2.4. SHEAR FORCE AND BENDING MOMENT DIAGRAM FOR OVER-HANGING BEAM:

The beam having its portion is extended beyond the support, such beam is known as overhanging beam. In case of overhanging beam, the BM is positive between the two supports, whereas the BM is negative for the overhanging portion. Hence at some point, the BM is zero after changing its sign from positive to negative or vice versa. That point is known as **point of contraflexure or point of inflexion.**

Problem 2.9: Draw the S.F. and B.M. diagram for the loaded beam shown in fig

Given Data: shown in figure.

To find: SFD and BMD, Max. BM

Solution:

Find the reaction at A and B as R_A and R_B

Step1: Take moment about A is equal to zero

For UDL, it will be converted into point load as (Point load = UDL x load acting distance) and the converted point load acting at its middle means divided by 2.

$$- (2 \times 7) + R_B \times 5 - [(2 \times 5) \times \frac{5}{2}] - (5.5 \times 2) = 0$$

$$R_B \times 5 = (2 \times 7) + [(2 \times 5) \times \frac{5}{2}] + (5.5 \times 2) \gg R_B = 10\text{kN}$$

Step2: Sum of upward force = Sum of downward force

$$R_A + R_B = 2 + (2 \times 5) + 5.5 \gg R_A = 17.5 - R_B$$

$$\gg R_A = 17.5 - 10 = 7.5 \text{ kN}$$

Shear Force Calculation: (Sum of vertical forces)

SF at B = + 2 = +2kN

SF at C = +2 - 10 = - 8 kN

When point load and UDL acting at a particular point, first we have not consider Point Load and next consider point load to find shear force.

SF at D = +2 - 10 + (2 x 3) = - 2 kN (Without consider PL)

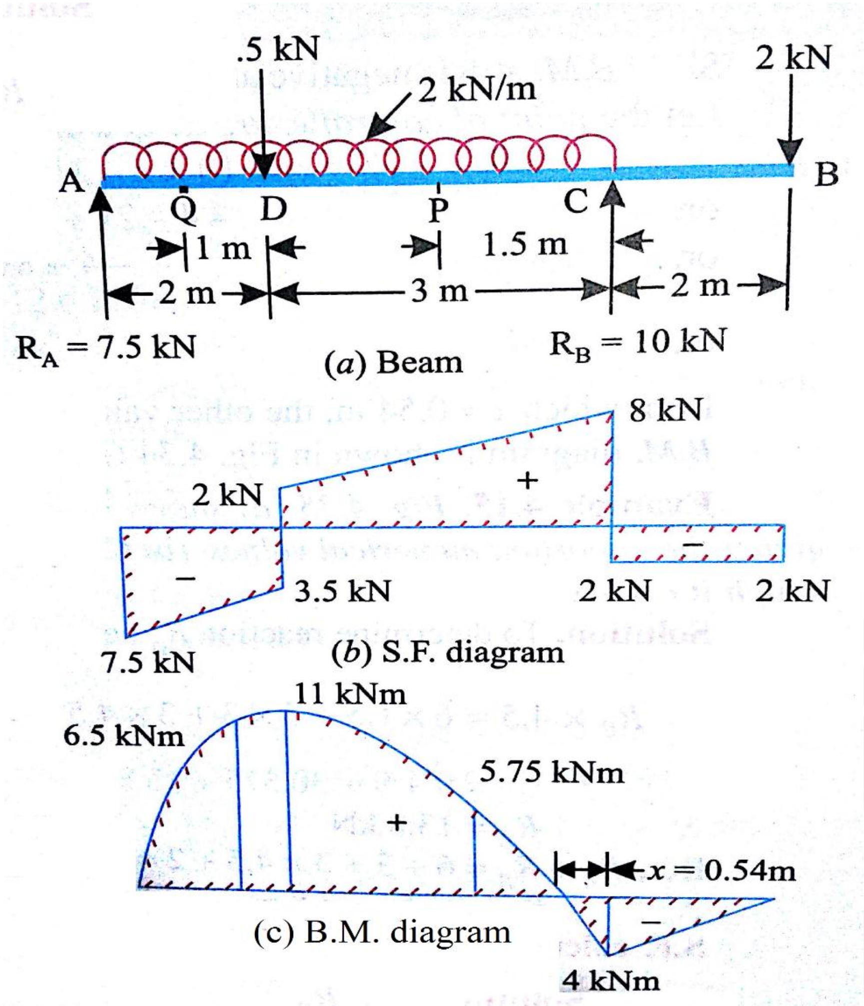
SF at D = +2 - 10 + 5.5 + (2 x 3) = +3.5 kN (With consider PL)

SF at A = +2 - 10 + 5.5 + (2 x 5) = + 7.5 kN

Shear Force Diagram:

Vertical upward reaction and downward load are drawn as vertical line based on sign.

