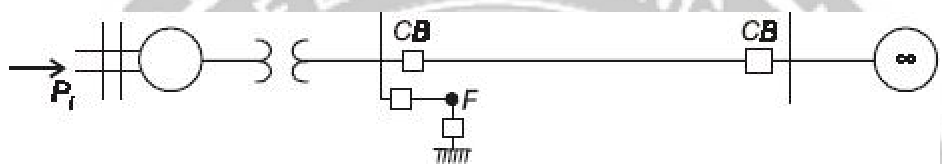
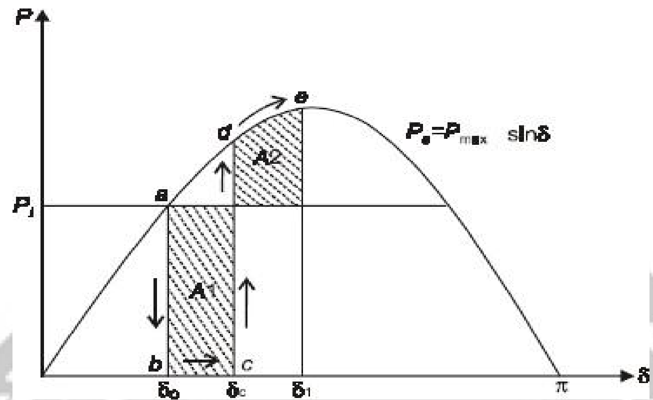
Critical Clearing Angle and Critical Clearing Time:-

If a fault occurs in a system, δ begins to increase under the influence of positive accelerating power, and the system will become unstable if δ becomes very large. There is a critical angle within which the fault must be cleared if the system is to remain stable and the equal area criterion is to be satisfied. This angle is known as the **critical clearing angle**.



(Fig. 8 Single machine infinite bus system)

Consider a system as shown in Fig. 8 operating with mechanical input P_i at steady angle δ_0 . $P_i = P_e$ as shown by point 'a' on the power angle diagram as shown in Fig. 9. Now if three phase short circuit occur at point F of the outgoing radial line , the terminal voltage goes to zero and hence electrical power output of the generator instantly reduces to zero i.e., $P_e = 0$ and the state point drops to 'b'. The acceleration area A_1 starts to increase while the state point moves along b-c. At time t_c corresponding clearing angle δ_C, the fault is cleared by the opening of the line circuit breaker. t_c is called clearing time and δ_C is called clearing angle. After the fault is cleared, the system again becomes healthy and transmits power $P_e = P_{max} \sin\delta$, i.e., the state point shifts to 'd' on the power angle curve. The rotor now decelerates and the decelerating area A_2 begins to increase while the state point moves along d-e. For stability, the clearing a ngle, δ_C, must be such that area A_1 = area A_2.



(Fig. 9 $P_e \sim \delta$ characteristics)

Expressing area A1 =Area A2 mathematically we have,

$$P_i(\delta_c - \delta_0) = \int_{\delta_c}^{\delta_1} (P_e - P_i) d\delta$$

$$\therefore \quad P_i(\delta_c - \delta_0) = \int_{\delta_c}^{\delta_1} P_{max} \sin\delta \, . \, d\delta - P_i(\delta_1 - \delta_c)$$

$$\therefore \quad P_i\delta_c - P_i\delta_0 = P_{max}(-\cos\delta_1 + \cos\delta_c) - P_i\delta_1 + P_i\delta_c$$

$$\therefore \quad P_{max}(\cos\delta_c - \cos\delta_1) = P_i(\delta_1 - \delta_0) \quad \ldots\ldots (60)$$

Also $$P_{max} = \sin\delta_0 \quad \ldots\ldots (61)$$

Using equation (60) and (61) we get,

$$P_{max}(\cos\delta_c - \cos\delta_1) = P_{max}(\delta_1 - \delta_0)\sin\delta_0$$

$$\therefore \quad \cos\delta_c = \cos\delta_1 + (\delta_1 - \delta_0)\sin\delta_0 \quad \ldots\ldots (62)$$

Where δ_c= clearing angle, δ_0 = initial power angle, and δ_1 = power angle to which the rotor advances (or overshoots) beyond δ_c.

