Hasse Diagram:

Pictorial representation of a Poset is called Hasse Diagram.

Example:

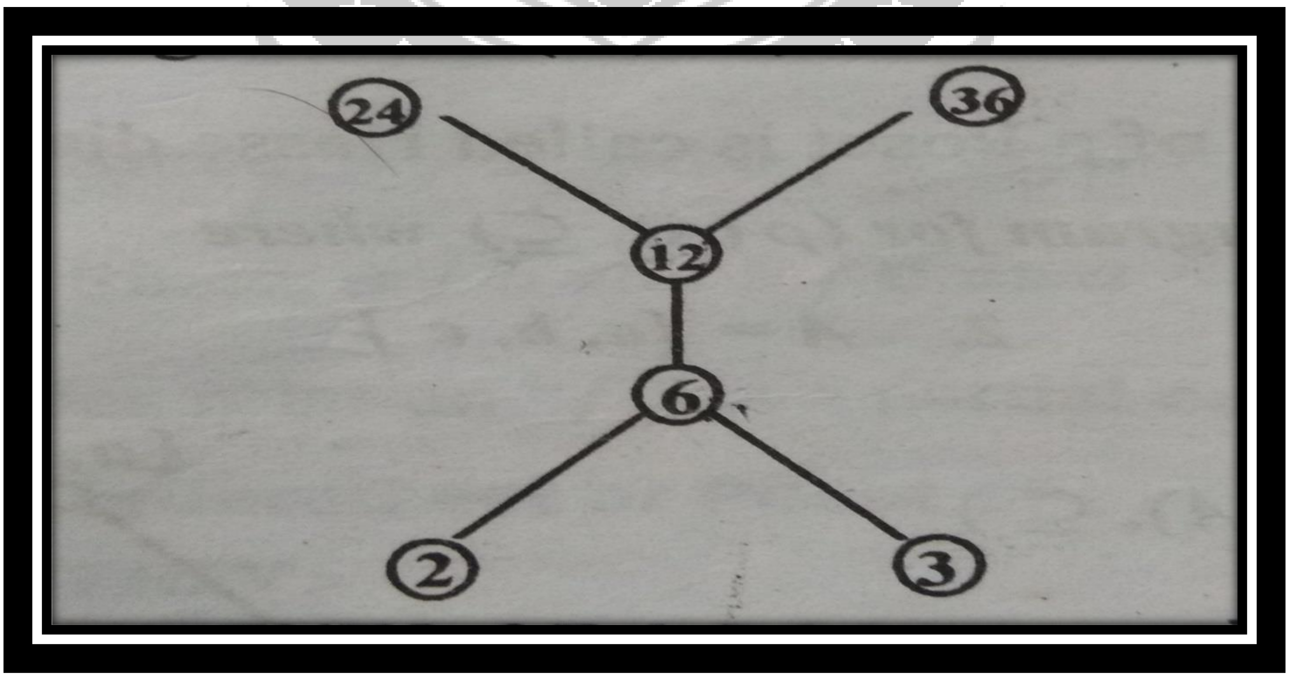
If $X = \{2, 3, 6, 12, 24, 36\}$ and the relation R defined on X by $R = \{\langle a, b\rangle / a / b\}$. Draw the Hasse diagram for (X, R).

Solution:

The relation

$$R = \left\{ \begin{matrix} \langle 2,6\rangle \langle 2,12\rangle \langle 2,24\rangle \langle 2,36\rangle \langle 3,6\rangle \langle 3,12\rangle \langle 3,24\rangle \langle 3,36\rangle \langle 6,12\rangle \\ \langle 6,24\rangle \langle 6,36\rangle \langle 12,24\rangle \langle 2,36\rangle \end{matrix} \right\}$$

The Hasse Diagram for (X, R) is



Special Elements of a Poset:

Let $(P, \leq)$ be a Poset. An element $a \in P$ is called least element in P, if $a \leq x$ for all $x \in P$.

