## UNIT:5 MASS TRANSFER

## Definition:

Transfer of mass as a result of particle concentration difference in a mixture.
Air is a mixture of various gases. Whenever we have a multi component system with a concentration gradient, one constituent of the mixture gets transported from the region of higher concentration to the region of lower concentration till the concentration gradient reduces to zero. This phenomenon of the transport of mass as a result of concentration gradient is called 'Mass Transfer'.

## Difference of Heat transfer and Mass Transfer

| Heat Transfer |  | Mass transfer |  |
| :--- | :---: | :--- | :--- |
| Temperature Gradient | $\neq \mathrm{a}$ | Concentration Gradient |  |
| Occurs from higher temperature to <br> lower temperature | $\nLeftarrow$ | b | Occurs from higher Concentration to lower <br> concentration |

## Modes of Mass Transfer

There are basically three modes of mass transfer:
i. Diffusion mass Transfer

* occurs due to concentration difference
* Transport of matter in microscopic level
* Occurs between higher concentration and lower concentration

Eg. Osmosis, Reverse osmosis, Leakage of air from automobile and leakage of LPG from tanks
ii. Convective Mass Transfer

* occurs due to concentration difference and velocity
* Concentration of particles at its surface differs from its concentration in a gas moving over the surface

Eg. Drying of clothes, evaporization of water from swimming pool.
iii. Phase change Mass Transfer

* occurs due to simultaneous effect of convection and diffusion mass transfer

Eg. Burnt gases from chimney rise by convection and then mixes with air by diffusion

## Important Terms in concentration:

a. Mass concentration or mass density ( $\rho$ )

$$
=\frac{\text { Mass of a component }}{\text { Unit volume }}=\frac{\mathrm{m}_{\mathrm{A}}}{\mathrm{~V}}
$$

b. Molar Concentration or molar density ( $C_{A}$ )

$$
=\frac{\text { Mass concentration of a component }}{\text { Molecular weight of a component }}=\frac{\rho_{\mathrm{A}}}{\mathrm{M}_{\mathrm{A}}}
$$

c. Mass fraction $\left(\mathrm{x}_{\mathrm{A}}\right)$

$$
=\frac{\text { Mass concentration of a component }}{\text { Mass density of mixture }}=\frac{\rho_{\mathrm{A}}}{\rho}
$$

d. Mole fraction $\left(\mathrm{m}_{\mathrm{A}}\right)$

$$
=\frac{\text { Mole concentration of a component }}{\text { Mole concentration of mixture }}=\frac{\mathrm{C}_{\mathrm{A}}}{\mathrm{C}}
$$

e. Mass flow rate $\left(\dot{m}_{A}\right) \quad=N_{A}\left(M_{A}\right)$
$P_{A} \quad$ where, $N_{A}=$ Molar mass rate of flow in $\mathrm{kg}-\mathrm{mol} / \mathrm{sec}$ We also Know that, $\left(\rho_{A}\right)=\frac{P_{A}}{\text { RT }}$, where, $R=$ Characteristic Gas Constant

$$
\text { also, }\left(\mathrm{C}_{\mathrm{A}}\right)=\frac{\mathrm{P}_{\mathrm{A}}}{\overline{\mathrm{R}} \mathrm{~T}} \text {, where, } \bar{R}=\text { Universal gas constant }=8314.3 \mathrm{j} / \mathrm{kg}-\mathrm{mol} \mathrm{~K}
$$

## Problem 1:

The composition of dry atmospheric air on a molar basis is $78.1 \% \mathrm{~N}_{2}, 20.9 \% \mathrm{O}_{2}$, and $1 \% \mathrm{Ar}$.
Neglecting other constituents, Assuming atmospheric pressure 1bar and tempe rature $27^{\circ} \mathrm{C}$.
Find the mass fractions of the constituents of air.

## Solution:

Since, | Mass fraction $\left(X_{A}\right)=\frac{\text { Mass concentration of a component }}{\text { Mass density of mixture }}=\frac{\rho_{A}}{\rho}$
To find Molar concentration of $\mathrm{N}_{2}, \mathrm{O}_{2}, \mathrm{Ar}$,

$$
\begin{aligned}
& \mathrm{C}_{\mathrm{N}_{2}}=\frac{P_{N_{2}}}{\bar{R} T}=\frac{0.781 \times 1 \times 10^{5}}{8314 \times 300}=0.0313 \mathrm{~kg} \mathrm{~mole} / \mathrm{m}^{3} \\
& \mathrm{C}_{\mathrm{O}_{2}}=\frac{P_{O_{2}}}{\bar{R} T}=\frac{0.209 \times 1 \times 10^{5}}{8314 \times 300}=0.0084 \mathrm{~kg} \mathrm{~mole} / \mathrm{m}^{3} \\
& \mathrm{C}_{\mathrm{Ar}}=\frac{P_{A_{r}}}{\bar{R} T}=\frac{0.01 \times 1 \times 10^{5}}{8314 \times 300}=0.0004 \mathrm{~kg} \mathrm{~mole} / \mathrm{m}^{3}
\end{aligned}
$$

