## UNIT – III– SOLUTION OF EQUATIONS AND EIGEN VALUE PROBLEMS

**Problems based on** Gauss-Jacobi method and Gauss-Seidal method

**Iterative methods**

The types of iterative methods are

(i) Gauss-Jacobi method

(ii) Gauss-Seidal method

**1.Solve the system of equations by (i) Gauss- Jacobi Method (ii) Gauss-Seidal Method** $\mathbf{27x + 6y - z = 85}$, $\mathbf{x + y + 54z = 110}$, $\mathbf{6x + 15y + 2z = 72}$

**Solution:**

As the coefficient matrix is not diagonally dominant we rewrite the equations

$$27x + 6y - z = 85, 6x + 15y + 2z = 72,$$

$x + y + 54z = 110,$

Since the diagonal elements are dominant in the coefficient matrix, we write, x, y, z as follows:

$$x = \frac{1}{27}[85 - 6y + z]$$

$$y = \frac{1}{15}[72 - 6x - 2z]$$

$$z = \frac{1}{54}[110 - x - y]$$

**Gauss- Jacobi Method**

Let the initial values be $x = 0, y = 0, z = 0$

**First iteration**

$$x^{(1)} = \frac{1}{27}[85] = 3.148$$

$$y^{(1)} = \frac{1}{15}[72] = 4.8$$

$$z^{(1)} = \frac{1}{54}[110] = 2.037$$

**Second iteration**

$$x^{(2)} = \frac{1}{27}\left[85 - 6y^{(1)} + z^{(1)}\right] = \frac{1}{27}[85 - 6(4.8) + (2.037)] = 2.157$$

$$y^{(2)} = \frac{1}{15}\left[72 - 6x^{(1)} - 2z^{(1)}\right] = \frac{1}{15}[72 - 6(3.148) - 2(2.037)] = 3.269$$

$$z^{(2)} = \frac{1}{54}\left[110 - x^{(1)} - y^{(1)}\right] = \frac{1}{54}[110 - 3.148 - 4.8] = 1.890$$

**Third iteration**

$$x^{(3)} = \frac{1}{27}\left[85 - 6y^{(2)} + z^{(2)}\right] = \frac{1}{27}[85 - 6(3.269) + (1.890)] = 2.492$$

$$y^{(3)} = \frac{1}{15}\left[72 - 6x^{(2)} - 2z^{(2)}\right] = \frac{1}{15}[72 - 6(2.157) - 2(1.890)] = 3.685$$

$$z^{(3)} = \frac{1}{54}\left[110 - x^{(2)} - y^{(2)}\right] = \frac{1}{54}[110 - 2.157 - 3.269] = 1.937$$

**Fourth iteration**

$$x^{(4)} = \frac{1}{27}\left[85 - 6y^{(3)} + z^{(3)}\right] = \frac{1}{27}[85 - 6(3.685) + (1.937)] = 2.401$$

$$y^{(4)} = \frac{1}{15}\left[72 - 6x^{(3)} - 2z^{(3)}\right] = \frac{1}{15}[72 - 6(2.492) - 2(1.937)] = 3.545$$

$$z^{(4)} = \frac{1}{54}\left[110 - x^{(3)} - y^{(3)}\right] = \frac{1}{54}[110 - 2.492 - 3.685] = 1.923$$

**Fifth iteration**

$$x^{(5)} = \frac{1}{27}\left[85 - 6y^{(4)} + z^{(4)}\right] = \frac{1}{27}[85 - 6(3.545) + (1.923)] = 2.432$$

$$y^{(5)} = \frac{1}{15}\left[72 - 6x^{(4)} - 2z^{(4)}\right] = \frac{1}{15}[72 - 6(2.401) - 2(1.923)] = 3.583$$

$$z^{(5)} = \frac{1}{54}\left[110 - x^{(4)} - y^{(4)}\right] = \frac{1}{54}[110 - 2.401 - 3.545] = 1.927$$

**Sixth iteration**

$$x^{(6)} = \frac{1}{27}\left[85 - 6y^{(5)} + z^{(5)}\right] = \frac{1}{27}[85 - 6(3.583) + (1.927)] = 2.423$$

$$y^{(6)} = \frac{1}{15}\left[72 - 6x^{(5)} - 2z^{(5)}\right] = \frac{1}{15}[72 - 6(2.4332) - 2(1.927)] = 3.570$$

$$z^{(6)} = \frac{1}{54}\left[110 - x^{(5)} - y^{(5)}\right] = \frac{1}{54}[110 - 2.432 - 3.583] = 1.926$$

**Seventh iteration**

$$x^{(7)} = \frac{1}{27}\left[85 - 6y^{(6)} + z^{(6)}\right] = \frac{1}{27}[85 - 6(3.570) + (1.926)] = 2.426$$

$$y^{(7)} = \frac{1}{15}\left[72 - 6x^{(6)} - 2z^{(6)}\right] = \frac{1}{15}[72 - 6(2.423) - 2(1.926)] = 3.574$$

$$z^{(7)} = \frac{1}{54}\left[110 - x^{(6)} - y^{(6)}\right] = \frac{1}{54}[110 - 2.423 - 3.570] = 1.926$$