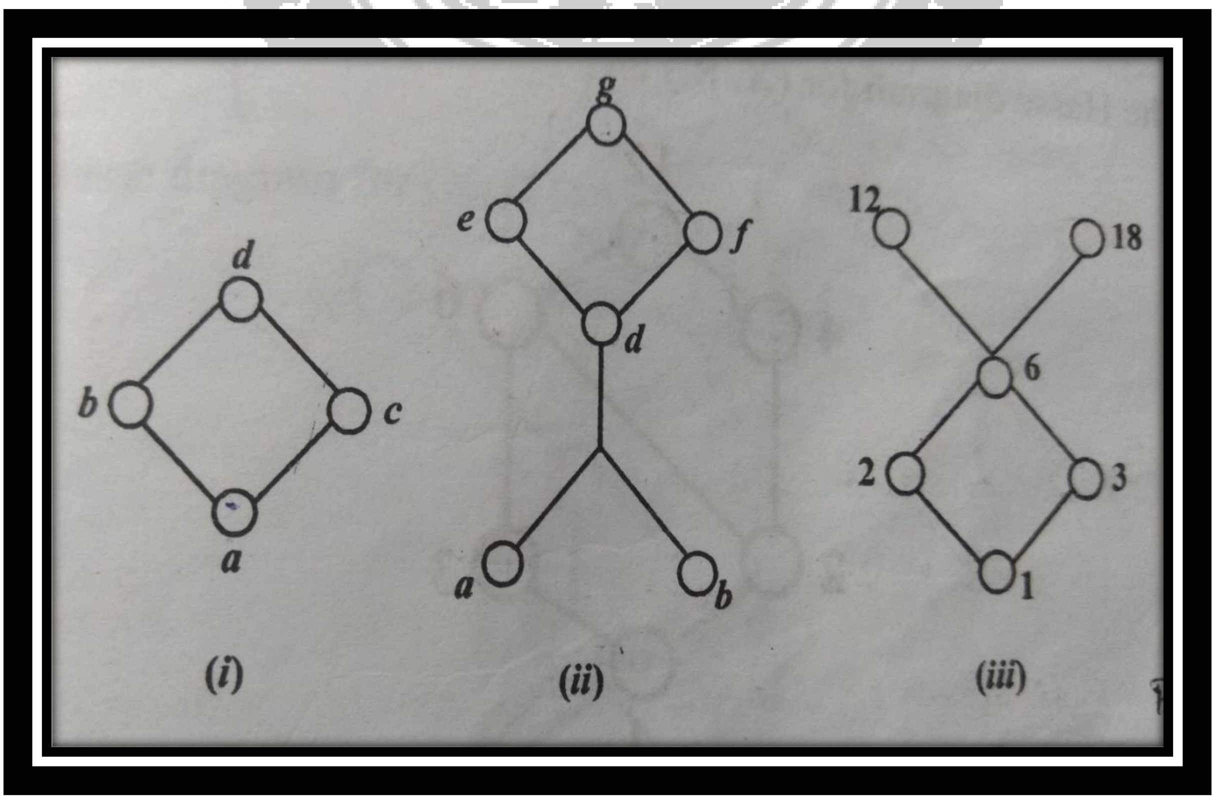
An element $b \in P$ is called greatest element in P, if $b \geq x$ for all $x \in P$

Note:

The least element is called "0" element and the greatest element is called "1" element.

Example:

Consider the following Hasse Diagram



In (i) "a" is the least element and "d" is the greatest element.

In (ii) "g" is the greatest element and there is no least element.

In (iii) "1" is the least element and there is no greatest element.

Definition:

Let $(P, \leq)$ be a Poset an A be any non - empty subset of P. An element $a \in P$ is an upper bound of A, if $a \geq x$ for all $x \in A$.

An element $b \in P$ is said to be lower bound in P, if $b \leq x$ for all $x \in A$.

