e and the rotor will once again be accelerated and synchronism will be lost as shown in Fig. 7. Therefore, the equal area criterion requires that equation (57) be satisfied with δm replacing δ_2.

From Fig. 7 $\delta_m = \pi - \delta_1$. Therefore equation (59) becomes

$$(\pi - \delta_1 - \delta_0)\sin\delta_1 + \cos(\pi - \delta_1) - \cos\delta_0 = 0$$

$$(\pi - \delta_1 - \delta_0)\sin\delta_1 - \cos\delta_1 - \cos\delta_0 = 0 \quad \text{..............................(i)}$$

Substituting $\delta_0 = 8^0 = 0.139$radian in equation (i) gives

$$(3 - \delta_1)\sin\delta_1 - \cos\delta_1 - 0.99 = 0 \quad \text{...(ii)}$$

Solving equation (ii) we get, $\delta_1 = 50^o$

Now $P_{ef} = P_{max}\sin\delta_1 = 500\ \sin 50^0 = 383.02\ \text{MW}$

Initial power developed by machine was 69.6MW. Hence without loss of stability, the system can accommodate a sudden increase of

$P_{ef} - P_{e0} = 383.02 - 69.6 = 313.42$ MW per phase
= 3x313.42 = 940.3 MW (3-φ) of input shaft power.