MULTIPROCESSOR NETWORK TOPOLOGIES

Multiprocessor system consists of multiple processing units connected via some interconnection network plus the software needed to make the processing units work together. There are two major factors used to categorize such systems:

- The processing units
- The interconnection network

A number of communication styles exist for multiprocessing networks. These can be broadly classified according to the communication model as shared memory (single address space) versus message passing (multiple address spaces).

Design Issues of Interconnection Networks

The important issue in the design of multiprocessor systems is how to cope with the problem of an adequate design of the interconnection network in order to achieve the desired performance at low cost. The choice of the interconnection network may affect several characteristics of the system such as node complexity, scalability and cost etc. The following are the issues which should be considered while designing an interconnection network.

- **Dimension and size of network:** It should be decided how many processing element are there in the network and what the dimensionality of the network is i.e. with how many neighbors, each processor is connected.
- **Symmetry of the network:** It is important to consider whether the network is symmetric or not i.e., whether all processors are connected with same number of processing elements or the processing elements of corners or edges have different number of adjacent elements.
- **Message Size:** Message size is dependent on the amount of data that can be transferred in one unit time.
- **Data transfer Time:** The time taken for a message to reach to another processor, Whether this time is a function of link distance between two processors or it depends upon the number of nodes coming in between are chief factors

- **Startup Time:** It is the time of initiation of the process.

Performance parameters

- **Number of nodes (N):** The number of nodes in a multiprocessor network plays a dynamic role by virtue of which the performance of the system is evaluated. Higher number of nodes means higher complexity but higher is the system performance. Therefore, number of processors should be optimal.
- **Node degree (D):** The node degree of the network is defined as the number of edges connected with the nodes. It is the connectivity among different nodes in a network. The connectivity of the nodes determines the complexity of the network. The greater number of links in the network means greater is the complexity. If the edge carries data from the node, it is called out degree and if this carries data into the node then it is called in degree.
- **Diameter (D):** The network diameter is defined as the maximum shortest path between the source and destination node. The path length is measured by the number of links traversed. This virtue is important in determining the distance involved in communication and hence the performance of parallel systems. The low diameter is always better because the diameter puts a lower bound on the complexity of parallel algorithms requiring communication between arbitrary pairs of nodes.
- **Cost (C):** It is defined as the product of the diameter and the degree of the node for a symmetric network.

  Cost (C) = Diameter * Degree = D * d

  Greater number of nodes means greater the cost of the network. It is good creation to measure the hardware cost and the performance of the multiprocessor network and gives more insight to design a cost-effective parallel system.

- **Extensibility**

  It is virtue which facilitates large sized system out of small ones with minimum changes in the configuration of the nodes. It is the smallest increment by which the system can be expanded in a useful way. A network with large number of links or a large node degree tends to increase the hardware cost. Expandability is an important parameter to evaluate the performance of a multiprocessor system. The feasibility to extend a system while retaining its topological characteristics enables to design large scale parallel systems.

Network Topologies

The multiprocessor networks are classified in two broad categories based on their topological properties. These are given below:

- Cube based network
- Linearly Extensible Network

Cube Based Network

- The cube based architectures are widely used networks in parallel systems. They have good topological properties such as symmetry, scalability and possess a rich interconnection topology. The types of cube based networks are:
- **Binary hypercube or n-cube:**
- This is a loosely coupled parallel multiprocessor based on the binary n-cube network.
- An n-dimensional hypercube contains 2^n nodes and has n edges per node.
- In hypercube, the number of communication links for each node is a logarithmic function of the total number of nodes.

  The hypercube organization has low diameter and high bisection width at the expense of the number of edges per node and the length of the longest edge.
- The length of the longest edge in a hypercube network increases as the number of nodes in the network increases.
- The node degree increases exponentially with respect to the dimension, making it difficult to consider the hypercube a scalable architecture.
- The major drawback of the hypercube is the increase in the number of communication links for each node with the increase in the total number of nodes.
- The bellow Fig 1 represents the Hypercube.