Vertical downward UDL are drawn as inclined line based on sign



Bending moment Calculation: [Sum of (Vertical force x Distance of load acting from required section)]

BM at B = -(2 x 0) = - 0 kN

BM at C = -(2 x 2) + (10 x 0) = - 4kN

BM at D = -(2 x 5) + (10 x 3) - (5.5 x 0) – [(2 x 3) x $\frac{3}{2}$]= +11kN

BM at A = -(2 x 7) + (10 x 5) - (5.5 x 2) – [(2 x 5) x $\frac{5}{2}$]= 0kN

Bending moment Diagram:

Vertical downward and upward PL are drawn as inclined line and UDL are drawn as parabolic curved line based on their sign.

Calculate the position of point of contraflexure or point of inflexion:

The point of contraflexure or point of inflexion lie at a point where BM become zero. In the fig. the BM act at X at a distance x m from support C.

BM at X = - [2 x (x+ 2)] + (10 xx) -[(2 x x) x $\frac{x}{2}$] = 0 ≫

= - 2x - 4 +10x - x^2 ≫ x^2 – 8x + 4 = 0

Then $x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \text{ x } 1 \text{ x } 4}}{2 \text{x } 1} = 0.54$or 7.45m

Now x = 0.54m and other value 7.45m being inadmissible.

Result:

The SFD and BMD are drawn as shown in fig.

Point of contraflexure or point of inflexion acts at a distance of **0.54m** from right support.

Problem 2.4.1:A beam AB 10 m long carries a UDL of 20 kN/m over its entire length together with concentrated load of 50 kN at the left end A and 80 kN at the right end B. The beam is to be supported at two props at the same level 6m apart, so that the reaction is the same at each. Determine the position of the supports and draw S.F. and B.M. diagram. Find the value of maximum B.M. Locate the point of contraflexure, if any.

Given Data: shown in figure.

To find: SFD and BMD, Max. BM and Locate the point of contraflexure

Solution:

Find the position of reactions:

Step1: Sum of upward force = Sum of downward force

$R_C + R_D$ = 80 + (20 x 10) + 50

Given $R_C = R_D$ ≫ $2R_C$ = 330 ≫ R_C = 330/2 ≫ R_C = 165 kN

≫ $R_C = R_D$ = 165 kN

Step2: Take moment about A is equal to zero. Let us take a distance x m form left end one reaction D will be acted.

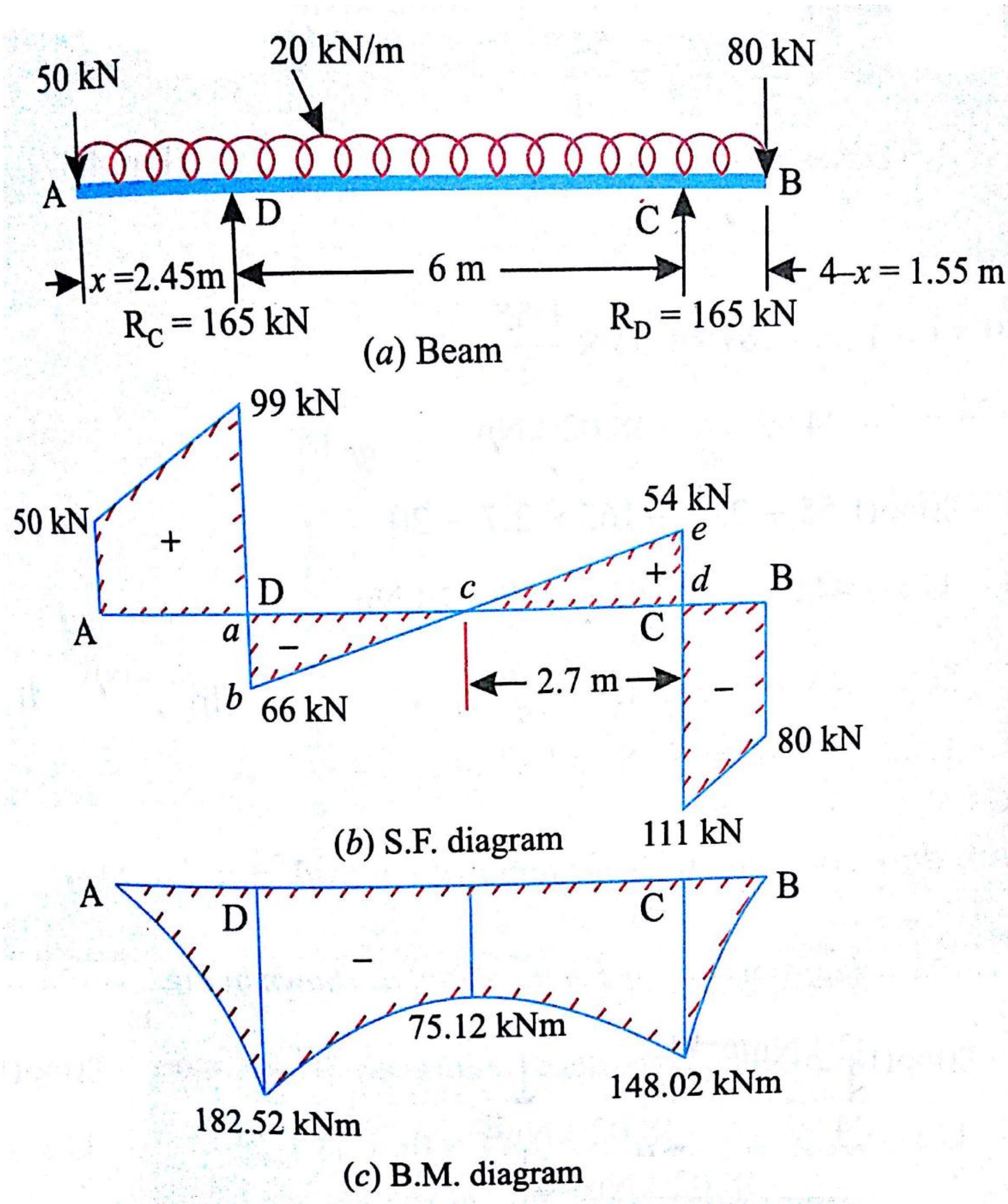
For UDL, it will be converted into point load as (Point load = UDL x load acting distance) and the converted point load acting at its middle means divided by 2.