To find mass Densities of $\mathrm{N}_{2}, \mathrm{O}_{2}, \mathrm{Ar}$,

$$
\begin{aligned}
\rho_{\mathrm{N} 2} & =M_{\mathrm{N}_{2}} \times C_{\mathrm{N} 2}=28 \times 0.0313=0.8764 \mathrm{~kg} / \mathrm{m} 3 \\
\rho_{\mathrm{O} 2} & =M_{\mathrm{O} 2} \times C_{02}=32 \times 0.0084=0.2688 \mathrm{~kg} / \mathrm{m} 3 \\
\rho_{\mathrm{Ar}} & =M_{\mathrm{Ar}} \times C_{A_{\mathrm{r}}}=18 \times 0.0004=0.0072 \mathrm{~kg} / \mathrm{m}^{3}
\end{aligned}
$$

Over all mass density are,

$$
\rho=\rho_{\mathrm{N} 2}+\rho_{\mathrm{O} 2}+\rho_{\mathrm{Ar}}=0.8764+0.2688+0.0072=1.1524 \mathrm{~kg} / \mathrm{m}^{3}
$$

Mass Fractions of Constituents of air are,

$$
\begin{aligned}
& \mathrm{x}_{\mathrm{N} 2}=\frac{\rho_{N_{2}}}{\rho}=\frac{0.8764}{1.1524}=0.7605 \\
& \mathrm{x}_{\mathrm{O} 2}=\frac{\rho_{O_{2}}}{\rho}=\frac{0.2688}{1.1524}=0.2334 \\
& \mathrm{x}_{\mathrm{Ar}_{\mathrm{r}}}=\frac{\rho_{A_{r}}}{\rho}=\frac{0.0072}{1.1524}=0.00625
\end{aligned}
$$

## Fick's law of diffusion

The molar flux (Rate of Mass transfer) is directly proportional to concentration difference and inversely proportional to separation.

## Molar flux $\propto \frac{\text { Concentration Difference }}{\text { Separation }}$

$$
\begin{aligned}
& \frac{N_{a}}{A} \propto \frac{\mathrm{C}_{\mathrm{a}_{2}}-\mathrm{C}_{\mathrm{a}_{1}}}{\mathrm{dx}} \\
& \frac{N_{a}}{A}=-\mathrm{D}_{\mathrm{ab}} \frac{\left(\mathrm{C}_{\mathrm{a}_{2}}-\mathrm{C}_{\mathrm{a}_{1}}\right)}{\mathrm{dx}}
\end{aligned}
$$

where, $D_{a b}=$ Diffusion coefficient or Diffusivity ( $\mathrm{m}^{2} / \mathrm{sec}$ )
And $\mathrm{C}_{\mathrm{a}}=$ concentration or molecules per unit volume of the particles

$$
=\text { Solubility x Pressure }
$$

$\mathrm{A}=$ Area through which the mass is flowing in m
-ve sign indicates that the diffusion takes place in the direction opposite to that of increasing concentration

## Types of Diffusion Mass transfer

Type A: Steady State diffusion of a component "a" through the membrane


## Problem 2:

Hydrogen diffuses through a plastic membrane of 1 mm thick. The molar concentration of hydrogen on either side of the plastic membrane are $0.02 \mathrm{~kg}-\mathrm{mol} / \mathrm{m}^{3}, 0.005 \mathrm{~kg}-\mathrm{mol} / \mathrm{m}^{3}$. Diffusion coefficient of $\mathrm{H}_{2}$ through plastic $10^{-9} \mathrm{~m}^{2} / \mathrm{sec}$. determine molar flux and mass flux.

Solution:
Molar flux:
From HMT data book, pg. no. 175

$$
\begin{aligned}
& \frac{N_{a}}{A}=-\mathrm{D}_{\mathrm{ab}} \frac{\left(\mathrm{C}_{\mathrm{a}_{2}}-\mathrm{C}_{\mathrm{a}_{1}}\right)}{\mathrm{dx}} \text { or } \frac{N_{a}}{A}=\mathrm{D}_{\mathrm{ab}} \frac{\left(\mathrm{C}_{\mathrm{a}_{1}}-\mathrm{C}_{\mathrm{a}_{2}}\right)}{\mathrm{dx}} \\
& \frac{N_{a}}{A}=10^{-9} \frac{(0.02-0.005)}{1 \times 10^{-3}}=\frac{\mathrm{N}_{\mathrm{a}}}{\mathrm{~A}}=1.5 \times 10^{-8} \frac{\mathrm{~kg}}{\mathrm{~m}^{2} \mathrm{~s}}
\end{aligned}
$$

Mass flux:

$$
\frac{\dot{\mathrm{m}}_{\mathrm{a}}}{\mathrm{~A}}=\left[\frac{\mathrm{N}_{\mathrm{a}}}{\mathrm{~A}}\right] \mathrm{M}_{\mathrm{wt}}=\frac{\dot{\mathrm{m}}_{\mathrm{a}}}{\mathrm{~A}}=1.5 \times 10^{-8} \times 2.016
$$

$$
\frac{\dot{\mathrm{m}}_{\mathrm{a}}}{\mathrm{~A}}=3.024 \times 10^{-8} \frac{\mathrm{~kg}}{\mathrm{~m}^{2} \mathrm{~s}}
$$



## Problem 3:

Oxygen at $25^{\circ} \mathrm{C}$ and pressure of 2 bar flows through a rubber pipe of inside diameter 25 mm and wall thickness 2.5 mm . The diffusivity of oxygen through the rubber tube is $0.21 \times 10^{-9}$ $\mathrm{m}^{2} / \mathrm{sec}$ and the solubility of oxygen in rubber is $3.12 \times 10^{-3} \mathrm{~kg} . \mathrm{mole} / \mathrm{m}^{3}$ bar. Find the loss of oxygen by diffusion / m length of the pipe. Molar proportion of oxygen in air is $21 \%$.

