UNIT IV DESIGN OF COLUMNS

Types of columns –Axially Loaded columns – Design of short Rectangular Square and circular columns –Design of Slender columns- Design for Uniaxial and Biaxial bending using Column Curves

Design of Slender columns

Slender Columns

Columns having both *lex/D* and *ley/b* less than twelve are designated as short and otherwise, they are slender, where *lex* and *ley* are the effective lengths with respect to major and minor axes, respectively; and *D* and *b* are the depth and width of rectangular columns, respectively.

Design of Slender Columns

The design of slender compression members shall be based on the forces and the moments determined from an analysis of the structure, including the effect of deflections on moments and forces. When the effect of deflections are not taken into account in the analysis, additional moment given in cl no 39.7.1 of IS 456:2000 shall be taken into account in the appropriate direction.

The additional moments *M,* and *My,* shall be calculated by the following formulae:

$$M_{ax} = (P_u D/2000)\,(l_{ex}/D)^2$$

$$M_{ay} = (P_u b/2000)\,(l_{ey}/b)^2$$

Where *Pu* = axial load on the member,
lex = effective length in respect of the major axis,
ley = effective length in respect of the minor axis,
D = depth of the cross-section at right angles to the major axis, and
b = width of the member.

Problem:

Determine the reinforcement required for a braced column against sideway with the following data: size of the column = 350 x 450 mm concrete and steel grades = M 30 and Fe 415, respectively; effective lengths *lex* and *ley* = 7.0 and 6.0 m, respectively; unsupported length *l* = 8 m; factored load *Pu* = 1700 kN; factored moments in the direction of larger dimension = 70 kNm at top and 30 kNm at bottom; factored moments in the direction of shorter dimension = 60 kNm at top and 30 kNm at bottom. The column is bent in double curvature. Reinforcement will be distributed equally on four sides.

Solution 1:

Step 1: Checking of slenderness ratios

lex/D = 7000/450 = 15.56 > 12,

ley/b = 6000/350 = 17.14 > 12.

Hence, the column is slender with respect to both the axes.

Step 2: Minimum eccentricities and moments due to minimum eccentricities

ex min = *l*/500 + *D*/30 = 8000/500 + 450/30 = 31.0 > 20 mm
ey min = *l*/500 + *b*/30 = 8000/500 + 350/30 = 27.67 > 20 mm

Mox (Min. ecc.) = *Pu*(*ex min*) = (1700) (31) (10-3) = 52.7 kNm
Moy (Min. ecc.) = *Pu*(*ey min*) = (1700) (27.67) (10-3) = 47.04 kNm

Step 3: Additional eccentricities and additional moments
Table I of SP-16

For *lex/D* = 15.56, Table I of SP-16 gives:

eax/D = 0.1214, which gives *eax* = (0.1214) (450) = 54.63 mm

For *ley/D* = 17.14, Table I of SP-16 gives:

eay/b = 0.14738, which gives *eay* = (0.14738) (350) = 51.583 mm

Step 4: Primary moments and primary eccentricities

M_{ox} = 0.6M_2 – 0.4M_1 = 0.6(70) – 0.4(30) = 30 kNm, which should be ≥ 0.4 M_2 (= 28 kNm). Hence, o.k.

M_{oy} = 0.6M_2 – 0.4M_1 = 0.6(60) – 0.4(30) = 24 kNm, which should be ≥ 0.4 M_2 (= 24 kNm). Hence, o.k.

Primary eccentricities:

$e_x = M_{ox}/P_u$ = (30/1700) (10^3) = 17.65 mm

$e_y = M_{oy}/P_u$ = (24/1700) (10^3) = 14.12 mm

Since, both primary eccentricities are less than the respective minimum eccentricities (see Step 2), the primary moments are revised to those of Step 2. So, M_{ox} = 52.7 kNm and M_{oy} = 47.04 kNm.

Step 5: Modification factors

To determine the actual modification factors, the percentage of longitudinal reinforcement should be known. So, either the percentage of longitudinal reinforcement may be assumed or the modification factor may be assumed which should be verified subsequently. So, we assume the modification factors of 0.55 in both directions.

Step 6: Total factored moments

$Mux = Mox$ + (Modification factor) (Max) = 52.7 + (0.55) (92.548)

= 52.7 + 50.9 = 103.6 kNm

$Muy = Moy$ + (Modification factor) (May) = 47.04 + (0.55) (87.43)

= 47.04 + 48.09 = 95.13 kNm

Step 7: Trial section

The trial section is determined from the design of uniaxial bending with P_u = 1700 kN and

$M_u = 1.15(M^2_{ux} + M^2_{uy})$

So, we have = (1.15){(103.6)^2 + (95.13)^2}$^{1/2}$ = 161.75 kNm.

With these values of Pu (= 1700 kN) and Mu (= 161.75 kNm),

we use chart of SP-16 for the d' / D= 0.134. We assume the diameters of longitudinal bar as 25 mm, diameter of lateral tie = 8 mm and cover = 40 mm, to get = 40 + 8 + 12.5 = 60.5 mm.

