MAEKAWA's ALGORITHM

- Maekawa's Algorithm is quorum based approach to ensure mutual exclusion in distributed systems.
- In permission based algorithms like Lamport's Algorithm, Ricart-Agrawala Algorithm etc. a site request permission from every other site but in quorum based approach, a site does not request permission from every other site but from a subset of sites which is called quorum.
- Three type of messages (REQUEST, REPLY and RELEASE) are used.
- A site send a REQUEST message to all other site in its request set or quorum to get their permission to enter critical section.
- A site send a REPLY message to requesting site to give its permission to enter the critical section.
- A site send a RELEASE message to all other site in its request set or quorum upon exiting the critical section.

Requesting the critical section:

(a) A site S_i requests access to the CS by sending REQUEST(i) messages to all sites in its request set R_i.

(b) When a site S_j receives the REQUEST(i) message, it sends a REPLY(j) message to S_i provided it hasn't sent a REPLY message to a site since its receipt of the last RELEASE message. Otherwise, it queues up the REQUEST(i) for later consideration.

Executing the critical section:

(c) Site S_i executes the CS only after it has received a REPLY message from every site in R_i.

Releasing the critical section:

(d) After the execution of the CS is over, site S_i sends a RELEASE(i) message to every site in R_i.

(e) When a site S_j receives a RELEASE(i) message from site S_i, it sends a REPLY message to the next site waiting in the queue and deletes that entry from the queue. If the queue is empty, then the site updates its state to reflect that it has not sent out any REPLY message since the receipt of the last RELEASE message.

Fig : Maekawa's Algorithm

The following are the conditions for Maekawa's algorithm:

M1 $(\forall i\ \forall j : i \neq j,\ 1 \leq i, j \leq N :: R_i \cap R_j \neq \phi)$.

M2 $(\forall i : 1 \leq i \leq N :: S_i \in R_i)$.

M3 $(\forall i : 1 \leq i \leq N :: |R_i| = K)$.

M4 Any site S_j is contained in K number of R_is, $1 \leq i, j \leq N$.

Maekawa used the theory of projective planes and showed that $N = K(K - 1) + 1$. This relation gives $|Ri| = \sqrt{N}$.

To enter Critical section:

- When a site S_i wants to enter the critical section, it sends a request message REQUEST(i) to all other sites in the request set R_i.
- When a site S_j receives the request message REQUEST(i) from site S_i, it returns a REPLY message to site S_i if it has not sent a REPLY message to the site from the time it received the last RELEASE message. Otherwise, it queues up the request.

To execute the critical section:

- A site S_i can enter the critical section if it has received the REPLY message from all the site in request set R_i

To release the critical section:

- When a site S_i exits the critical section, it sends RELEASE(i) message to all other sites in request set R_i
- When a site S_j receives the RELEASE(i) message from site S_i, it send REPLY message to the next site waiting in the queue and deletes that entry from the queue
- In case queue is empty, site S_j update its status to show that it has not sent any REPLY message since the receipt of the last RELEASE message.

Correctness

Theorem: Maekawa's algorithm achieves mutual exclusion.

Proof: Proof is by contradiction.

- Suppose two sites Si and Sj are concurrently executing the CS.
- This means site Si received a REPLY message from all sites in Ri and concurrently site Sj was able to receive a REPLY message from all sites in Rj .
- If $Ri \cap Rj = \{Sk\}$, then site Sk must have sent REPLY messages to both Si and Sj concurrently, which is a contradiction

Message Complexity:

Maekawa's Algorithm requires invocation of $3\sqrt{N}$ messages per critical section execution as the size of a request set is $\sqrt{N}$. These $3\sqrt{N}$ messages involves.

- $\sqrt{N}$ request messages
- $\sqrt{N}$ reply messages
- $\sqrt{N}$ release messages

Drawbacks of Maekawa's Algorithm:

This algorithm is deadlock prone because a site is exclusively locked by other sites and requests are not prioritized by their timestamp.

Performance:

Synchronization delay is equal to twice the message propagation delay time. It requires $3\sqrt{n}$ messages per critical section execution.