- (80 x 10) + R_c x (6 + x) + R_D x x – [(20 x 10) x$\frac{10}{2}$] = 0

2 x 165x = 800 + 1000 – (6 x 165)

x = 810/330 = 2.45m

Then distance of another reaction from right support = 4 – 2.45 = 1.55m



Shear Force Calculation: (Sum of vertical forces)

SF at B = + 80 = +80kN

When point load or reaction and UDL acting at a particular point, first we have not consider Point Load and next consider point load to find shear force.

SF at C = +80 + (20 x 1.55) = 111kN (Without consider PL)

SF at C = +80 - 165 + (20 x 1.55) = - 54 kN (With consider PL)

SF at D = +80 - 165 + (20 x 7.55) =+ 66kN (Without consider PL)

SF at D = +80 - 165 – 165 + (20 x 7.55) = - 99kN (With consider PL)

SF at A = +80 - 165 – 165 + (20 x 10) =- 50kN

Shear Force Diagram:

Vertical upward reaction and downward load are drawn as vertical line based on sign.Vertical downward UDL are drawn as inclined line based on sign

Bending moment Calculation: [Sum of (Vertical force x Distance of load acting from required section)]

BM at B = -(80 x 0) = 0 kN

BM at C = -(80 x 1.55) + (165 x 0) – [(20 x 1.55) x $\frac{1.55}{2}$] = - 148.02kNm

BM at D = -(80 x 7.55)+(165 x 6)– (165 x 0) -[(20 x 7.55) x $\frac{7.55}{2}$] = - 182.52kNm

BM at A = -(80 x 10)+(165 x 8.45)– (165 x 2.45) - [(20 x 10) x $\frac{10}{2}$] = - 0 kNm

Bending moment Diagram:

UDL are drawn as parabolic curved line based on their sign.

Maximum B.M. Calculation:

Maximum B.M. available at the reaction D which is equal to **182.52kNm**

Calculate the position of point of contraflexure or point of inflexion:

The B.M remains negative throughout, hence there is no point of contraflexure or point of inflexion.

Result: The SFD and BMD are drawn as shown in fig.

There is no Point of contraflexure or point of inflexion.

Maximum B.M. = 182.52kNm

2.4.2. S.F AND B.M. DIAGRAM FOR BEAMS CARRYING INCLINED LOAD AND SUPPORTS

The shear force is defined as the algebraic sum of the vertical forces at any section of a beam to the right or left of the section. But when a beam carries inclined loads, then these inclined loads are resolved into their vertical and horizontal components. The vertical components only will cause shear force and bending moments.

The horizontal components of the inclined loads will introduce axial force or thrust in the beam. The variation of axial force for all sections of the beam can be shown by a diagram known as thrust diagram or axial force diagram.

In most of the cases, one end of the beam is hinged and the other end is supported on rollers. The roller support cannot provide any horizontal reaction. Hence only the hinged end will provide the horizontal reaction.

Problem 2.4.3:A beam AB 4 m long is hinged at A and supported on roller at B. The beam carries inclined loads of 100N, 200N and 300N inclined at 60^o, 45^o and 30^o to the horizontal at a distance of 1m 2m and 3m respectively form left support. Draw the S.F., B.M. and thrust diagram for the beam.