Accordingly, d' / D = 60.5/450 = 0.134 and d' / b = 60.5/350 = 0.173.

$$P_u/f_{ck}bD = 1700(10^3)/(30)(350)(450) = 0.3598$$

$$M_u/f_{ck}bD^2 = 161.75(10^6)/(30)(350)(450)(450) = 0.076$$

We have to interpolate the values of p/f_{ck} for $d'/D = 0.134$ obtained from Charts 44 (for $d'/D = 0.1$) and 45 ($d'/D = 0.15$). The values of p/f_{ck} are 0.05 and 0.06 from Charts 44 and 45, respectively. The corresponding values of p are 1.5 and 1.8 per cent, respectively. The interpolated value of p for = 0.134 is 1.704 per cent, which gives A_{sc} = (1.704)(350)(450)/100 = 2683.8 mm^2. We use 4-25 + 4-20 (1963 + 1256 = 3219 mm^2), to have p provided = 2.044 per cent giving p/f_{ck} = 0.068.

Step 8: Calculation of balanced loads P_b

The values of P_{bx} and P_{by} are determined using Table 60 of SP-16. For this purpose, two parameters k_1 and k_2 are to be determined first from the table. We have
p/f_{ck}

= 0.068, d'/D= 0.134 and d'/b= 0.173. From Table 60,
k_1 = 0.19952 and k_2 = 0.243 (interpolated for d'/D = 0.134) for P_{bx}

So, we have: $P_{bx}/f_{ck}bD = k_1 + k_2(p/f_{ck})$ = 0.19952 + 0.243(0.068) = 0.216044,

which gives P_{bx}= 0.216044(30)(350)(450)(10^{-3}) = 1020.81 kN.

Similarly, for P_{by} : d'/D= 0.173, p/f_{ck}= 0.068. From Table 60 of SP-16, k_1= 0.19048 and k_2

= 0.1225 (interpolated for d'/b= 0.173). This gives $P_{by}/f_{ck}bD$ = 0.19048 + 0.1225(0.068) = 0.19881,

which gives P_{by}= (0.19881)(30)(350)(450)(10^{-3}) = 939.38 kN.

Since, the values of P_{bx} and P_{by} are less than P_u , the modification factors are to be used.

Step 9: Determination of *Puz*

$Puz = 0.45\, fck\, Ag + (0.75\, fy - 0.45\, fck)\, Asc$

= 0.45(30)(350)(450) + {0.75(415) – 0.45(30)}(3219) = 3084.71 kN

Step 10: Determination of modification factors

$kax = (Puz - Pu)/(Puz - Pubx)$

or
kax = (3084.71 – 1700)/(3084.71 – 1020.81) = 0.671 and

$kay = (Puz - Pu)/(Puz - Puby)$

or
kay = (3084.71 – 1700)/(3084.71 – 939.39) = 0.645

The values of the two modification factors are different from the assumed value of 0.55 in Step 5. However, the moments are changed and the section is checked for safety.

Step 11: Total moments incorporating modification factors

Mux = *Mox* (from Step 4) + (*kax*) *Max* (from Step 3)
= 52.7 + 0.671(92.548) = 114.8 kNm

Muy = *Moy* (from Step 4) + *kay* (*May*) (from Step 3)
= 47.04 + (0.645)(87.43) = 103.43 kNm.

Step 12: Uniaxial moment capacities

The two uniaxial moment capacities *M ux*1 and *Muy*1 are determined as stated: (i) For *M ux*1 , by interpolating the values obtained from Charts 44 and 45, knowing the values of *Pu/fckbD* =0.3598 (see Step 7),

p/fck= 0.068 (see Step 7), *d'*/ *D*= 0.134 (see Step 7), (ii) for M*uy*1, by interpolating the values obtained from Charts 45 and 46, knowing the same values of *Pu/fckbD* and *p/fck* as those of (i) and *d'*/ *D*= 0.173 (see Step 7). The results are given below:

(i) $Mux1/fckbD^2$ = 0.0882 (interpolated between 0.095 and 0.085)

(ii) $Muy1/fckbb^2$= 0.0827 (interpolated between 0.085 and 0.08)

So, we have, *Mux*1= 187.54 kNm and *Muy*1= 136.76 kNm.

Step 13: Value of α_n

We have P_u / P_{uz}

= 1700/3084.71 = 0.5511.

We have $_n\alpha$ = 0.67 + 1.67 (P_u/P_{uz}) = 1.59.

Step 14: Checking of column for safety

$$(M_{ux} / M_{ux1})^{\alpha^n} + (M_{uy} / M_{uy1})^{\alpha^n} \leq 1$$

Here, putting the values of M_{ux}, M_{ux1}, M_{uy}, M_{uy1} and $n\alpha$, we get: $(114.8/187.54)^{1.5452}$ + $(103.43/136.76)^{1.5852}$ = 0.4593 + 0.6422 = 1.1015. Hence, the section or the reinforcement has to be revised.