1.6 ELECTRIC FIELD DUE TO DISCRETE AND CONTINUOUS CHARGES

If the charge are distributed instead of concentrated at one point, it is better to define charge distribution in terms of charge density. It is also possible to have continuous charge distribution along line, on a surface or in a volume.

It is customary to denote the line charge density by $\boldsymbol{\rho_l}$ in$(\boldsymbol{C/m})$, surface charge density by $\boldsymbol{\rho_s}$ in $(\boldsymbol{C/m^2})$ and volume charge density by $\boldsymbol{\rho_v}$ in $(\boldsymbol{C/m^3})$ respectively.

LINE OR LINEAR CHARGE DENSITY:

It is defined as the total charge distributed over a line or curve.

$$\boldsymbol{\rho_l = \lim_{\Delta l \to 0} \left(\frac{\Delta Q}{\Delta l}\right)}$$

This gives the total charge per length. It is given by

$$\boldsymbol{\rho_l = \frac{Q}{l} \quad Coulomb/meter(c/m)}$$

SURFACE CHARGE DENSITY:

It is defined as the total charge distributed over a surface.

$$\boldsymbol{\rho_s = \lim_{\Delta s \to 0} \left(\frac{\Delta Q}{\Delta s}\right)}$$

This gives the total charge per area. It is given by

$$\boldsymbol{\rho_s = \frac{Q}{S} = \frac{Q}{A} \quad Coulomb/squaremeter(c/m^2)}$$

VOLUME CHARGE DENSITY:

It is defined as the total charge distributed over a volume.

$$\boldsymbol{\rho_{sv} = \lim_{\Delta v \to 0} \left(\frac{\Delta Q}{\Delta v}\right)}$$

This gives the total charge per volume. It is given by

$$\boldsymbol{\rho_v = \frac{Q}{V} \quad Coulomb/cubicmeter(c/m^3)}$$

ELECTRIC FILED OR ELECTRIC FIELD INTENSITY:

The electric field or electric field intensity is defined as the electric force per unit charge .It is given by

$$\boldsymbol{E = \frac{F}{q}}$$

According to coulomb's law

$$\boldsymbol{F = \frac{Qq}{4\pi\varepsilon r^2}}$$

Electric Filed

$$\boldsymbol{E = \frac{F}{q}}$$

Substitute $\boldsymbol{F}$ value in above equation

$$\boldsymbol{E = \frac{\frac{Qq}{4\pi\varepsilon r^2}}{q}}$$

$$\boldsymbol{E = \frac{Q\cancel{q}}{4\pi\varepsilon r^2 \cancel{q}}}$$

$$\boldsymbol{E = \frac{Q}{4\pi\varepsilon r^2}\ V/m}$$

The another unit of electric field is $\boldsymbol{Volts/meter}$

ELECTRIC POTENTIAL DIFFERENCE AND POTENTIAL:

Consider a uniform electric field $\boldsymbol{E}$ and a unit positive charge $\boldsymbol{q}$ .There is a force act on the charge due to the electric field. The force is given by

$$\boldsymbol{F = qE}$$

There is a movement of charge in the electric field from one point $\boldsymbol{r_1}$ to another $\boldsymbol{r_2}$, there will be work done against the force

$$\boldsymbol{W = -\int_{r_1}^{r_2} qE.dr}$$

$$\boldsymbol{W = -q\int_{r_1}^{r_2} E.dr}$$

Potential difference ($\boldsymbol{V}$) is defined as the work done in moving a unit positive charge from one point to another in an electric field.

Work done on unit positive charge per charges

$$V = \frac{W}{q}$$

$$V = -\int_{r_1}^{r_2} E.dr \quad Joules/Coulomb$$

But

$$E = \frac{Q}{4\pi\varepsilon r^2}$$

Substitute E in V

$$V = -\frac{Q}{4\pi\varepsilon}\int_{r1}^{r2} \frac{1}{r^2}\ dr$$

$$V = -\frac{Q}{4\pi\varepsilon}\left[\frac{1}{r}\right]_{r_1}^{r_2}$$

$$V = -\frac{Q}{4\pi\varepsilon}\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

$$V = \frac{Q}{4\pi\varepsilon}\left[\frac{1}{r_1} - \frac{1}{r_2}\right] \quad Volts$$

This is the potential difference between two points r_1 and r_2

$$V = \left[\frac{Q}{4\pi\varepsilon r_1} - \frac{Q}{4\pi\varepsilon r_2}\right] \quad Volts$$

$$V_1 = \frac{Q}{4\pi\varepsilon r_1}$$

$$V_2 = \frac{Q}{4\pi\varepsilon r_2}$$

$$V = V_1 + V_2$$

If the charge is moving form infinity to a given point in the electric filed

$$V_2 = 0$$

Then

$$V = V_1 + 0$$

$$V = V_1$$

Absolute potential or potential at a point is defined s the work done in moving a unit positive charge from infinity to a given point in an electric field.

$$V = \frac{Q}{4\pi\varepsilon r} \quad Volts$$

Any field where the closed line integral of the field is zero, is said to be a conservative field

$$\boldsymbol{E.dr = 0}$$

Thus the electric field strength at any points just the negative of the potential gradient at that point. The negative sign shows that the direction of E is opposite to the direction in which V increases.

$$\boldsymbol{E = -\nabla V}$$