Given Data: shown in figure.

To find: SFD and BMD and Thrust force diagram.

Solution:

First of all resolve the inclined force into their vertical and horizontal components

Inclined load at C is having horizontal component = 300 x cos 30^o= 259.8N

Vertical component = 300 x sin30^o= 150N

Inclined load at D is having horizontal component = 200 x cos 45^o= 141.4N

Vertical component = 200 x sin45^o= 141.4 N

Inclined load at E is having horizontal component = 100 x cos 60^o= 50N

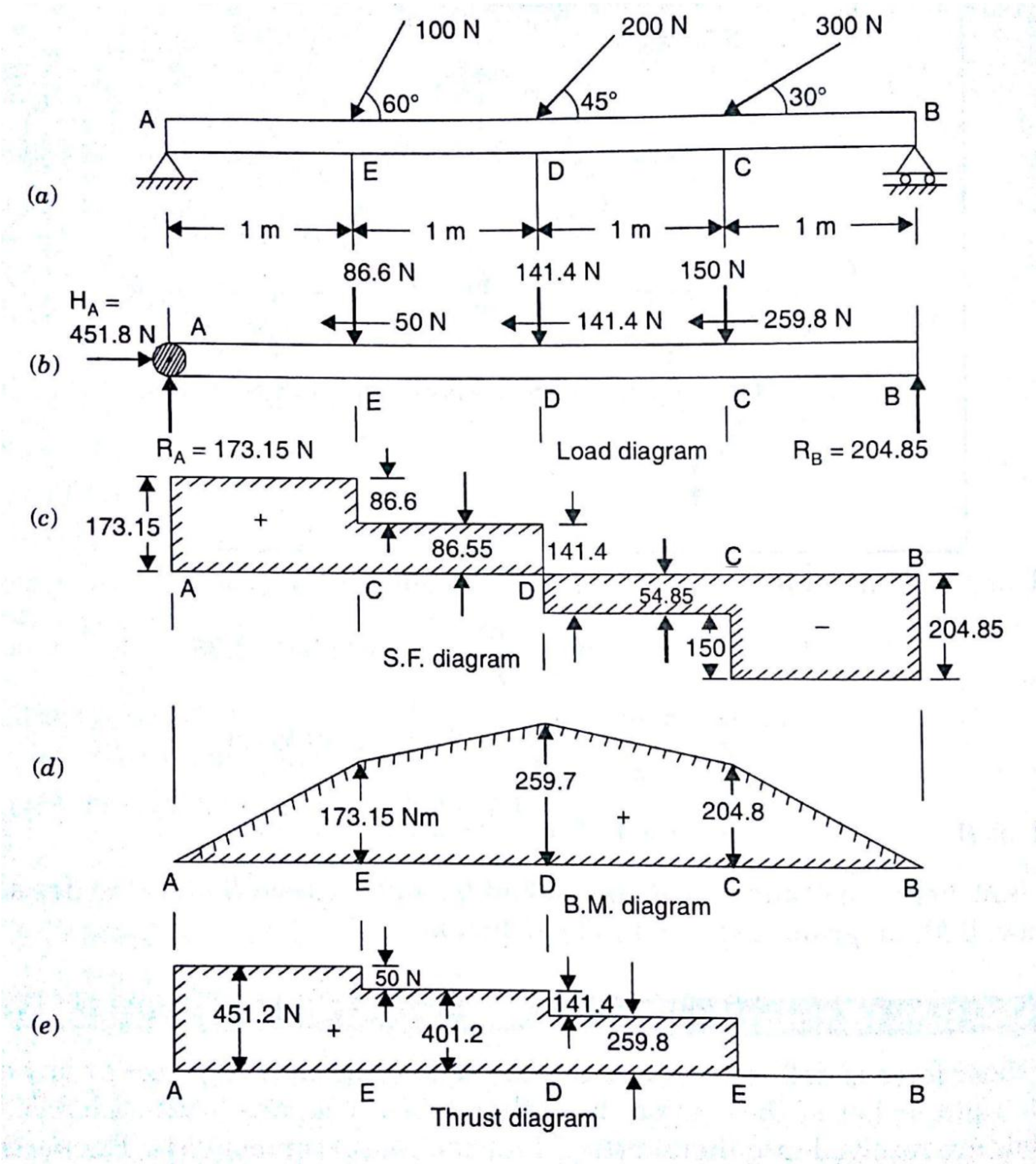
Vertical component = 100 x sin60^o= 86.6N

As the beam is supported on roller at B, hence have vertical reaction as R_B. But at A having hinged support, hence Horizontal and vertical reaction are there as R_A and H_B

Horizontal Reaction at B = H_B = Sum all horizontal component of inclined loads

= 50 + 141.4 + 259.8 = 451.20 N

(Based on horizontal force direction put + or – sign)