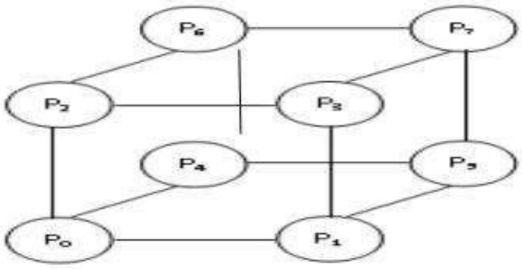

Fig 1: Hypercube

Source: Miles J. Murdocca and Vincent P. Heuring, — "Computer Architecture and Organization: An Integrated approach"

Cube Connected Cycle (CCC)

- The CCC architecture is an attractive parallel computation network suitable for VLSI implementation while preserving all the desired features of hypercube.
- The CCC is constructed from the n- dimensional hypercube by replacing each node in hypercube with a ring containing n node.
- Each node in a ring then connects to a distinct node to one of the n dimensions.
- The advantage of the cube- connected cycles is that node's degree is always 3, independent of the value of n. This architecture is modified from hypercube i.e. a 3-cube is modified to form a 3-cube-connected cycles (CCC) restricted the node degree to 3.
- The idea is to replace the corner nodes (vertices) of the 3-cube with a ring of 3-nodes.
- In general one can construct k-cube-connected cycles from a k-cube with $n=2^k$ rings nodes.

Folded Hyper Cube (FHC)

- The FHC is the variation of the hypercube network and constructed by introducing some extra links to the hypercube.
- Halved diameter, better average distance, shorter delay in communication links, less message traffic density, lower cost make it very promising.
- The hardware overhead is almost 1/n, n being the dimensionality of the hypercube,

which is negligible for large n.

- Optimal routing algorithms are developed are developed and proven to be remarkably more efficient than those of the conventional n-cube.
- A folded hypercube of dimension n is called FHC (n).
- The FHC (n) is a regular network of node connectivity (n+1)and the hypercube of degree 3 is converted to FHC (n) network. The bellow Fig 2 represents Folded Hypercube.
- Extended versions of FHC (n) is called Extended Folded Cube (EFC). The EFC has better properties than the other variations of basic hypercube in terms of parameter.
- It has constant node degree, smaller diameter, and lower cost and also it maintains several numerous desirable characteristics including symmetry, hierarchical, expansive, recursive.

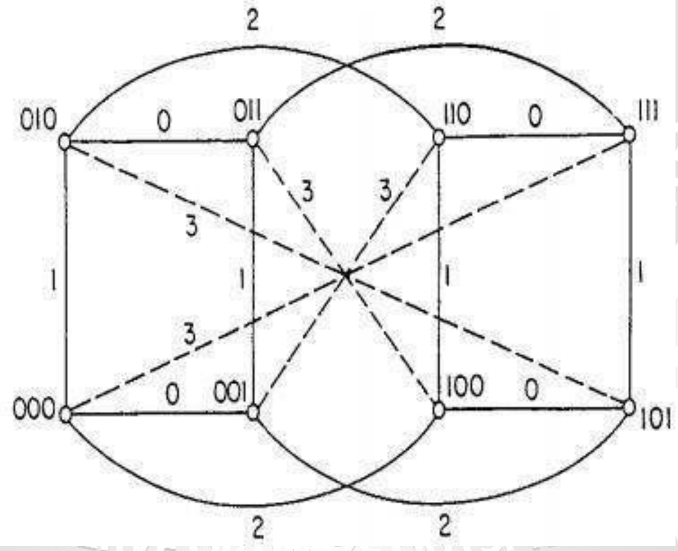

Fig 2: Folded Hypercube

Source: Miles J. Murdocca and Vincent P. Heuring, — "Computer Architecture and Organization: An Integrated approach"

Crossed Cube

- The Crossed Cube (CC) has the same node and link complexity as the hypercube and has most of its desirable properties including regularity, recursive structure, partition ability, strong connectivity and ability to simulate other architectures.

- Its diameter is only half of the diameter of the hypercube.
- Mean distance between vertices is smaller and it can simulate a hypercube through dilation 2 embedding.
- The basic properties of the CC, optimal routing and broadcasting algorithms are developed. The below fig 3 shows crossed cube.
- The CC is derived from a hypercube by changing the way of connection of some hypercube links.
- The diameter of CC is almost half of that of its corresponding hypercube.