Given: $\mathrm{D}_{\mathrm{ab}}=0.21 \times 10^{-9} \mathrm{~m}^{2} / \mathrm{sec} ; \mathrm{dx}=2.5 \mathrm{~mm}=2.5 \times 10^{-3} \mathrm{~m} ; \mathrm{D}=25 \mathrm{~mm}=25 \times 10^{-3} \mathrm{~m}$
Solution:

$$
\mathrm{C}_{\mathrm{a} 1}=\text { Solubility } \times \text { Pressure }=3.12 \times 10^{-3} \times 2=\mathrm{C}_{\mathrm{a}_{1}}=6.24 \times 10^{-3} \frac{\mathrm{~kg}-\mathrm{mol}}{\mathrm{~m}^{3}}
$$

Since, molar proportion of oxygen in air is given in percentage, and we know that the atmospheric pressure is 1 bar,

$$
\mathrm{P}_{\mathrm{O} 2}=0.21 \times 1=0.21 \mathrm{bar}
$$

Therefore,

$$
C_{a 2}=\text { Solubility } \times \text { Pressure }=3.12 \times 10^{-3} \times 0.21=C_{a_{2}}=6.552 \times 10^{-4} \frac{\mathrm{~kg}-\mathrm{mol}}{\mathrm{~m}^{3}}
$$

Since,
Molar Flux,

$$
\begin{gathered}
\frac{N_{a}}{A}=\mathrm{D}_{\mathrm{ab}} \frac{\left(\mathrm{C}_{\mathrm{a}_{1}}-\mathrm{C}_{\mathrm{a}_{2}}\right)}{\mathrm{dx}} \text { or } \frac{\mathrm{N}_{\mathrm{a}}}{\mathrm{~A}}=0.21 \times 10^{-9} \frac{\left(6.24 \times 10^{-3}-6.552 \times 10^{-4}\right)}{2.5 \times 10^{-3}} \\
\frac{\mathrm{~N}_{\mathrm{a}}}{\mathrm{~A}}=34.6914 \times 10^{-10} \frac{\mathrm{~kg}-\mathrm{mol}}{\mathrm{~m}^{2} \mathrm{~s}}
\end{gathered}
$$

Since the surface area of cylinder is,

$$
A=\pi D L=\pi \times 25 \times 10^{-3} \times 1 \text { or } A=0.07854 \mathrm{~m}^{2}
$$

Therefore,

$$
\begin{aligned}
& \mathrm{N}_{\mathrm{a}}=34.6914 \times 10^{-10} \times 0.07854 \\
& \mathrm{~N}_{\mathrm{a}}=3.6846 \times 10^{-10} \frac{\mathrm{~kg}-\mathrm{mol}}{\mathrm{~s}}
\end{aligned}
$$

But, mass flow rate,

$$
\dot{\mathrm{m}}_{\mathrm{a}}=\mathrm{M}_{\mathrm{wt}} \mathrm{~N}_{\mathrm{a}}=32 \times 3.66846 \times 10^{-11}=1.1788 \times 10^{-9} \mathrm{~kg} / \mathrm{s}
$$

$\therefore$ Loss of oxygen per meter length $=1.1788 \times 10^{-9} \mathrm{~kg} / \mathrm{s}$

Type B: Steady State equimloar counter diffusion of a component "a" and "b"


## Problem 4:

Ammonia and air experiences diffusion through 3 mm diameter, 20 mm long pipe. Total pressure is 1 atm and temperature $25^{\circ} \mathrm{C}$. Determine the diffusion rate of ammonia and air

Solutior
Place 1


## Given that,

$$
\mathrm{P}_{\mathrm{a} 1}=1 \mathrm{~atm}=1 \mathrm{bar}, \overline{\mathrm{R}}=8314.3 \mathrm{~J} / \mathrm{kg} \text { mole. } \mathrm{K} \text { (known), } \mathrm{T}=25^{\circ} \mathrm{C}
$$

Therefore,

$$
\mathrm{C}_{\mathrm{a}_{1}}=\frac{\mathrm{P}_{\mathrm{a}_{1}}}{\overline{\mathrm{R} T}}=\frac{1 \times 10^{5}}{8314.3 \times 298}=0.04036 \frac{\mathrm{~kg}-\text { mole }}{\mathrm{m}^{3}}, \mathrm{C}_{\mathrm{a}_{2}}=0
$$

From HMT Data book, pg No. 181

$$
\text { Diffusion Coefficient [for Ammonia and Air] } \mathrm{D}_{\mathrm{ab}}=21.60 \times 10^{-6} \mathrm{~m}^{2} / \mathrm{s}
$$