Least Upper Bound: (LUB)

Let $(P, \leq)$be a Poset and $A \subseteq P$. An element $a \in P$ is said to be least upper bound (LUB) or supremum (sup) of A, if a is a upper bound of A.

$a \leq c$, where c is any other upper bound of A.

Greatest Lower Bound: (GLB)

Let $(P, \leq)$be a Poset and $A \subseteq P$. An element $b \in P$ is said to be least upper bound (GLB) or infimum (inf) of A, if b is a lower bound of A.

$b \geq d$, where d is any other lower bound of A.

Examples:

1. If $X = \{1, 2, 3, 4, 6, 12\}$ and the relation R defined on X by $R = \{\langle a, b\rangle / a / b\}$. Find LUB and GLB for the Poset (X, R).

Solution:

The relation

$$R = \left\{ \langle 1, 2\rangle\ \langle 1, 3\rangle\ \langle 1, 4\rangle\ \langle 1, 6\rangle\ \langle 1, 12\rangle\ \langle 2, 4\rangle\ \langle 2, 6\rangle\ \langle 2,12\rangle\ \langle 3, 6\rangle\ \langle 3, 12\rangle\ \langle 4, 12\rangle \right\}$$

The Hasse Diagram for (X, R) is

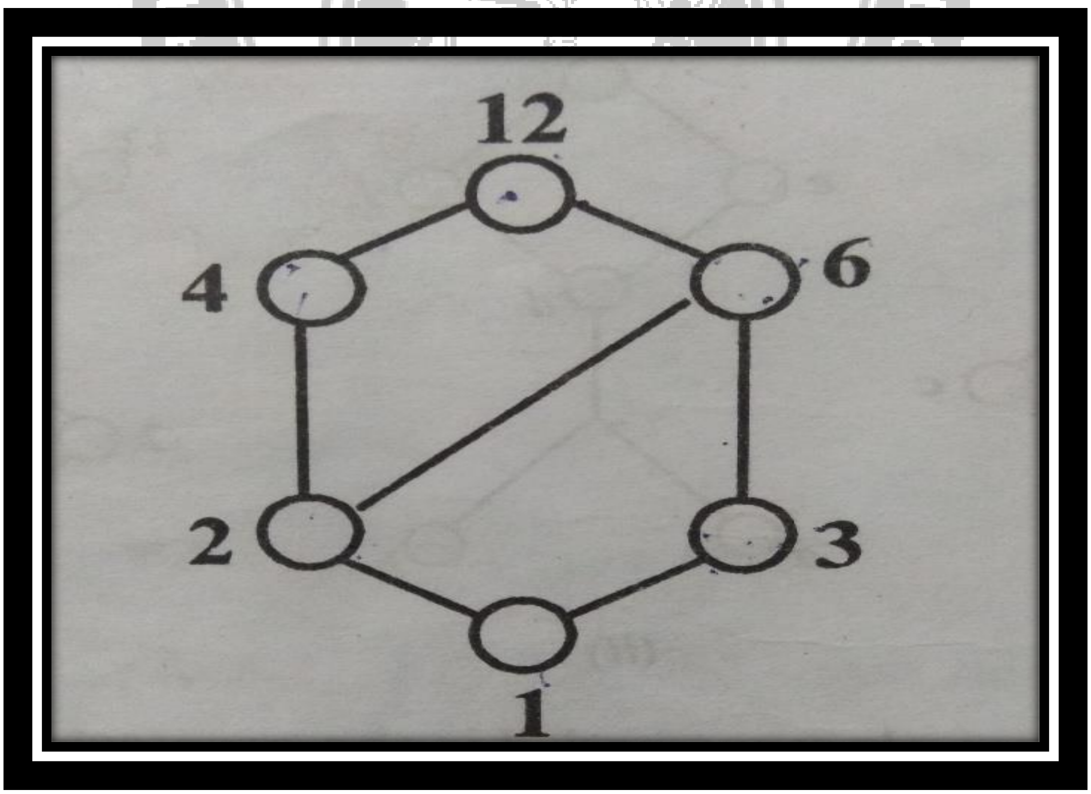


Table for LUB and GLB

1	$UB\{1,3\} = \{3,6,12\}$ $LUB\{1,3\} = 3$	**1**	$LB\{1,3\} = \{1\}$ $GLB\{1,3\} = 1$
