3.4 SIMILITUDES AND MODEL LAWS

Similitude is basically defined as the similarity between model and its prototype in each and every respect. It suggests us that model and prototype will have similar properties or we can say that similitude explains that model and prototype will be completely similar.

Three types of similarities must exist between model and prototype and these similarities are as mentioned here.

Geometric similarity

Kinematic similarity

Dynamic similarity

Geometric similarity

Geometric similarity is the similarity of shape. Geometric similarity is said to exist between model and prototype, if the ratio of all respective linear dimension in model and prototype are equal.

Ratio of dimension of model and corresponding dimension of prototype will be termed as scale ratio i.e. Lr.

Let us assume the following linear dimension in model and prototype.

$$\boxed{\frac{L_P}{L_m} = \frac{B_P}{B_m} = \frac{D_P}{D_m} = L_r} \qquad \boxed{\frac{A_P}{A_m} = L_r^2} \qquad \boxed{\frac{V_P}{V_m} = L_r^3}$$

where L_r is Scale Ratio

L_m = Length of model, L_P = Length of prototype

B_m = Breadth of model, B_P = Breadth prototype

D_m = Diameter of model, D_P = Diameter of prototype

A_m = Area of model, A_P = Area of prototype

V_m = Volume of model, V_P = Volume of prototype

Kinematic Similarity

The Kinemetic similarity is said to exist between model and prototype, if the ratios of velocity and acceleration at a point in model and at the respective point in the prototype are the same.

We must note it here that the direction of velocity and acceleration in the model and prototype must be identical.

$$\boxed{\frac{V_P}{V_m} = V_r}$$

$$\boxed{\frac{a_P}{a_m} = a_r}$$

where V_r is Velocity Ratio

where a_r is Acceleration Ratio

V_m = Velocity of fluid at a point in model

V_P = Velocity of fluid at respective point in prototype

a_m = Acceleration of fluid at a point in model

a_P = Acceleration of fluid at respective point in prototype

Dynamic Similarity

The dynamic similarity is said to exist between model and prototype, if the ratios of corresponding forces acting at the corresponding points are the same.

We must note it here that the direction of forces at the corresponding points in the model and prototype must be same.

$$\boxed{\frac{F_P}{F_m} = F_r}$$

where F_r is Force Ratio

F_m = Force at a point in model, F_P = Force at respective point in prototype

Model laws or similarity laws

For the dynamic similarity between the model and the prototype, ratio of corresponding forces acting on corresponding points in the model and the prototype should be same.

Ratios of the forces are dimensionless numbers. Therefore we can say that for the dynamic similarity between the model and the prototype, dimensionless numbers should be equal for the model and the prototype.

However, it is quite difficult to satisfy the condition that all the dimensionless numbers should be equal for the model and the prototype.

However for practical problems, it is observed that one force will be most significant as compared to others and that force is considered as predominant force. Therefore for dynamic similarity, predominant force will be considered in practical problems.

Therefore, models are designed on the basis of ratio of force which is dominating in the phenomenon.

Hence, we can define the model laws or similarity laws as the law on which models are designed for the dynamic similarity.

There are following types of model laws

Reynold's Model law

Froude Model law

Euler Model law

Weber Model law

Mach Model law

Reynold's Model law

Reynold's model law could be defined as a model law or similarity law where models are designed on the basis of Reynold's numbers.

According to the Reynold's model law, for the dynamic similarity between the model and the prototype, Reynold's number should be equal for the model and the prototype.

In simple, we can say that Reynold's number for the model must be equal to the Reynold's number for the prototype.

As we know that Reynold's number is basically the ratio of inertia force and viscous force, therefore a fluid flow situation where viscous forces are alone predominant, models will be designed on the basis of Reynold's model law for the dynamic similarity between the model and the prototype.

$$(\mathrm{Re})_P = (\mathrm{Re})_m \; or \; \frac{V_P L_P}{\upsilon_P} = \frac{V_m L_m}{\upsilon_m}$$

Where,

V_m = Velocity of the fluid in the model

L_m = Length of the model

ν_m = Kinematic viscosity of the fluid in the model

V_P = Velocity of the fluid in the prototype

L_P = Length of the prototype

ν_P = Kinematic viscosity of the fluid in the prototype

Models based on the Reynold's model law

Pipe flow

Resistance experienced by submarines, airplanes etc.

Froude Model law

Froude model law could be defined as a model law or similarity law where models are designed on the basis of Froude numbers.