Since molar flux,

$$
\frac{\mathbf{N}_{\mathrm{a}}}{\mathbf{A}}=\mathbf{D}_{\mathrm{ab}}\left\{\frac{\mathrm{C}_{\mathrm{a}_{1}}-\mathbf{C}_{\mathrm{a}_{2}}}{\mathrm{dx}}\right\}
$$

$$
\begin{aligned}
& \frac{\mathbf{N}_{\mathrm{a}}}{\mathbf{A}}=2.161 \times 10^{-5} \frac{(0.04036-0)}{20} \\
&=4.3611 \times 10^{-8} \frac{\mathrm{~kg}-\text { mole }}{\mathrm{m}^{2} \mathrm{sec}}
\end{aligned}
$$

Since the cross-sectional area,

$$
\mathrm{A}=\frac{\pi \mathrm{d}^{2}}{4}=\frac{\pi\left(3 \times 10^{-3}\right)^{2}}{4}=7.0685 \times 10^{-6} \mathrm{~m}^{2}
$$

Therefore,

$$
\begin{aligned}
\mathrm{N}_{\mathrm{a}}=4.3611 \times 10^{-8} \times 7.0685 \times 10^{-6} \\
=3.0827 \times 10^{-13} \frac{\mathrm{~kg}-\mathrm{mol}}{\mathrm{~s}}=\mathrm{N}_{\mathrm{b}}
\end{aligned}
$$

Therefore, diffusion rate of Ammonia,

$$
\begin{aligned}
& \dot{\mathrm{m}}_{\mathrm{a}}=\left(\mathrm{M}_{\mathrm{wt}}\right)_{\mathrm{a}} \mathrm{~N}_{\mathrm{a}}=\left(\mathrm{M}_{\mathrm{wt}}\right)_{\mathrm{NH}_{3}} \mathrm{~N}_{\mathrm{NH}_{3}}=3.0827 \times 10^{-13} \times 17.03 \\
&= 5.248 \times 10^{-12} \frac{\mathrm{~kg}}{\mathrm{~s}} \\
& \quad \begin{array}{r}
\because \text { From HMT Data book Pg.No } \\
184
\end{array} \\
& \text { Molecular wt of } \mathrm{NH}_{3}=17.03
\end{aligned}
$$

$$
\begin{aligned}
& \dot{\mathbf{m}}_{\mathrm{b}}=\left(\mathbf{M}_{\mathrm{wt}}\right)_{\mathrm{b}} \mathbf{N}_{\mathrm{b}}=\left(\mathrm{M}_{\mathrm{wt}}\right)_{\mathrm{air}} \mathbf{N}_{\mathrm{air}}=3.0827 \times 10^{-13} \times 28.96 \\
&=8.92 \times 10^{-12} \frac{\mathrm{~kg}}{\mathrm{~s}}
\end{aligned}
$$

## Problem 5:

Two large tanks, maintained at the same temperature and pressure are connected by a circular 0.15 m diameter direct, which is 3 m in length. One tank contains a uniform mixture of 60 mole $\%$ ammonia and 40 mole $\%$ air and the other tank contains a uniform mixture of 20 mole $\%$ air and the other tank contains a uniform mixture of 20 mole $\%$ ammonia and 80 mole $\%$ air. The system is at 273 K and $1.013 \times 10^{5} \mathrm{~Pa}$. Determine the rate of ammonia transfer between the two tanks. Assuming a steady state mass transfer.


## Tank a,

$$
P_{a_{1}}=\frac{60}{100} \times 1.1013=0.6078 \text { bar } \quad P_{b_{1}}=\frac{40}{100} \times 1.1013=0.4052 \mathrm{bar}
$$

Tank b,

$$
\begin{gathered}
\mathrm{P}_{\mathrm{a}_{2}}=\frac{20}{100} \times 1.1013=0.2026 \mathrm{bar} \quad \mathrm{P}_{\mathrm{b}_{2}}=\frac{80}{100} \times 1.1013=0.8104 \mathrm{bar} \\
\mathrm{~A}=\frac{\pi \mathrm{d}^{2}}{4}=\frac{\pi(0.15)^{2}}{4}=0.017671 \mathrm{~m}^{2} \\
\mathrm{C}_{\mathrm{a}_{1}}=\frac{\mathrm{P}_{\mathrm{a}_{1}}}{\overline{\mathrm{R}} \mathrm{~T}}=\frac{0.6078 \times 10^{5}}{8314.3 \times 273}=0.02677 \frac{\mathrm{~kg}-\mathrm{mole}}{\mathrm{~m}^{3}} \\
\mathrm{C}_{\mathrm{a}_{2}}=\frac{\mathrm{P}_{\mathrm{a}_{2}}}{\overline{\mathrm{R}} \mathrm{~T}}=\frac{0.2026 \times 10^{5}}{8314.3 \times 273}=0.008925 \frac{\mathrm{~kg}-\mathrm{mole}}{\mathrm{~m}^{3}}
\end{gathered}
$$

## From HMT Data book, pg No. 181

Diffusion Coefficient [for Ammonia and Air] $D_{a b}=21.60 \times 10^{-6} \mathrm{~m}^{2} / \mathrm{s}$