Find the reaction at A and B as R_A and R_B

Step1: Take moment about A is equal to zero

+ R_B x4 - (150 x 3) - (141.4 x 2) – (86.6 x 1)= 0

R_B x4 = (150 x 3) + (141.4 x 2)+(86.6 x 1) ≫ R_B = 204.85 N

Step2: Sum of upward force = Sum of downward force

R_A + R_B = 150 + 141.4 + 86.6≫ R_A = 378 - R_B

≫ R_A = 378 – 204.85 = 173.15 N

Shear Force Calculation: (Sum of vertical forces)

SF at B = -R_B = - 204.85N

SF at C = - 204.85 +150 = - 54.85 N

SF at D =- 204.85 +150 + 141.4= +86.55 N

SF at E = - 204.85 +150 + 141.4 + 86.6 = +173.15 N

SF at A = - 204.85 +150 + 141.4 + 86.6 = +173.15N

Shear Force Diagram:

Vertical upward reaction and downward load are drawn as vertical line based on sign.

Bending moment Calculation: [Sum of (Vertical force x Distance of load acting from required section)]

BM at B = -+ (R_B x 0) = 0Nm

BM at C = +(204.85 x 1) – (150 x 0) = + 204.85Nm

BM at D = + (204.85 x 2) –(150 x 1) – (141.4 x 0) = + 259.7Nm

BM at E = +(204.85 x 3) – (150 x 2) – (141.4 x 1) –(86.6 x 0) = +173.15Nm

BM at A = +(204.85 x 4) – (150 x 3) – (141.4 x 2) – (86.6 x 1) = 0Nm

Bending moment Diagram:

PL are drawn as inclined line based on their sign.

Thrust or Axial force calculation :(Sum of horizontal component of forces)

TF at B = -0 kN = 0N

TF at C = 0 + 259.8= +259.8 N

TF at D =0+ 259.8+ 141.4 = + 401.2 N

TF at E = 0+ 259.8 + 141.4+ 50 = +451.2 N

TF at A = 0+ 259.8 + 141.4+ 50 = +451.2 N

Thrust or Axial force Diagram:

PL are drawn as straight line based on their sign.

Result:

The SFD,BMD and TFD are drawn as shown in fig.

2.4.4. S.F AND B.M. DIAGRAM FOR BEAMS CARRYING INCLINED LOAD, SUPPORTS AND SUBJECTED TO COUPLE

When a beam subjected to couple at a section, only the B.M. at the section of the couple changes suddenly in magnitude equal to that of the couple. But the S.F. does not changes at the section of the couple as there is no change in load due to couple at the section. But while calculating the reactions, the magnitude of the couple is taken into account. Hence the B.M. calculate at the couple acting section with or without consider the couple.

Problem 2.4.5: Draw the SF and BM diagram for a beam shown in fig.

Given Data: shown in figure.

To find: SFD and BMD

Solution:

The horizontal force 5 kN acting on the top of a 1m lever at D causes a clockwise moment of 5 kNm at D and a horizontal thrust of 5kN in the beam. Since the horizontal thrust n beam effects neither the S.F in the beam nor the B.M. in it, for purpose of analyzing the shear and the bending moment in the beam the given force of 5kN can be replaced by a clockwise moment of 5kNm

Find the reaction at A and B as R_A and R_B

Step1: Take moment about A is equal to zero

-(10 x 8)+ (R_C x 6) - 5 – (2 x 4 x $\frac{4}{2}$)= 0

R_C x6 = (10 x 8) + 5+(2 x 4) ≫ R_C = 101/6 = 16.83kN

Step2: Sum of upward force = Sum of downward force

R_A + R_C = 10 + (2 x 4)≫ R_A = 18 - R_C

≫ R_A = 18 – 16.83 = 1.17kN

Shear Force Calculation: (Sum of vertical forces)