**Eighth iteration**

$$x^{(8)} = \frac{1}{27}\left[85 - 6y^{(7)} + z^{(7)}\right] = \frac{1}{27}[85 - 6(3.574) + (1.926)] = 2.425$$

$$y^{(8)} = \frac{1}{15}\left[72 - 6x^{(7)} - 2z^{(7)}\right] = \frac{1}{15}[72 - 6(2.426) - 2(1.926)] = 3.573$$

$$z^{(8)} = \frac{1}{54}\left[110 - x^{(7)} - y^{(7)}\right] = \frac{1}{54}[110 - 2.426 - 3.574] = 1.926$$

**Ninth iteration**

$$x^{(9)} = \frac{1}{27}\left[85 - 6y^{(8)} + z^{(8)}\right] = \frac{1}{27}[85 - 6(3.573) + (1.926)] = 2.426$$

$$y^{(9)} = \frac{1}{15}\left[72 - 6x^{(8)} - 2z^{(8)}\right] = \frac{1}{15}[72 - 6(2.425) - 2(1.926)] = 3.573$$

$$z^{(9)} = \frac{1}{54}\left[110 - x^{(8)} - y^{(8)}\right] = \frac{1}{54}[110 - 2.425 - 3.573] = 1.926$$

**Tenth iteration**

$$x^{(10)} = \frac{1}{27}\left[85 - 6y^{(9)} + z^{(9)}\right] = \frac{1}{27}[85 - 6(3.573) + (1.926)] = 2.426$$

$$y^{(10)} = \frac{1}{15}\left[72 - 6x^{(9)} - 2z^{(9)}\right] = \frac{1}{15}[72 - 6(2.426) - 2(1.926)] = 3.573$$

$$z^{(10)} = \frac{1}{54}\left[110 - x^{(9)} - y^{(9)}\right] = \frac{1}{54}[110 - 2.426 - 3.573] = 1.926$$

Hence, $x = 2.426, y = 3.573, z = 1.926,$

Correct to three decimal places.

**Gauss- Seidal Method**

Let the initial values be $y = 0, z = 0$

**First iteration**

$$x^{(1)} = \frac{1}{27}\left[85 - 6y^{(0)} + z^{(0)}\right] = \frac{1}{27}[85 - 6(0) + (0)] = 3.148$$

$$y^{(1)} = \frac{1}{15}\left[72 - 6x^{(1)} - 2z^{(0)}\right] = \frac{1}{15}[72 - 6(3.148) - 0] = 3.541$$

$$z^{(1)} = \frac{1}{54}\left[110 - x^{(1)} - y^{(1)}\right] = \frac{1}{54}[110 - 3.148 - 3.541] = 1.913$$

**Second iteration**

$$x^{(2)} = \frac{1}{27}\left[85 - 6y^{(1)} + z^{(1)}\right] = \frac{1}{27}[85 - 6(3.541) + (1.913)] = 2.432$$

$$y^{(2)} = \frac{1}{15}\left[72 - 6x^{(2)} - 2z^{(1)}\right] = \frac{1}{15}[72 - 6(2.432) - 2(1.913)] = 3.572$$

$$z^{(2)} = \frac{1}{54}\left[110 - x^{(2)} - y^{(2)}\right] = \frac{1}{54}[110 - 2.432 - 3.572] = 1.926$$

**Third iteration**

$$x^{(3)} = \frac{1}{27}\left[85 - 6y^{(2)} + z^{(2)}\right] = \frac{1}{27}[85 - 6(3.572) + (1.926)] = 2.426$$

$$y^{(3)} = \frac{1}{15}\left[72 - 6x^{(3)} - 2z^{(2)}\right] = \frac{1}{15}[72 - 6(2.426) - 2(1.926)] = 3.573$$

$$z^{(3)} = \frac{1}{54}\left[110 - x^{(3)} - y^{(3)}\right] = \frac{1}{54}[110 - 2.426 - 3.573] = 1.926$$

**Fourth iteration**

$$x^{(4)} = \frac{1}{27}\left[85 - 6y^{(3)} + z^{(3)}\right] = \frac{1}{27}[85 - 6(3.573) + (1.926)] = 2.426$$

$$y^{(4)} = \frac{1}{15}\left[72 - 6x^{(4)} - 2z^{(3)}\right] = \frac{1}{15}[72 - 6(2.426) - 2(1.926)] = 3.573$$

$$z^{(4)} = \frac{1}{54}\left[110 - x^{(4)} - y^{(4)}\right] = \frac{1}{54}[110 - 2.426 - 3.573] = 1.926$$

Hence, $x = 2.426, y = 3.573, z = 1.926,$

**2.Solve the system of equations by Gauss-Seidal Method**

$$\mathbf{4x + 2y + z = 85},\ \ \mathbf{x + 5y - z = 110},\ \ \ \mathbf{x + y + 8z = 20}$$