For a three phase fault with P_e =0,

$$\frac{d^2\delta}{dt^2} = \frac{fP_i}{H} \quad \ldots\ldots (63)$$

Integrating equation (63) twice and utilizing the fact that $\frac{d\delta}{dt} = 0$ and t = 0 yields

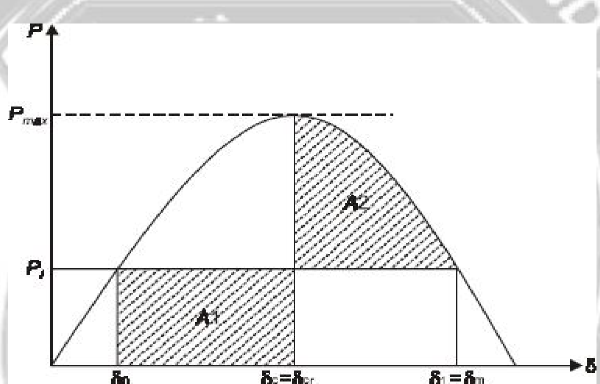
$$\delta = \frac{fP_i}{2H} t^2 + \delta_0 \quad \ldots\ldots (64)$$

If t_c is the clearing time corresponding to a clearing angle δ_c, then we obtain from equation (64),

$$\delta_c = \frac{\pi f P_i}{2H} t_c^2 + \delta_0$$

So
$$t_c = \sqrt{\frac{2H(\delta_c - \delta_0)}{fP_i}} \quad \text{.............................. (65)}$$

Note that δ_c can be obtained from equation (62). As the clearing of faulty line is delayed, A1 increases and so does δ_1 to find A2=A1 till $\delta_1 = \delta_m$ as shown in Fig. 10.



(Fig. 10 Critical clearing angle)

For a clearing angle (clearing time) larger than this value, the system would be unstable. The maximum allowable value of the clearing angle and clearing time for the system to remain stable are known as critical clearing angle and critical clearing time respectively.

From Fig. 10, $\delta_m = \pi - \delta_0$. Substituting this in equation (62) we have,

