2.8 GUMMEL–POON MODEL

The Gummel–Poon model of the BJT considers more physics of the transistor than the Ebers–Moll model. This model can be used if, for example, there is a nonuniform doping concentration in the base. The electron current density in the base of an npn transistor can be written as

$$J_n = e\mu_n n(x)\mathrm{E} + eD_n \frac{dn(x)}{dx}$$

An electric field will occur in the base if nonuniform doping exists in the base.

$$\mathrm{E} = \frac{kT}{e} \cdot \frac{1}{p(x)} \cdot \frac{dp(x)}{dx}$$

where p(x) is the majority carrier hole concentration in the base. Under low injection, the hole concentration is just the acceptor impurity concentration. The electric field is negative (from the collector to the emitter). The direction of this electric field aids the flow of electrons across the base.

$$J_n = e\mu_n n(x) \cdot \frac{kT}{e} \cdot \frac{1}{p(x)} \cdot \frac{dp(x)}{dx} + eD_n \frac{dn(x)}{dx}$$

Using Einstein's relation,

$$J_n = \frac{eD_n}{p(x)}\left[n(x)\frac{dp(x)}{dx} + p(x)\frac{dn(x)}{dx}\right] = \frac{eD_n}{p(x)} \cdot \frac{d(pn)}{dx}$$

Above can be written in the form

$$\frac{J_n p(x)}{eD_n} = \frac{d(pn)}{dx}$$

Integrating above Equation through the base region while assuming that the electron current density is essentially a constant and the diffusion coeffi cient is a constant,

$$\frac{J_n}{eD_n}\int_0^{x_B} p(x)\,dx = \int_0^{x_B}\frac{dp(x)}{dx}\,dx = p(x_B)n(x_B) - p(0)n(0)$$

Assuming that the B–E junction is forward biased and the B–C junction is reverse biased, $n(0) = n_{B0} \exp(V_{BE}/V_t)$ and $n(x_B) = 0$. Note that $n_{B0}p = ni^2$ so that Equation written as

$$J_n = \frac{-eD_n n_i^2 \exp(V_{BE}/V_t)}{\int_0^{x_B} p(x)\,dx}$$

The integral in the denominator is the total majority carrier charge in the base and is known as the base Gummel number, defined as Q_B. The hole current density in the emitter of an npn transistor can be expressed as

$$J_p = \frac{-eD_p n_i^2 \exp(V_{BE}/V_t)}{\int_0^{x_E} n(x')\,dx'}$$

The integral in the denominator is the total majority carrier charge in the emitter and is known as the emitter Gummel number, defi ned as QE.

Since the currents in the Gummel–Poon model are functions of the total integrated charges in the base and emitter, these currents can easily be determined for nonuniformly doped transistors.

The Gummel–Poon model can also take into account nonideal effects, such as the Early effect and high-level injection. As the B–C voltage changes, the neutral base width changes so that the base Gummel number Q_B changes. The change in Q_B with B–C voltage then makes the electron current density given by a function of the B–C voltage.

If the B–E voltage becomes too large, low injection no longer applies, which leads to high-level injection. In this case, the total hole concentration in the base increases because of the increased excess hole concentration. This means that the base Gummel number will increase.

The Gummel–Poon model can then be used to describe the basic operation of the transistor as well as to describe nonideal effects.