**Solution:**

As the coefficient matrix is diagonally dominant solving x,y,z, we get

$$x = \frac{1}{4}[14 - 2y - z]$$

$$y = \frac{1}{5}[10 - x + z]$$

$$z = \frac{1}{8}[20 - x - y]$$

Let the initial values be $y = 0, z = 0$

**First iteration**

$$x^{(1)} = \frac{1}{4}\left[14 - 2y^{(0)} - z^{(0)}\right] = \frac{1}{4}[14 - 2(0) - (0)] = 3.5$$

$$y^{(1)} = \frac{1}{5}\left[10 - x^{(1)} + z^{(0)}\right] = \frac{1}{5}[10 - 3.5 + 0] = 1.3$$

$$z^{(1)} = \frac{1}{8}\left[20 - x^{(1)} - y^{(1)}\right] = \frac{1}{8}[20 - 3.5 - 1.3] = 1.9$$

**Second iteration**

$$x^{(2)} = \frac{1}{4}\left[14 - 2y^{(1)} - z^{(1)}\right] = \frac{1}{4}[14 - 2(1.3) - (1.9)] = 2.375$$

$$y^{(2)} = \frac{1}{5}\left[10 - x^{(2)} + z^{(1)}\right] = \frac{1}{5}[10 - 2.375 + 1.9] = 1.905$$

$$z^{(2)} = \frac{1}{8}\left[20 - x^{(2)} - y^{(2)}\right] = \frac{1}{8}[20 - 2.375 - 1.905] = 1.965$$

**Third iteration**

$$x^{(3)} = \frac{1}{4}\left[14 - 2y^{(2)} - z^{(2)}\right] = \frac{1}{4}[14 - 2(1.905) - (1.965)] = 2.056$$

$$y^{(3)} = \frac{1}{5}\left[10 - x^{(3)} + z^{(2)}\right] = \frac{1}{5}[10 - 2.056 + 1.965] = 1.982$$

$$z^{(3)} = \frac{1}{8}\left[20 - x^{(3)} - y^{(3)}\right] = \frac{1}{8}[20 - 2.056 - 1.982] = 1.995$$

**Fourth iteration**

$$x^{(4)} = \frac{1}{4}\left[14 - 2y^{(3)} - z^{(3)}\right] = \frac{1}{4}[14 - 2(1.982) - (1.995)] = 2.010$$

$$y^{(4)} = \frac{1}{5}\left[10 - x^{(4)} + z^{(3)}\right] = \frac{1}{5}[10 - 2.010 + 1.995] = 1.997$$

$$z^{(4)} = \frac{1}{8}\left[20 - x^{(4)} - y^{(4)}\right] = \frac{1}{8}[20 - 2.010 - 1.997] = 1.999$$

**Fifth iteration**

$$x^{(5)} = \frac{1}{4}\left[14 - 2y^{(4)} - z^{(4)}\right] = \frac{1}{4}[14 - 2(1.997) - (1.999)] = 2.002$$

$$y^{(5)} = \frac{1}{5}\left[10 - x^{(5)} + z^{(4)}\right] = \frac{1}{5}[10 - 2.002 + 1.999] = 1.999$$

$$z^{(5)} = \frac{1}{8}\left[20 - x^{(5)} - y^{(5)}\right] = \frac{1}{8}[20 - 2.002 - 1.999] = 2$$

**Sixth iteration**

$$x^{(6)} = \frac{1}{4}\left[14 - 2y^{(5)} - z^{(5)}\right] = \frac{1}{4}[14 - 2(1.999) - (2)] = 2.001$$

$$y^{(6)} = \frac{1}{5}\left[10 - x^{(6)} + z^{(5)}\right] = \frac{1}{5}[10 - 2.001 + 2] = 2$$

$$z^{(6)} = \frac{1}{8}\left[20 - x^{(6)} - y^{(6)}\right] = \frac{1}{8}[20 - 2.001 - 2] = 2$$

**Seventh iteration**

$$x^{(7)} = \frac{1}{4}\left[14 - 2y^{(6)} - z^{(6)}\right] = \frac{1}{4}[14 - 2(2.001) - (2)] = 2$$

$$y^{(7)} = \frac{1}{5}\left[10 - x^{(7)} + z^{(6)}\right] = \frac{1}{5}[10 - 2 + 2] = 2$$

$$z^{(7)} = \frac{1}{8}\left[20 - x^{(7)} - y^{(7)}\right] = \frac{1}{8}[20 - 2 - 2] = 2$$

**Eighth iteration**

$$x^{(8)} = \frac{1}{4}\left[14 - 2y^{(7)} - z^{(7)}\right] = \frac{1}{4}[14 - 2(2) - (2)] = 2$$

$$y^{(8)} = \frac{1}{5}\left[10 - x^{(8)} + z^{(7)}\right] = \frac{1}{5}[10 - 2 + 2] = 2$$

$$z^{(8)} = \frac{1}{8}\left[20 - x^{(8)} - y^{(8)}\right] = \frac{1}{8}[20 - 2 - 2] = 2$$

Hence, $x = 2, y = 2, z = 2$.