$$\cos \delta_{cr} = \cos \delta_m + (\delta_m - \delta_0) \sin \delta_0$$

$$\cos \delta_{cr} = \cos \delta_m + (\pi - \delta_0 - \delta_0) \sin \delta_0$$

$$\cos \delta_{cr} = \cos(\pi - \delta_0) + (\pi - 2\delta_0) \sin \delta_0$$

$$\cos \delta_{cr} = (\pi - 2\delta_0) \sin \delta_0 - \cos \delta_0$$

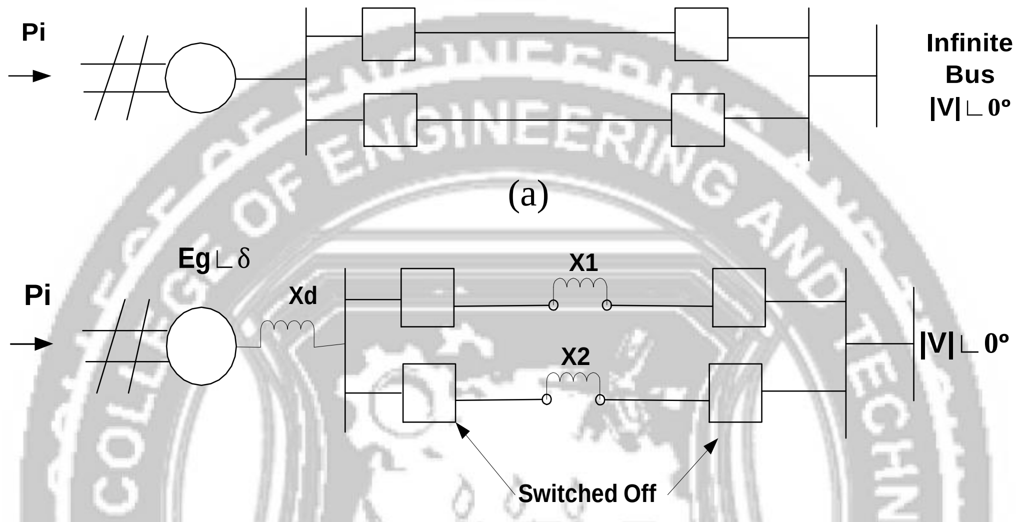
$$\delta_{cr} = \cos^{-1}(\pi - 2\delta_0) \sin \delta_0 - \cos \delta_0 \quad \text{.............................. (66)}$$

Using equation (65) critical clearing angle can be obtained as

$$t_{cr} = \sqrt{\frac{2H(\delta_{cr} - \delta_0)}{fP_i}} \quad \text{.............................. (67)}$$

Application of the Equal Area Criterion:-

(1) Sudden Loss of One of parallel Lines:-



(a)

(b)

(Fig. 11 Single machine tied to infinite bus through two parallel lines)

Consider a single machine tied to infinite bus through parallel lines as shown in Fig. 11(a). The circuit model of the system is given in Fig. 11(b).

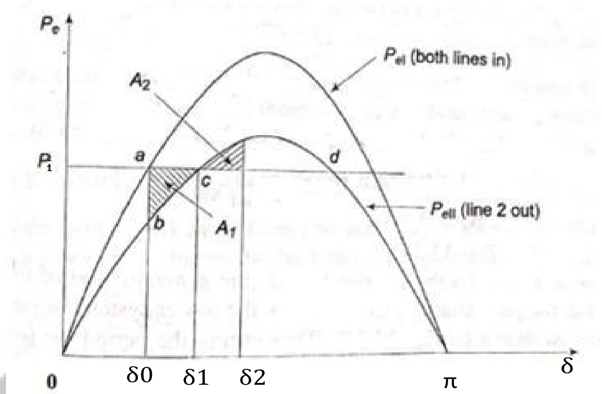
Let us study the transient stability of the system when one of the lines is suddenly switched off with the system operating at a steady load. Before switching off, power angle curve is given by

$$P_{eI} = \frac{|E_g|\ |V|}{X_d + X_1 \parallel X_2} \sin\delta = P_{maxI} \sin\delta$$

Immediately on switching of line 2, power angle curve is given by

$$P_{eII} = \frac{|E_g|\ |V|}{X_d + X_1} \sin\delta = P_{maxII} \sin\delta$$

In Fig. 12, wherein $P_{maxII} < P_{maxI}$ as $X_d + X_1 > X_d + X_1 \parallel X_2$. The system is operating initially with a steady state power transfer $P_e = P_i$ at a torque angle δ_0 on curve I.



(Fig. 12 Equal area criterion applied to the opening of one of the two lines in parallel)

On switching off line2, the electrical operating point shifts to curve II (point b). Accelerating energy corresponding to area $A_1$is put into rotor followed by decelerating energy for $\delta > \delta_1$. Assuming that an area A_2 corresponding to decelerating energy (energy out of rotor) can be found such that $A_1 = A_2$, the system will be stable and will finally operate at c corresponding to a new rotor angle is needed to transfer the same steady power.

If the steady load is increased (line P_i is shifted upwards) a limit is finally reached beyond which decelerating area equal to A_1 cannot be found and therefore, the system behaves as an unstable one. For the limiting case, δ_1 has a maximum value given by

$$\delta_1 = \delta_{max} = \pi - \delta_0$$