Since molar flux,

$$
\begin{aligned}
\frac{\mathrm{N}_{\mathrm{a}}}{\mathrm{~A}}=\mathrm{D}_{\mathrm{ab}} & \left\{\frac{\mathrm{C}_{\mathrm{a}_{1}}-\mathrm{C}_{\mathrm{a}_{2}}}{\mathrm{dx}}\right\} \\
& =21.61 \times 10^{-6}\left\{\frac{0.02677-0.008925}{3}\right\} \\
& =1.28543 \times 10^{-7} \frac{\mathrm{~kg}-\mathrm{mol}}{\mathrm{~m}^{2} \mathrm{sec}}
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
\mathrm{Na}=1.28543 \times 10^{-7} \times \mathrm{A} & =1.28543 \times 10^{-7} \times \mathrm{A} 0.017671= \\
\mathrm{N}_{\mathrm{a}} & =2.27149 \times 10^{-9} \frac{\mathrm{~kg}-\mathrm{mole}}{\mathrm{~m}^{3}}=\mathrm{N}_{\mathrm{b}}
\end{aligned}
$$

Therefore the rate of ammonia transfer between two tanks,

$$
\begin{aligned}
\dot{\mathrm{m}}_{\mathrm{a}}= & \dot{m}_{N H_{3}}=\mathrm{M}_{\mathrm{wt}} \mathrm{~N}_{\mathrm{a}}=2.27149 \times 10^{-9} \times 17.03 \\
& =3.8683 \times 10^{-8} \frac{\mathrm{~kg}}{\mathrm{~s}} \quad \begin{array}{r}
\text { From HMT Data book Pg. No } 184 \\
\quad \begin{array}{l}
\text { Molecular wt of } \mathrm{NH}_{3}=17.03
\end{array}
\end{array}
\end{aligned}
$$

Therefore the rate of air transfer between two tanks,

$$
\begin{aligned}
\dot{\mathrm{m}}_{\mathrm{b}}=\dot{\mathrm{m}}_{\mathrm{air}} & =\left(\mathrm{M}_{\mathrm{wt}}\right)_{\mathrm{b}} \mathrm{~N}_{\mathrm{b}}=2.27149 \times 10^{-9} \times 28.96 \\
& =65.782 \times 10^{-9} \frac{\mathrm{~kg}}{\mathrm{~s}} \quad \begin{array}{r}
\because \text { From HMT Data book Pg.No } 184 \\
\text { Molecular wt of air }=28.96
\end{array}
\end{aligned}
$$

Type C: Steady State evaporation of a component "a" into a stagnant air

## Assumptions:

* Water vapor and air behaves as ideal gases
* System is held at isothermal conditions
* Evaporation Process is steady


## Problem 6:

Determine the rate of water from bottom of a test tube of 10 mm diameter, 150 mm long, into a dry stagnant air at $25^{\circ} \mathrm{C}$

## Points to be remembered:

From Dalton's law,

Position '2' $\quad$ Dry stagnated air 'b'


Since the Temperature of air is given as $25^{\circ} \mathrm{C}$,
The partial pressure of water vapor $\left(\mathrm{p}_{\mathrm{a} 1}\right)$ corresponds to the saturation pressure at $25^{\circ} \mathrm{C}$ in steam tables.

Therefore, $\mathrm{P}_{\mathrm{a} 1}=0.03166 \mathrm{bar}$
Since,
Dry air or $\phi=0$ or no humidity

$$
P_{\mathrm{a} 2}=0
$$

And since,

$$
P_{a 1}+P_{b 1}=P_{\mathrm{atm}}
$$

Therefore,

$$
P_{b 1}=0.96834 \text { bar }
$$

From HMT Data book Pg.No. 181, Diffusion Coefficient
[for Water and Air at $25^{\circ} \mathrm{C}$ ] $\mathrm{D}_{\mathrm{ab}}=\mathbf{2 5 . 8 3 \times 1 0 ^ { - 6 }} \mathrm{m}^{2} / \mathrm{s}$
From HMT Data book, pg No. 175,

$$
\frac{N_{a}}{\mathrm{~A}}=\frac{\mathrm{D}_{\mathrm{ab}}}{\mathrm{dy}} \frac{\mathrm{P}_{\mathrm{atm}}}{\overline{\mathrm{R}} \mathrm{~T}} \ln \left[\frac{\mathrm{P}_{\mathrm{b}_{2}}}{\mathrm{P}_{\mathrm{b}_{1}}}\right]
$$