2	$UB\{1,2,3\} = \{6,12\}$ $LUB\{1,2,\ 3\} = 6$	**2**	$LB\{1,2,3\} = \{1\}$ $GLB\{1,2,\ 3\} = 1$
3	$UB\{2,3\} = \{3,6,12\}$ $LUB\{2,3\} = 6$	**3**	$LB\{2,3\} = \{1\}$ $GLB\{2,3\} = 1$
4	$UB\{2,3,6\} = \{6,12\}$ $LUB\{2,3,6\} = 6$	**4**	$LB\{2,3,6\} = \{1\}$ $GLB\{2,3,6\} = 1$
5	$UB\{4,6\} = \{12\}$ $LUB\{4,6\} = 12$	**5**	$LB\{4,6\} = \{1,2\}$ $GLB\{4,6\} = 2$

2. If $X = \{2,3,6,12,24,36\}$ and the relation R defined on X by $R = \{\langle a,b\rangle / a/\ b\}$. Draw the Hasse diagram for (X,R).

Solution:

The relation

$$R = \begin{Bmatrix} \langle 2,6\rangle\ \langle 2,12\ \rangle\ \langle 2,24\rangle\ \langle 2,36\rangle\ \langle 3,6\rangle\ \langle 3,12\rangle\ \langle 3,24\rangle\ \langle 3,36\rangle\ \langle 6,12\rangle \\ \langle 6,24\rangle\ \langle 6,36\rangle\ \langle 12,24\rangle\ \langle 2,36\rangle \end{Bmatrix}$$

The Hasse Diagram for (X,R) is

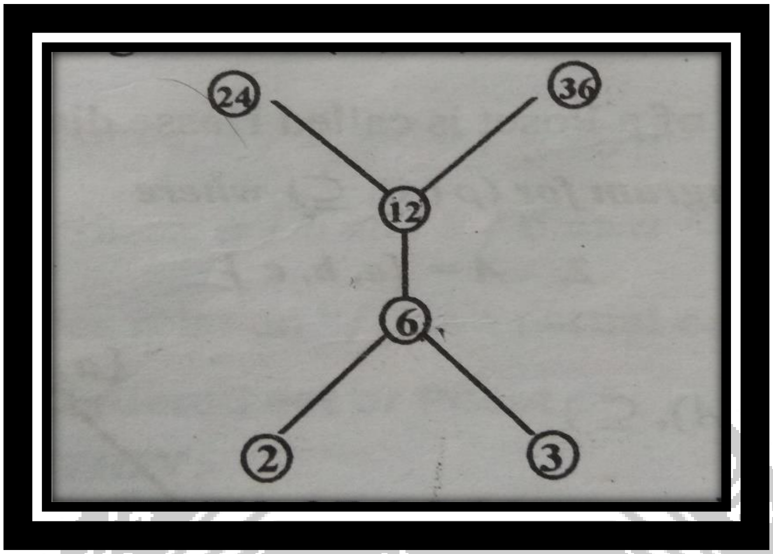


Table of LUB and GLB

1	$UB\{2,3\} = \{6,12,24,36\}$ $LUB\{2,3\} = 6$	**1**	$LB\{2,3\}$ =does not exists $GLB\{2,3\}$ =does not exists
2	$UB\{24,36\} =$ does not exists $LUB\{24,36\} =$ does not exists	**2**	$LB\{24,36\} = \{2,3,6,12\}$ $GLB\{24,36\} = 12$