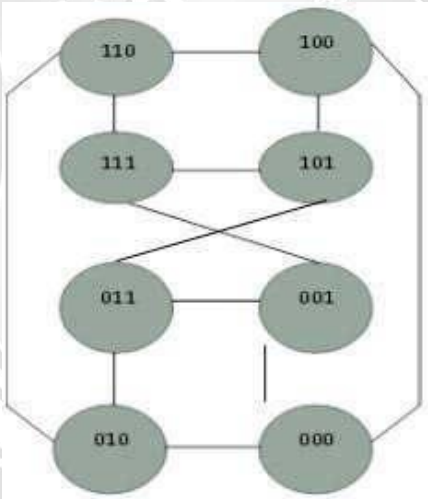

Fig 3: Crossed Cube

Source: Miles J. Murdocca and Vincent P. Heuring, — "Computer Architecture and Organization: An Integrated approach"

Reduced Hypercube (RHC)

- The RH (k, m) is obtained from the n- dimensional hypercube by reducing node edges in hypercube by following rules where k+2m= n.
- The lower VLS) complexity of R(s permit the construction of systems with more processing elements than are found in conventional hypercube.
- There are clusters and each cluster is a conventional k- dimensional hypercube.
- Of the higher n-k=2m dimensions, a node has only one direct connection is decided by the leftmost m bits in the k-bits field, i.e., the (2i + k) dimension, where i is the value of the

m-bit binary number.

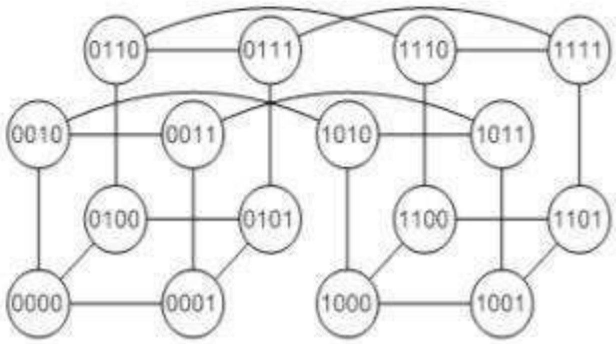

Fig 4: Reduced Hypercube Hierarchal Cube Network (HCN)

Source: Miles J. Murdocca and Vincent P. Heuring, — "Computer Architecture and Organization: An Integrated approach"

- The Hierarchical Cube Network (HCN) is interconnection network for large-scale distributed memory multiprocessors. The above fig 4 represents the Reduced Hypercube Hierarchal Cube Network(HCN).
- HCN has about three-fourths the diameter of a comparable hypercube, although it uses about half as many links per node-a fact that has positive ramifications on the implementation of HCN-connected systems.
- The HCN (n, n)has 2n clusters, where each cluster is an n-cube.
- Each node in the HCN (n, n) has n+1 links connected to it. n links are used inside the cluster. The additional links are used to connect nodes among clusters.
- The advantage of HCN is that the number of links required is reduced approximately to half as many links per node and the diameter is reduced to about three-fourth of a corresponding hypercube. The below Fig 5 shows the Hierarchical Cube Network.

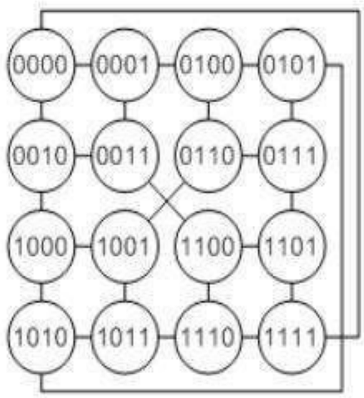

Fig 5: Hierarchical Cube Network

Source: Miles J. Murdocca and Vincent P. Heuring, — "Computer Architecture and Organization: An Integrated approach"

Dual Cube (DC)

- The DC is a new interconnection topology for large-scale distributed memory Multiprocessors that reduces the problem of increasing number of links in the large-scale hypercube network.
- This preserves most of the topological properties of the hypercube network.
- The DC shares the desired properties of the hypercube, however increases tremendously the total number of nodes in the system with limited links per node.
- The key properties of hypercube are also true in the dual-cube: each node can be represented by unique binary number such that two nodes are connected by an edge if and only if the two binary numbers differ in one bit only.
- However, the size of the dual-cube can be as large as eight thousands with up to eight links per node.
- A dual-cube uses binary hypercube as basic components. Each such hypercube component is referred to as a **cluster.**
- Assume that the number of nodes in a cluster is 2^m. In a dual cube, there are two
- Classes with each class consisting of 2^m clusters.
- The total number of nodes is 2^m or 2^{m+1}. Therefore, the nodes address has 2m+1 bits

- The leftmost bit is used to indicate the type of the class (class 0 and class 1).
- For the class 0, the rightmost m bits are used as the node ID within the cluster.
- Each node in cluster of class 0 has one and only one extra connection to a node in a cluster of class 1.