Saturated Water and Steam

| $\frac{T}{\left[{ }^{\circ} \mathrm{C}\right]}$ | $\frac{p_{5}}{[\text { bar }]}$ | $\frac{v_{3}}{\left[\mathrm{~m}^{3} / \mathrm{kg}\right]}$ |
| :---: | :---: | :---: |
| 0.01 | 0.006112 | 206.1 |
| $\begin{aligned} & 1 \\ & 2 \\ & 3 \\ & 4 \end{aligned}$ | $\begin{aligned} & 0.006566 \\ & 0.007054 \\ & 0.007575 \\ & 0.008129 \end{aligned}$ | $\begin{aligned} & 192.6 \\ & 179.9 \\ & 168.2 \\ & 157.3 \end{aligned}$ |
| $\begin{aligned} & 5 \\ & 6 \\ & 7 \\ & 8 \\ & 9 \end{aligned}$ | $\begin{aligned} & 0.008719 \\ & 0.009346 \\ & 0.01001 \\ & 0.01072 \\ & 0.01147 \end{aligned}$ | $\begin{aligned} & 147.1 \\ & 137.8 \\ & 129.1 \\ & 121.0 \\ & 113.4 \end{aligned}$ |
| $\begin{aligned} & 10 \\ & 11 \\ & 12 \\ & 13 \\ & 14 \end{aligned}$ | $\begin{aligned} & 0.01227 \\ & 0.01312 \\ & 0.01401 \\ & 0.01497 \\ & 0.01597 \end{aligned}$ | $\begin{gathered} 106.4 \\ 99.90 \\ 93.83 \\ 88.17 \\ 82.89 \end{gathered}$ |
| $\begin{aligned} & 15 \\ & 16 \\ & 17 \\ & 18 \\ & 19 \end{aligned}$ | $\begin{aligned} & 0.01704 \\ & 0.01817 \\ & 0.01936 \\ & 0.02063 \\ & 0.02196 \end{aligned}$ | $\begin{aligned} & 77.97 \\ & 73.38 \\ & 69.09 \\ & 65.08 \\ & 61.34 \end{aligned}$ |
| $\begin{aligned} & 20 \\ & 21 \\ & 22 \\ & 23 \\ & 24 \\ & \hline \end{aligned}$ | $\begin{aligned} & 0.02337 \\ & 0.02486 \\ & 0.02642 \\ & 0.02808 \\ & 0.02982 \end{aligned}$ | $\begin{aligned} & 57.84 \\ & 54.56 \\ & 51.49 \\ & 48.62 \\ & 45.92 \end{aligned}$ |
| 25 | 0.03166 | 43.40 |
| $\xrightarrow{26}$ | 0.03360 กn2<64 | 41.05 |

$$
\begin{aligned}
& \therefore \frac{\mathrm{N}_{\mathrm{a}}}{\mathrm{~A}}=\frac{2.583 \times 10^{-5}}{0.15} \times \frac{1 \times 10^{5}}{8314.4 \times 298} \ln \left[\frac{1}{0.96834}\right] \\
& =2.23598 \times 10^{-7} \frac{\mathrm{~kg}-\mathrm{mol}}{\mathrm{~m}^{2} \mathrm{~s}} \\
& A=\frac{\pi d^{2}}{4}=\frac{\pi(0.01)^{2}}{4}=78.539 \times 10^{-6} \mathrm{~m}^{2}
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
& \mathrm{N}_{\mathrm{a}}=2.2359 \times 10^{-7} \times 78.539 \times 10^{-6} \\
&= 1.75607 \times 10^{-11} \frac{\mathrm{~kg}-\mathrm{mol}}{\mathrm{~s}}
\end{aligned}
$$

From HMT data Book, Pg.No. 184, Molecular Weight of steam $=18.016$
Therefore, Rate of Evaporation is,

$$
\begin{gathered}
\dot{\mathrm{m}}_{\mathrm{a}}=\mathrm{N}_{\mathrm{a}}\left(\mathrm{M}_{\mathrm{wt}}\right)_{\mathrm{a}}=1.75607 \times 10^{-11} \times 18.016 \\
=3.106929 \times 10^{-10} \frac{\mathrm{~kg}}{\mathrm{~s}}
\end{gathered}
$$

## Problem 7:

A well of 40 m deep 9 m diameter is exposed to atmospheric air at $25 \mathrm{C}, 50 \%$ R.H. determine the rate of atmospheric evaporation of water from well.

## Solution:

Since the Temperature of air is given as $25^{\circ} \mathrm{C}$,
The partial pressure of water vapor $\left(p_{a 1}\right)$ corresponds to the saturation pressure at $25^{\circ} \mathrm{C}$ in steam tables.

Therefore, $\mathrm{P}_{\mathrm{a} 1}=0.03166$ bar and

$$
\mathrm{P}_{\mathrm{a} 2}=\phi \mathrm{P}_{\mathrm{a} 1}
$$

Since,
R.H or $\phi=50 \%$

$$
P_{\mathrm{a} 2}=0.5 \times 0.03166=0.01583 \mathrm{bar}
$$

And since,

$$
P_{a 11}+P_{b 1}=P_{a t m}=1
$$

Therefore,

$$
P_{b 1}=0.96834 \text { bar }
$$

Dry stagnated air 'b'


Since,

$$
P_{a 2}+P_{b 2}=P_{\mathrm{atm}}
$$

Therefore,

$$
P_{b 2}=0.98417 \text { bar }
$$

From HMT Data book Pg.No. 181, Diffusion Coefficient [for Water and Air at $25^{\circ} \mathrm{C}$ ]

$$
D_{a b}=25.83 \times 10^{-6} \mathrm{~m}^{2} / \mathrm{s}
$$

From HMT Data book, pg No. 175,

$$
\begin{gathered}
\frac{\mathrm{N}_{\mathrm{a}}}{\mathrm{~A}}=\frac{\mathrm{D}_{\mathrm{ab}}}{\mathrm{dy}} \frac{\mathrm{P}_{\mathrm{atm}}}{\overline{\mathrm{R}} \mathrm{~T}} \ln \left[\frac{\mathrm{P}_{\mathrm{b}_{2}}}{\mathrm{P}_{\mathrm{b}_{1}}}\right] \\
\therefore \frac{\mathrm{N}_{\mathrm{a}}}{\mathrm{~A}}=\frac{2.583 \times 10^{-5}}{40} \times \frac{1 \times 10^{5}}{8314.4 \times 298} \ln \left[\frac{0.9847}{0.96834}\right] \\
=4.226 \times 10^{-10} \frac{\mathrm{~kg}-\mathrm{mol}}{\mathrm{~m}^{2} \mathrm{~s}}
\end{gathered}
$$