SF at B = + 10 kN

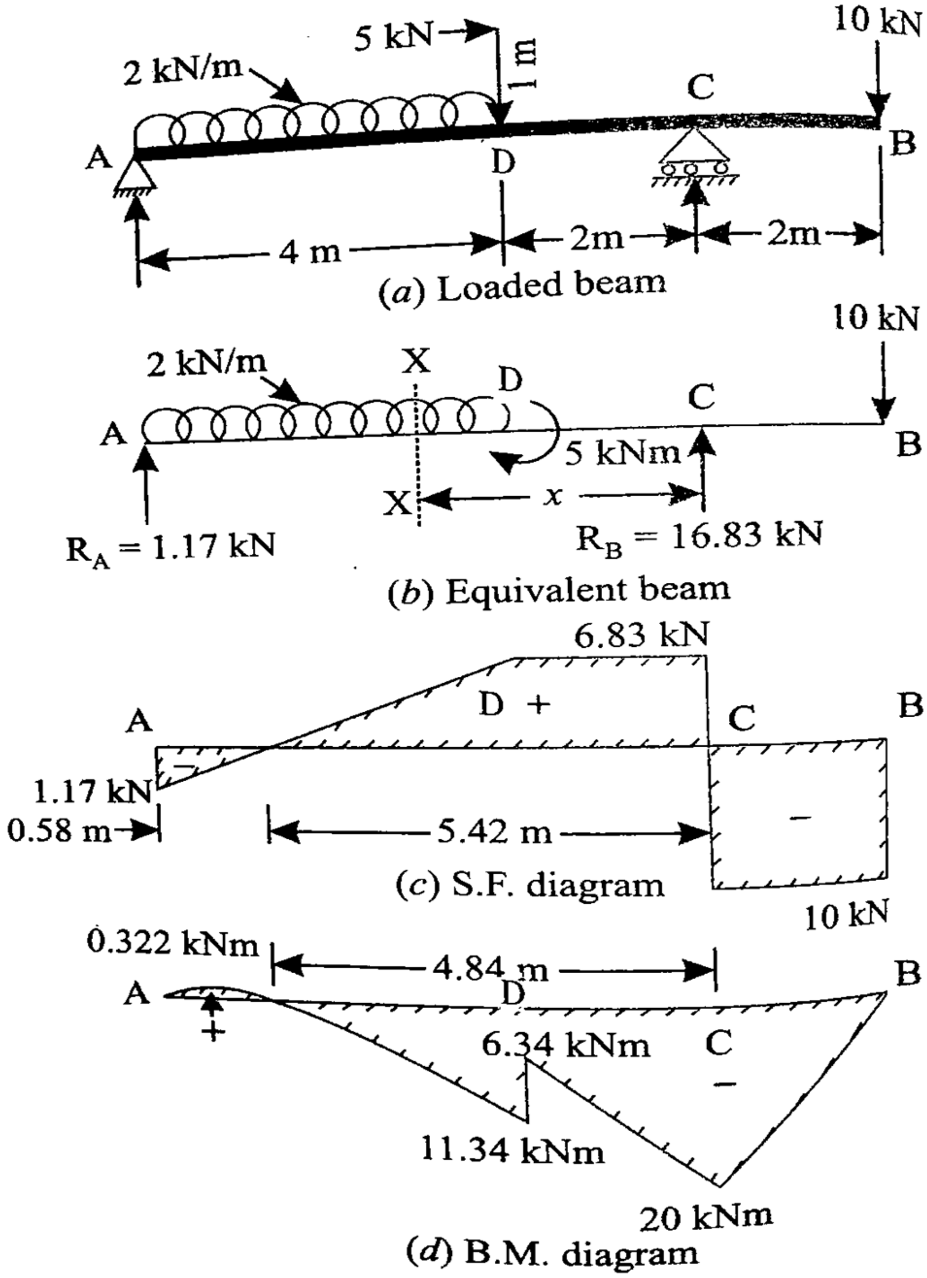
SF at C = + 10 – 16.83 = - 6.83 N

SF at D =+ 10 – 16.83 + 0 = - 6.83 N

SF at A = + 10 – 16.83 + (2 x 4) = 1.17 N

Shear Force Diagram:

Vertical upward reaction and downward load are drawn as vertical line based on sign. UDL are drawn as inclined line based on their sign.



Bending moment Calculation: [Sum of (Vertical force x Distance of load acting from required section)]

BM at B = - (10 x 0)= 0 Nm

BM at C = - (10 x 2) + (16.83 x 0) = - 6.83 N = + 204.85 Nm

BM at D = - (10 x 4) + (16.83 x 2) - 5 = + 259.7 Nm

BM at A = - (10 x 4) + (16.83 x 2) - 5 – (2 x 4 x $\frac{4}{2}$) = 0 Nm

Bending moment Diagram:

PL are drawn as inclined line and UDL drawn as parabolic curved line based on their sign.

Calculate Maximum Bending Moment:

The maximum BM will occur where the shear force becomes zero. Let us take a point X at a distance of *x* from D where SF become zero is shown in fig.

SF at X =+ 10 – 16.83 + (2 x *x*)= 0 ≫x = (16.83 – 10)/2 = 3.42 m

Then BM at X = - [10 x (4+3.42)] + [16.83 x (2 + 3.42)] - 5 – [2 x 3.42 x $\frac{3.42}{2}$]]

= 0.322kNm

Calculate the position of point of contraflexure or point of inflexion:

The point of contraflexure or point of inflexion lie at a point where BM become zero. In the fig. the BM act at Y at a distance y m from B.