Meta Cube (MC)

- The MC is an interconnection topology for a very large parallel system. Meta cube network has two level cube structures. An MC (k, m) network can connect $2k+m^{2k}$ nodes with (k+m) links per node where k is the dimension of the high-level cubes (classes) and m is the dimension of the low-level cubes (clusters).

In this network, the number of nodes is much larger than the hypercube with a small number of links per node

- An MC network is a symmetric network with short diameter, easy and efficient routing.
- Similar to that of the hypercube.
- The meta cube has tremendous potential to be used as an interconnection network for very large scale parallel computers since the meta cube can connect hundreds of millions nodes with up to six links per node and it keeps some desired properties of the hypercube that are useful efficient communication among the nodes.

Folded Dual Cube (FDC)

- The FDC is a new cube based Interconnection topology for parallel systems with reduced diameter, cost and constructed from DC and FHC.
- The FDC is a graph Fr (V, E), where V represents a set of vertices and E represent a set of links.
- The FDC is to be slightly greater than Dual cube but quite less than HC and FHC.
- Diameter of FDC is found to be smaller than that of Dual cube and with the comparison of Dual cube, HC and FHC.
- FDC exhibits quite a good improvement in broadcast time over its parent networks

with millions of nodes.

- The cost of the FDC topology is found to be less. The FDC will help to speed up the overall operation of large scale parallel systems.

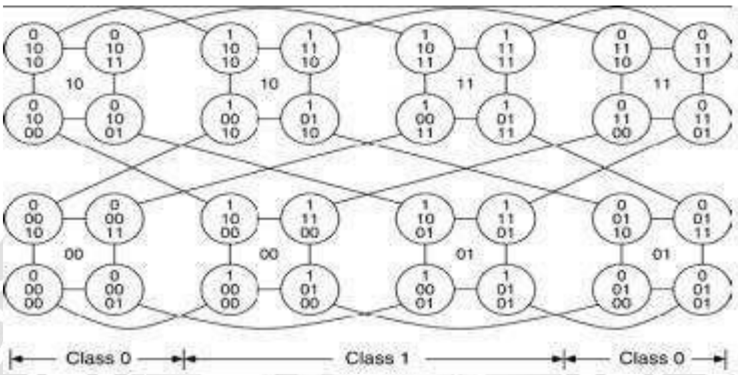

Fig 6: Folded Dual Cube

Folded Meta cube (FMC)

- The FMC is an efficient large scale parallel interconnection topology with better features such as reduced diameter, cost, improved broadcast time and constructed from MC. The above fig 6 shows Folded Dual Cube.
- The FMC is a graph G (V, E), where V represents a set of vertices and E represent a set of links.
- The FMC is to be slightly greater than Meta cube but quite less than HC and FHC.
- Diameter of FMC is found to be smaller than that of Meta cube.
- FMC exhibits quite a good improvement in broadcast time over its parent network while connecting millions of nodes.
- The cost of the FMC is found to be less and will help to speed the overall operation of large scale parallel systems.

Necklace Hypercube (NH)

- NH is an array of processors attached to each two adjacent nodes of the hypercube network.
- It is highly scalable architecture while preserving most of the desirable properties of hypercube such as logarithmic diameter, fault tolerance etc.

- It has also some other properties such as hardware scalability and efficient VLSI layout that make it more attractive than an equivalent hypercube network.
- The Necklace-Hypercube is an undirected graph which has a necklace of processors to each edge of hypercube. The below Fig 7 shows the Necklace Hypercube.
- The necklace length may be fixed or variable for different edge necklaces.