According to the Froude model law, for the dynamic similarity between the model and the prototype, Froude number should be equal for the model and the prototype.

In simple, we can say that Froude number for the model must be equal to the Froude number for the prototype.

As we know that Froude number is basically the ratio of inertia force and gravity force, therefore a fluid flow situation where gravity forces are alone predominant, models will be designed on the basis of Froude model law for the dynamic similarity between the model and the prototype.

$$(Fe)_P = (Fe)_m \text{ or } \frac{V_P}{\sqrt{g_P L_P}} = \frac{V_m}{\sqrt{g_m L_m}}$$

Where,

V_m = Velocity of the fluid in the model

L_m = Length of the model

g_m = Acceleration due to gravity at a place where model is tested

V_P = Velocity of the fluid in the prototype

L_P = Length of the prototype

g_P = Acceleration due to gravity at a place where prototype is tested

Models based on the Froude model law

Free surface flows such as flow over spillways, weirs, sluices, channels etc,

Flow of jet from an orifice or from a nozzle,

Where waves are likely to be formed on surface

Where fluids of different densities flow over one another

Euler's Model law

Euler's model law could be defined as a model law or similarity law where models are designed on the basis of Euler's numbers.

According to the Euler's model law, for the dynamic similarity between the model and the prototype, Euler's number should be equal for the model and the prototype.

In simple, we can say that Euler's number for the model must be equal to the Euler's number for the prototype.

As we know that Euler's number is basically the ratio of pressure force and inertia force, therefore a fluid flow situation where pressure forces are alone predominant, models will be designed on the basis of Euler's model law for the dynamic similarity between the model and the prototype.

$$\frac{V_m}{\sqrt{p_m / \rho_m}} = \frac{V_p}{\sqrt{p_p / \rho_p}}$$

Where,

V_m = Velocity of the fluid in the model

P_m = Pressure of fluid in the model

ρ_m = Density of the fluid in the model

V_P = Velocity of the fluid in the prototype

P_P = Pressure of fluid in the prototype

ρ_P = Density of the fluid in the prototype

Models based on the Euler's model law

Euler's model law will be applicable for a fluid flow situation where flow is taking place in a closed pipe, in which case turbulence will be fully developed so that viscous forces will be negligible and gravity force and surface tension force will be absent.

Weber Model law

Weber model law could be defined as a model law or similarity law where models are designed on the basis of Weber numbers.

According to the Weber model law, for the dynamic similarity between the model and the prototype, Weber number should be equal for the model and the prototype.

In simple, we can say that Weber number for the model must be equal to the Weber number for the prototype.

As we know that Weber number is basically the ratio of inertia force and surface tension force, therefore a fluid flow situation where surface tension forces are alone predominant, models will be designed on the basis of Weber model law for the dynamic similarity between the model and the prototype.

$$\frac{V_m}{\sqrt{\sigma_m / \rho_m L_m}} = \frac{V_p}{\sqrt{\sigma_p / \rho_p L_p}}$$

Where,

V_m = Velocity of the fluid in the model

σ_m = Surface tension force in the model

ρ_m = Density of the fluid in the model

L_m = Length of surface in the model

V_P = Velocity of the fluid in the prototype

σ_P = Surface tension force in the prototype

ρ_P = Density of the fluid in the prototype

L_P = Length of surface in the prototype

Models based on the Weber model law

Capillary rise in narrow passage

Capillary movement of water in soil

Capillary waves in channels

Flow over weirs for small heads

Mach Model law

Mach model law could be defined as a model law or similarity law where models are designed on the basis of Mach numbers.

According to the Mach model law, for the dynamic similarity between the model and the prototype, Mach number should be equal for the model and the prototype.

In simple, we can say that Mach number for the model must be equal to the Mach number for the prototype.

As we know that Mach number is basically the ratio of inertia force and Elastic force, therefore a fluid flow situation where elastic forces are alone predominant, models will be designed on the basis of Mach model law for the dynamic similarity between the model and the prototype.

$$\frac{V_m}{\sqrt{K_m / \rho_m}} = \frac{V_P}{\sqrt{K_p / \rho_p}}$$

Where,

V_m = Velocity of the fluid in the model

K_m = Elastic stress for model

ρ_m = Density of the fluid in the model

V_P = Velocity of the fluid in the prototype

K_P = Elastic stress for prototype

ρ_P = Density of the fluid in the prototype

Models based on the Mach model law

Water hammer problems

Under water testing of torpedoes

Aerodynamic testing

Flow of aeroplane and projectile through air at supersonic speed