BM at Y = - (10 x y) + [16.83 x (y - 2)] - 5 – [2 x (y-4) x $\frac{(y-4)}{2}$] = 0

≫ = - 10y+ 16.83y - 37.66 – 5 - y^2 +8y– 16 = 0

≫ y^2 – 14.83y– 58.66 = 0

Then y = $\frac{-(-14.83) \pm \sqrt{(-14.83)^2 - 4 \text{ x } 1 \text{ x } (-58.66)}}{2\text{x } 1}$ = 6.84m or 8.45m

Now x = 6.84m and other value 8.45m being inadmissible.

Result:

The SFD, BMD are drawn as shown in fig.

Point of contraflexure act at a distance of **6.84m** from B

Problem 2.4.6: Calculate the reaction at A and C and position of point of contraflexure of the given beam with load shown in fig. Also Draw the SF and BM diagram for the beam.

Given Data: shown in figure.

To find: SFD, BMD and distance of point of contraflexure.

Solution:

In this figure roller support has one reactionR_A and hinged support has horizontal and vertical reactionR_{C_H} and R_{C_v}. A force applied 4kN through a bracket 0.5m away from the point D. Now apply equal and opposite load of 4kN at D. This will be equivalent to a anticlockwise couple of value (4 x 0.5) = 2kNm acting at D together with a vertical downward load of 4 kN at D.

The inclined load 4kN resolved into vertical and horizontal component of forces.

Horizontal component of force = B_H = 4 x cos30^o = 2$\sqrt{3}$kN

Vertical component of force = B_V = 4 x sin30^o = 2 kN

The horizontal force balanced by a force at the hinged support. Which gives

Find the reaction at A and C as R_A and R_C

Step1: Take moment about A is equal to zero

- (2x 7) + (R_{C_v} x 6) –(4 x 4) + 2 – (1 x 2$\times \frac{2}{2}$)= 0

R_{C_v} x6 = (2 x 7) + (4 x 4) - 2+(1 x 2 $\times \frac{2}{2}$) $\gg$ R_{C_v} = 30/6 = 5kN

Step2: Sum of upward force = Sum of downward force

R_A + R_{C_v} = 2 + 4 + (1 x 2) $\gg$ R_A = 8 - R_{C_v}

$\gg$ R_A = 8 – 5 = 3kN

Total reaction at C (R_C)= $\sqrt{{R_{C_H}}^2 + {R_{C_v}}^2}$ = $\sqrt{\left(2\sqrt{3}\right)^2 + 5^2}$ = 6.08 kN

Location with horizontal θ = $\tan^{-1}\frac{R_{C_v}}{R_{C_H}}$ = $\tan^{-1}\frac{5}{2\sqrt{3}}$ = 55.3°

Shear Force Calculation: (Sum of vertical forces)