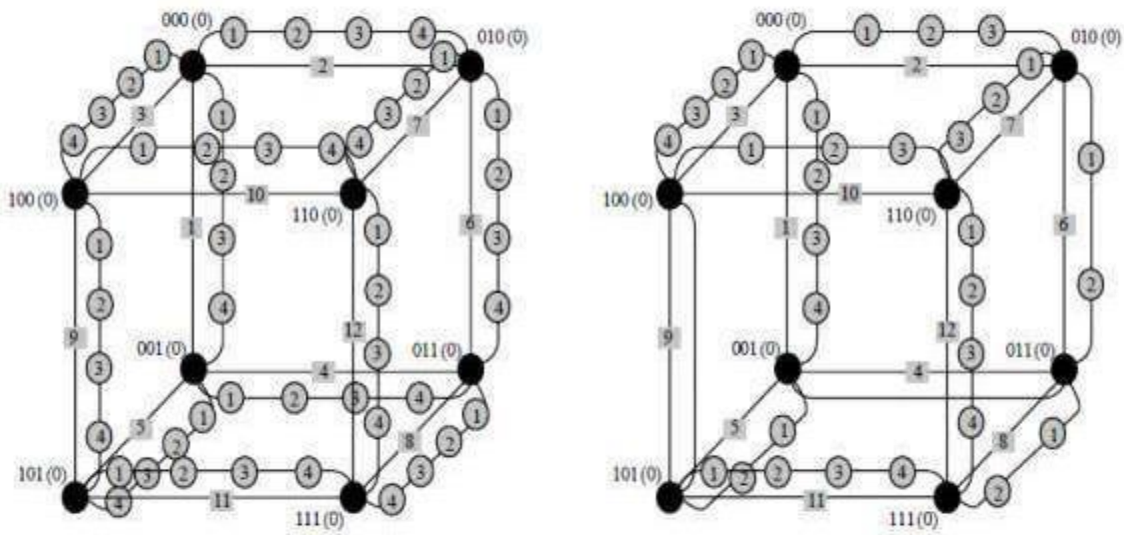

Fig 7: Necklace Hypercube

Source: Miles J. Murdocca and Vincent P. Heuring, — "Computer Architecture and Organization: An Integrated approach"

Linearly Extensible Network

The Linearly Extensible Networks is another class of multiprocessor architectures which reduces some of the drawbacks of HC architectures. The complexity of these networks is lesser as they do not have exponential expansion. Besides the scalability, other parameters to evaluate the performance of such networks are degree, number of nodes, diameter, bisection width and fault tolerance. Selection of a better interconnection network may have several applications with lesser complexities and improved power-efficiency.

Linear Array (LA)

- It is one dimensional network having the simplest topology with n-nodes having n-1 communication links.

- The internal nodes have degree 2 and the termination nodes have degree1.
- The diameter is n-1, which is long for large n and the bisection width is 1.
- It is asymmetric network. Linear array are the simplest connection topology.
- As the diameter increases linearly with respect to n, it should not be used for large n. For every small n, it is rather economical to implement a linear array.

Binary Tree (BT)

- A binary tree is either empty or consists of node called the root together with two binary trees called left sub tree and the right sub tree.
- When h is equal to height of a binary tree then maximum leaves are equal to 2^h and maximum nodes are 2^{h+1}-1.
- In a binary tree network there is only one path between any two nodes.
- The binary tree is scalable architecture with a constant node degree and constant bisection width. In general, an n-level, complexity balanced binary tree should have N=2n-1 nodes.
- The maximum node degree is 3 and the diameter is 2(n-1). But has a poor bisection width of 1.

Ring (R)

- This is a simple linear array where the end nodes are connected. It is equivalent to mesh with wrap around connections.
- The data transfer in a ring is normal one direction. A ring is obtained by connecting the two terminal nodes of a linear array with one extra link.
- A ring network can be uni-or bidirectional and it is symmetric with a constant.
- It has a constant node degree of d=2, the diameter is N/2for a bidirectional ring and N for unidirectional ring.
- A ring network has a constant width 2.

Linearly Extensible Tree (LET)

- The Linearly Extensible Tree (LET) architecture exhibits better connectivity, lesser number of nodes over cube based networks.
- The LET network has low diameter, hence reduce the average path length traveled by all message and contains a constant degree per node. The below Fig 8 shows Linearly Extensible Tree.
- The LET network grows linearly in a binary tree like shape.
- In a binary tree the number of nodes at level n is 2n whereas in LET network the number is (n+1).

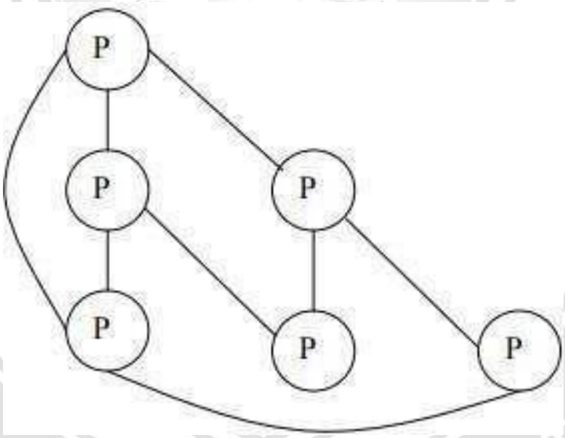

Fig 8: **Linearly Extensible Tree**