and the area of the well,

$$
A=\frac{\pi d^{2}}{4}=\frac{\pi(9)^{2}}{4}=63.6175 \mathrm{~m}^{2}
$$

Therefore,

$$
\mathrm{N}_{\mathrm{a}}=2.6880 \times 10^{-8} \frac{\mathrm{~kg}-\mathrm{mole}}{\mathrm{~m}^{3}}
$$

From HMT data Book, Pg.No. 184, Molecular Weight of steam = 18.016

Therefore, Rate of Evaporation is,

$$
\begin{gathered}
\dot{\mathrm{m}}_{\mathrm{a}}=\mathrm{N}_{\mathrm{a}}\left(\mathrm{M}_{\mathrm{wt}}\right)_{\mathrm{a}}=2.688 \times 10^{-8} \times 18.016 \\
=4.8441 \times 10^{-7} \frac{\mathrm{~kg}}{\mathrm{~s}}
\end{gathered}
$$

## Covective Mass Transfer

## Definition:

Mass transfer between surface and liquid / gas due to concentration difference.


## Terms used in Connective mass Transfer:

Sherwood number: [ HMT Data Book Pg.No. 112]
The ratio of concentration gradient at the boundary by diffusion to concentration gradient at the boundary by convection

$$
\mathrm{S}_{\mathrm{h}}=\frac{\mathrm{h}_{\mathrm{m}} \mathrm{~L}}{\mathrm{D}_{\mathrm{ab}}} \text { (for plates) and } \mathrm{S}_{\mathrm{h}}=\frac{\mathrm{h}_{\mathrm{m}} \mathrm{~d}}{\mathrm{D}_{\mathrm{ab}}} \quad \text { (for Tubes) }
$$

Where,

$$
\begin{aligned}
& \mathbf{h}_{m}=\text { Mass transfer coefficient }(\mathrm{m} / \mathrm{sec}) ; \mathrm{L}=\text { Length }(\mathrm{m}) ; \mathrm{d}=\text { Diameter }(\mathrm{m}) ; \\
& \mathrm{D}_{\mathrm{ab}}=\text { Diffusion Coefficient }\left(\mathrm{m}^{2} / \mathrm{sec}\right)
\end{aligned}
$$

Schmidt number: [HMT Data Book Pg.No. 112]
The ratio of Molecular diffusivity of momentum to the molecular diffusivity of mass.

$$
\mathrm{S}_{\mathrm{c}}=\frac{v}{\mathrm{D}_{\mathrm{ab}}}=\frac{\mu}{\rho \mathrm{D}_{\mathrm{ab}}}
$$

Where,

$$
v=\text { Kinematic viscosity }\left(\mathrm{m}^{2} / \mathrm{sec}\right), \mathrm{D}_{\mathrm{ab}}=\text { Diffusion Coefficient }\left(\mathrm{m}^{2} / \mathrm{sec}\right)
$$

## Reynolds number: [ HMT Data Book Pg.No. 112]

The ratio of Inertia force to viscous force

Where,

$$
\mathrm{R}_{\mathrm{e}}=\frac{\mathrm{uL}}{v} \quad \text { (for plates) and } \quad \mathrm{R}_{\mathrm{e}}=\frac{\mathrm{ud}}{v} \quad \text { (for Tubes) }
$$

$u=$ Velocity ( $\mathrm{m} / \mathrm{sec}$ ) $; \mathbf{v}=$ Kinematic viscosity ( $\mathrm{m}^{2} / \mathrm{sec}$ ) ; L = Length (m); d = Diameter (m)

It used to classify the type of flow

| Flat Plate | Tubes |
| :--- | :--- |
| if $R_{e}<5 \times 10^{5}$ flow is laminar | if $R_{e}<2000$ flow is laminar |
| if $R_{e}>5 \times 10^{5}$ flow is turbulent | if $R_{e}>2000$ flow is turbulent |

Lewis number: [ HMT Data Book Pg.No. 112]
The ratio heat diffusivity to mass diffusivity

Where,

$$
\mathbf{L}_{\mathrm{e}}=\frac{\mathbf{S}_{\mathrm{c}}}{\mathbf{P}_{\mathrm{r}}}
$$

Pr = Prandtl Number

## Problem 1:

Air at $25^{\circ} \mathrm{C}, 50 \%$ R.H, flows over a swimming pool at a surface temperature of $25^{\circ} \mathrm{C}$ of $12 \mathrm{~m} \times 6 \mathrm{~m}$. The velocity of air in the length direction is $2 \mathrm{~m} / \mathrm{sec}$. Determine the (a) mass transfer coefficient (b) mass rate of water evaporation

Given:
$T \propto=25^{\circ} \mathrm{C}, \phi=50 \%, \mathrm{~T}_{\mathrm{w}}=25^{\circ} \mathrm{C}$ $\mathrm{L}=12 \mathrm{~m}, \mathrm{w}=6 \mathrm{~m}, \mathrm{u}=2 \mathrm{~m} / \mathrm{s}$

## Solution:



Since velocity is given in the problem, it is a convection mass transfer.