SF at B = + 2kN

SF at C = + 2 – 5 = - 3 kN

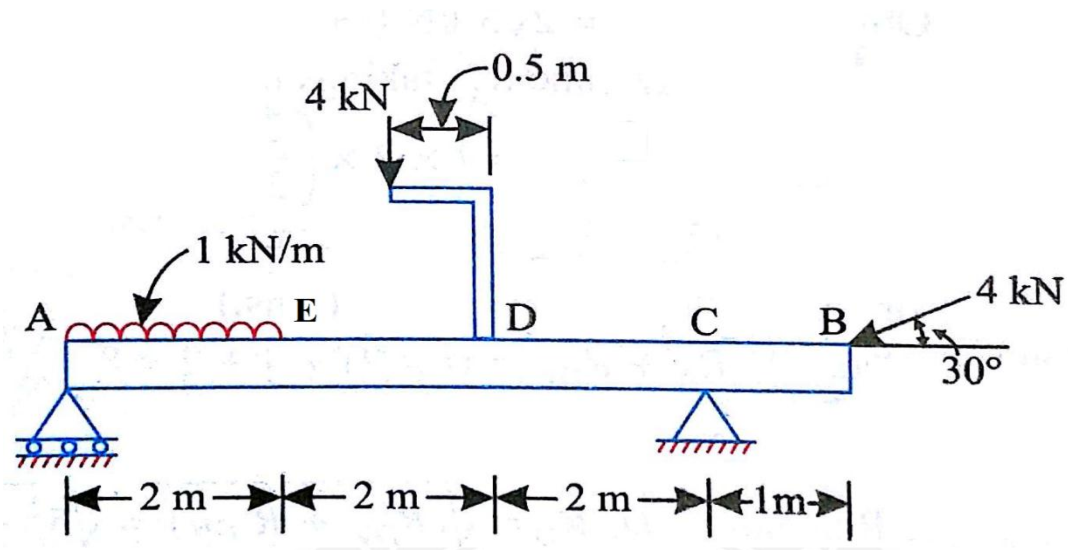
SF at D = + 2 – 5 + 4 = + 1kN

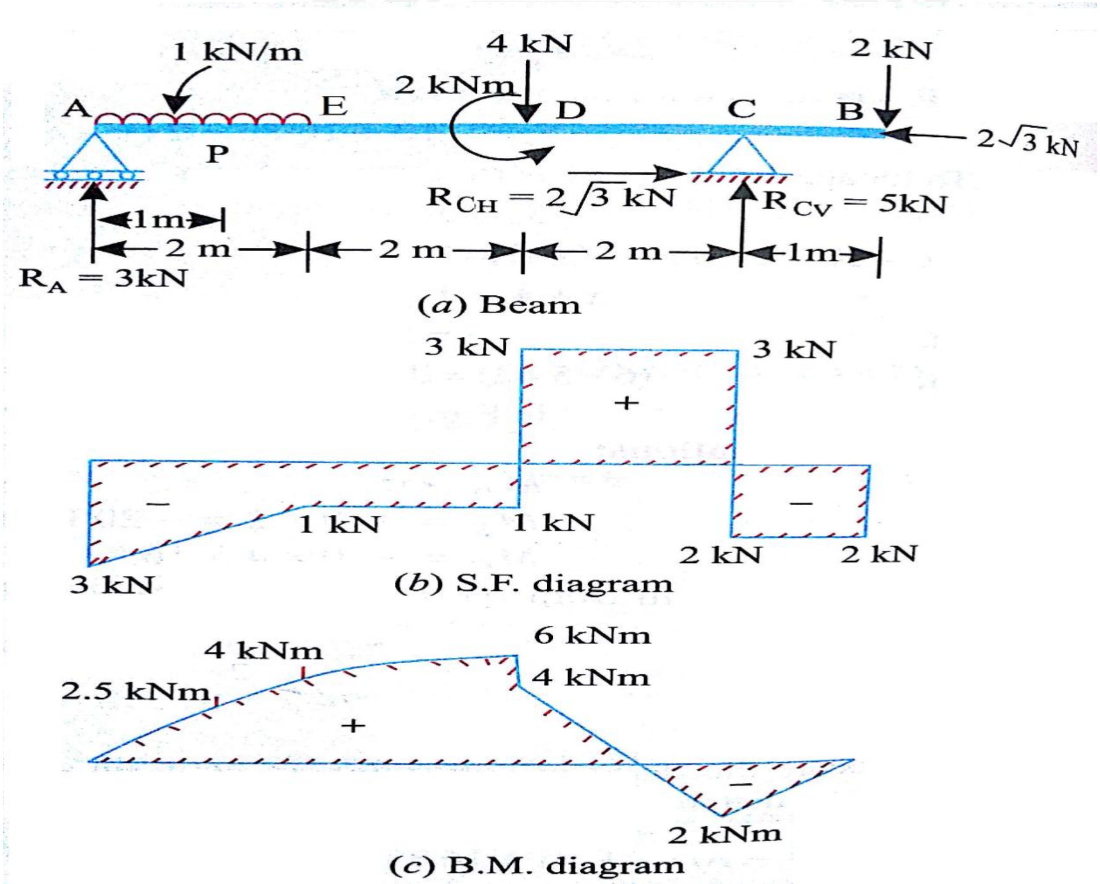
SF at E = + 2 – 5 + 4 = + 1kN

SF at A = + 2 – 5 + 4 + (1 x 2) = + 3 kN

Shear Force Diagram:

Vertical upward reaction and downward load are drawn as vertical line based on sign. UDL are drawn as inclined line based on their sign.





(a) Beam
(b) S.F. diagram
(c) B.M. diagram

Bending moment

Calculation: [Sum of (Vertical force x Distance of load acting from required section)]

BM at B = - (2 x 0) = 0kNm

BM at C = - (2 x 1) + (5 x 0) = - 2kNm

BM at D = - (2 x 3) + (5 x 2) -(4 x 0) +2 = + 6kNm

BM at E = - (2 x 5) + (5 x 4) - (4 x 2)+ 2 = + 4 kNm

BM at E = - (2 x 7) + (5 x 6) - (4 x 4) + 2 – (1 x 2 x $\frac{2}{2}$) = 0 kNm

Bending moment Diagram:

PL are drawn as inclined line and UDL drawn as parabolic curved line based on their sign. Couples are drawn as vertical line

Calculate the position of point of contraflexure or point of inflexion:

The point of contraflexure or point of inflexion lie at a point where BM become zero. In the fig. the BM act at X at a distance x m from B.

BM at X = - (2 x x) + [5 x (x-1)] = 0 kNm

= - 2.x +5.x -5 = 0 ≫ 3x = 5 ≫ x = 5/3 = 1.67m

Result:

The SFD, BMD are drawn as shown in fig.

Reaction forces $R_A = 3$ kN

R_C = 6.08 kN which makes an angle with horizontal θ= 55.3^{o}

Point of contraflexure act at a distance of **1.67m** from B