2.3 LATIN SQUARE:

Steps in constructing Latin Square

Step:1

Square the Grand total (T) and divide it by the number of observations (N).

i.e), Find $\frac{T^2}{N}$ which is called the correction factor (C.F)

Step:2

Add the squares of the individual observations ($X_i's$) and substract the C.F from it to get the total sum of squares. i.e)., Find Total sum of squares TSS

i.e)., TSS $=\sum_i (X_i)^2 - \frac{T^2}{N}$

Step:3

Add the squares of the row sums (R_i) divide it by the number of items in a row and substract the C.F from the result to get the row sum of squares.

Row sum of squares $SSR = \frac{(\sum R_i)^2}{n_1} - C.F$

Where n_1 is the number of items in a row.

Step:4

Add the squares of the columns sums (C_i) divide it by the number of items and substract the C.F from the result to get the column sum of squares.

Column sum of squares $SSC = \frac{(\sum C_j)^2}{n_2} - C.F$

Where n_2 is the number of items in a column.

Step:5

Sum of the squares of the treatment sums (T_i) divide it by the number of treatments and substract the C.F from it to get the treatment sum of squares, i.e., Treatment sum of squares.

$$SST = \frac{(\sum T_i)^2}{n_i} - C.F$$

Where n_i is the number of treatments.

Step:6

Substract the sum obtained in steps 3, 4, and 5 from 2 we get residual.

i.e)., Residual $SSE = TSS - (SSR + SSC + SST)$

Step:7

Prepare the ANOVA table using all these and calculate the various mean squares as follows.

Source of variation	Sum of Degrees	Degrees of Freedom	Mean Square	F - Ratio
Between Rows	SSR	n – 1	MSR = $\frac{SSR}{n-1}$	$F_R = \frac{MSR}{MSE}$ if MSR> MSE $F_R = \frac{MSE}{MSR}$ if MSE> MSR
Between Columns	SSC	n - 1	MSC = $\frac{SSC}{n-1}$	$F_c = \frac{MSC}{MSE}$ if MSC> MSE $F_c = \frac{MSE}{MSC}$ if MSE> MSC
Treatments	SST	n - 1	MST = $\frac{SST}{n-1}$	$F_T = \frac{MST}{MSE}$ if MST> MSE $F_T = \frac{MSE}{MST}$ if MSE> MST
Residual or Error	SSE	(n – 1)(n – 2)	MSE = $\frac{SSE}{(n-1)(n-2)}$	

Step:8

Compute the F-ratio and find out whether the differences are significant or not according to the given level of significance.

1. Set up the analysis of variance for the following results of a Latin square design.

A 12	**C 19**	**B 10**	**D 8**
C 18	**B 12**	**D 6**	**A 7**
B 22	**D 10**	**A 5**	**C 21**
C 12	**A 7**	**C 27**	**B 17**

Solution:

Set the null hypothesis H_0: There is no significance difference between the rows, columns and treatments.

Table I (To find TSS, SSR and SSC)

	C_1	C_2	C_3	C_4	Row Total R_i	${R_i}^2/4$
R_1	12	19	10	8	49	600.25
R_2	18	12	6	7	43	462.25
R_3	22	10	5	21	58	841
R_4	12	7	27	17	63	992.25
Column Total C_j	64	48	48	53	213 (T)	2895.75 $\sum {R_i}^2/4$
${C_j}^2/4$	1024	576	576	702.25	2895.75 $\sum {C_j}^2/4$	

Table II (To find SST)

	1	2	3	4	Row Total T_i	${T_i}^2/4$
A	12	7	5	7	31	240.25
B	10	12	22	17	61	930.25
C	19	18	21	27	85	1806.25
D	8	6	10	12	36	324
						3300.75= $\sum {T_i}^2/4$

Step:1

Grand total (T) =213

Step:2

Correction factor (C.F)= $\frac{T^2}{N} = \frac{(213)^2}{16} = 2835.56$

Step:3

Sum of squares of individual observations

$$= (\mathbf{12})^2 + (\mathbf{7})^2 + (\mathbf{5})^2 + (\mathbf{7})^2 + (\mathbf{10})^2 + (\mathbf{12})^2 + (\mathbf{22})^2 + (\mathbf{17})^2 + (\mathbf{19})^2 + (\mathbf{18})^2 + (\mathbf{21})^2 + (\mathbf{27})^2 + (\mathbf{8})^2 + (\mathbf{6})^2 + (\mathbf{10})^2 + (\mathbf{12})^2$$

$$= \mathbf{3483}$$

Step:4

TSS =$sum\ of\ squares\ of\ individual\ observations - C.F$

$$= \sum_i (X_i)^2 - \frac{T^2}{N} = 3486 - 2835.56 = 647.44$$

Step:5

Row sum of squares $SSR = \frac{(\sum R_i)^2}{4} - C.F = 2895.75 - 2835.56 = 60.19$

Step:6

Column sum of squares $SSC = \frac{(\sum C_j)^2}{4} - C.F = 2878.25 - 2835.56$

$$= 42.69$$

Step:7

Sum of squares of Treatment

$$SST = \frac{(\sum T_i)^2}{n_i} - C.F = 3300.75 - 2835.56 = 465.19$$

Step:8

Residual $SSE = TSS - (SSR + SSC + SST)$

$$= 647.44 - (60.19 + 42.69 + 465.19) = 79.37$$

Step:9

Prepare the ANOVA table using all these and calculate the various mean squares as follows.

Source of variation	Sum of Degrees	Degrees of Freedom	Mean Square	F - Ratio

Between Rows	SSR=60.19	4 – 1 =3	MSR = $\frac{SSR}{n-1}$ =20.06	$F_R = \frac{MSR}{MSE}$=1.52
Between Columns	SSC=42.69	4 – 1 =3	MSC = $\frac{SSC}{n-1}$ =14.23	$F_c = \frac{MSC}{MSE}$ =1.08
Treatments	SST=465.19	4 – 1 =3	MST = $\frac{SST}{n-1}$ =155.06	$F_T = \frac{MST}{MSE}$ =11.73
Residual or Error	SSE=79.37	(4 – 1)(4 – 2) =6	MSE = $\frac{SSE}{(n-1)(n-2)}$ =13.22	

Step: 10

d.f for (3, 6) at 5% level of significance is 4.76

Step: 11 Conclusion:

Calculated value $F_c <$ Table value, then we accept null hypothesis.

There is no significance difference between the columns.

Calculated value $F_R <$Table value, then we accept null hypothesis.

There is no significance difference between the rows.

Calculated value $F_T >$Table value, then we reject null hypothesis.

There is a significance difference between the rows.