Step 1: Determination of film temperature $\left(\mathrm{T}_{\mathrm{f}}\right)$

$$
\mathrm{T}_{\mathrm{f}}=\frac{\mathrm{T}_{\mathrm{w}}+\mathrm{T}_{\alpha}}{2}=\frac{25+25}{2}=25^{\circ} \mathrm{C}
$$

Step 2: Taking properties of air, [ from HMT Data book, Pg.No. 34]

Corresponding to $\mathrm{T}_{\mathrm{f}}=25^{\circ} \mathrm{C}$,

$$
\begin{aligned}
& v=15.53 \times 10^{-6} \frac{\mathrm{~m}^{2}}{\mathrm{~s}} \\
& \mathrm{P}_{\mathrm{r}}=0.702
\end{aligned}
$$

Step 3: Determination of type of flow:

$$
\mathrm{R}_{\mathrm{e}}=\frac{\mathrm{uL}}{v}=\frac{2 \times 12}{15.53 \times 10^{-6}}=1545396>5 \times 10^{5}
$$

Since greater than $5 \times 10^{5}$, the flow can be assumed as turbulent or Laminar - turbulent

Here we assume the flow is Laminar - turbulent.
Step 4: Determination of Diffusion coefficient, [ from HMT Data book, Pg.No. 181]

Corresponding to the medium, (water-air) at $\mathrm{T}_{\mathrm{f}}=25^{\circ} \mathrm{C}$

$$
\mathrm{D}_{\mathrm{ab}}=25.83 \times 10^{-6} \mathrm{~m}^{2} / \mathrm{s}
$$

Step 5: Determination of Schmidt Number ( $\mathbf{S}_{\mathrm{c}}$ ),

$$
\mathrm{S}_{\mathrm{c}}=\frac{v}{\mathrm{D}_{\mathrm{ab}}}=\frac{15.53 \times 10^{-6}}{25.83 \times 10^{-5}}=0.60123
$$

Step 6: Determination of Sherwood Number ( $\mathrm{S}_{\mathrm{h}}$ ), From HMT data book, Pg.No. 177,

$$
\begin{aligned}
& S_{\mathrm{h}}=\left[0.037 \mathrm{R}_{\mathrm{e}}^{0.8}-871\right] \mathrm{S}_{\mathrm{c}}^{0.33} \\
& \mathrm{~S}_{\mathrm{h}}=[0.037 \times 1545396-871] 0.60123^{0.33} \\
& \quad=2059.4906
\end{aligned}
$$

But we know that, [ HMT Data Book Pg.No. 112]

$$
\mathrm{S}_{\mathrm{h}}=\frac{\mathrm{h}_{\mathrm{m}} \mathrm{~L}}{\mathrm{D}_{\mathrm{ab}}}=2059.4906
$$

Step 7: Determination of mass transfer coefficient $\left(h_{m}\right)$,

$$
\begin{gathered}
\mathrm{h}_{\mathrm{m}}=\frac{\mathrm{S}_{\mathrm{h}} \mathrm{D}_{\mathrm{ab}}}{\mathrm{~L}}=\frac{2059.49 \times 2.583 \times 10^{-5}}{12} \\
=4.43305 \times 10^{-3} \frac{\mathrm{~m}}{\mathrm{~s}}
\end{gathered}
$$

Step 8: Mass of flow rate evaporated ( $\dot{m}_{w}$ ),

$$
\dot{\mathbf{m}}_{\mathrm{w}}=\mathbf{h}_{\mathrm{m}} \mathbf{A}\left(\rho_{\mathrm{aw}}-\varphi \rho_{\mathrm{a} \alpha}\right)
$$

Where,

$$
\begin{aligned}
& \rho_{\mathrm{aw}}=\text { Density of water vapor at } \mathrm{T}_{\mathrm{w}}=\frac{1}{v_{\mathrm{g}}} \\
& \rho_{\mathrm{a} \alpha}=\text { Density of water vapor at } \mathrm{T}_{\alpha}=\frac{1}{v_{\mathrm{g}}}
\end{aligned}
$$

From steam tables, corresponding to $\mathrm{T}_{\mathrm{w}}=\mathrm{T}_{\boldsymbol{\alpha}}=25^{\circ} \mathrm{C}$,

$$
\begin{gathered}
v_{\mathrm{g}}=43.402 \frac{\mathrm{~m}^{3}}{\mathrm{~kg}} \\
\rho_{\mathrm{aw}}=\frac{1}{v_{\mathrm{g}}}=0.02304 \frac{\mathrm{~m}^{3}}{\mathrm{~kg}}
\end{gathered}
$$

$\therefore$ Mass of flow rate evaporated

$$
\begin{aligned}
\dot{\mathrm{m}}_{\mathrm{w}}=4 . & 43305 \times 10^{-3} \times(12 \times 6)(0.02304 \\
& -0.5 \times 0.02304)=3.6769 \times 10^{-3} \frac{\mathrm{~kg}}{\mathrm{sec}}
\end{aligned}
$$

