Uniformly spaced array with uniform excitation amplitude
N Element Uniform Linear Arrays

At higher frequencies, for point to point communications it is necessary to have a pattern with single beam radiation. Such highly directive single beam pattern can be obtained by increasing the point sources in the arrow from 2 to n say. An array of n elements is said to be linear array if all the individual elements are spaced equally along a line. An array is said to be uniform array if the elements in the array are fed with currents with equal magnitudes and with uniform progressive phase shift along the line. Consider a general n element linear and uniform array with all the individual elements spaced equally at distance d from each other and all elements are fed with currents equal in magnitude and uniform progressive phase shift along line as shown in the Fig.
9.

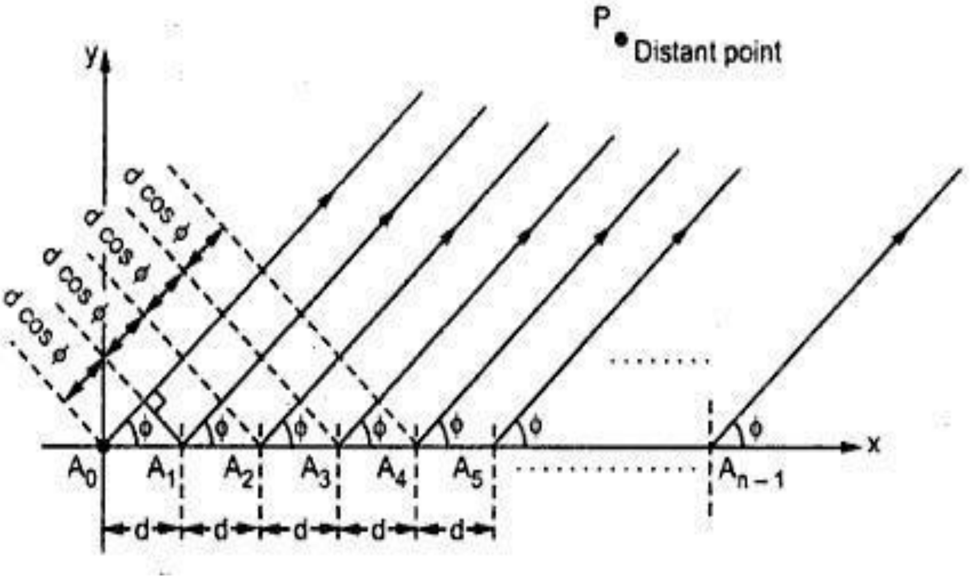


Fig. 9 Uniform, linear array of n elements

The total resultant field at the distant point P is obtained by adding the fields due to n individual sources vectorically. Hence we can write,

$$E_T = E_0 \cdot e^{j0} + E_0\, e^{j\psi} + E_0\, e^{2j\psi} + \ldots + E_0\, e^{j(n-1)\psi}$$

$$\therefore \quad \boxed{E_T = E_0\,[1 + e^{j\psi} + e^{j2\psi} + \ldots + e^{j(n-1)\psi}]} \quad \ldots (1)$$

Note that υ= (βdcosυ + α) indicates the total phase difference of the fields from adjacent sources calculated at point P. Similarly α is the progressive phase shift between two adjacent

0 0 0 point sources. The value of α may lie between 0 and 180 . If α = 0 we get n element uniform

0 linear broadside array. If α = 180 we get n element uniform linear endfire array.

j^{υ}, we get,

Multiplying equation (1) by e

$$E_T e^{j\psi} = E_0 [e^{j\psi} + e^{j2\psi} + e^{j3\psi} + \ldots + e^{jn\psi}] \quad \ldots(2)$$

Subtracting equation (2) from (1), we get,

$$E_T - E_T e^{j\psi} = E_0 \{[1 + e^{j\psi} + e^{j2\psi} + \ldots + e^{j(n-1)\psi}] - [e^{j\psi} + e^{j2\psi} + \ldots + e^{jn\psi}]\}$$

$$E_T (1 - e^{j\psi}) = E_0 (1 - e^{jn\psi})$$

$$\therefore \quad \boxed{E_T = E_0 \left[\frac{1 - e^{jn\psi}}{1 - e^{j\psi}}\right]} \quad \ldots(3)$$

.

Simply mathematically, we get

$$E_T = E_0 \left[\frac{e^{j\frac{n\psi}{2}}\left(e^{-j\frac{n\psi}{2}} - e^{j\frac{n\psi}{2}}\right)}{e^{j\frac{\psi}{2}}\left(e^{-j\frac{\psi}{2}} - e^{j\frac{\psi}{2}}\right)}\right]$$

According to trigonometric identity,

$$e^{-j\theta} - e^{j\theta} = -2j \sin\theta,$$

The resultant field is given by,

$$E_T = E_0 \left[\frac{\left(-j2\sin\frac{n\psi}{2}\right)e^{j\frac{n\psi}{2}}}{\left(-j2\sin\frac{\psi}{2}\right)e^{j\frac{\psi}{2}}}\right]$$

$$\therefore \quad \boxed{E_T = E_0 \left[\frac{\sin n\frac{\psi}{2}}{\sin\frac{\psi}{2}}\right] e^{j\left(\frac{n-1}{2}\right)\psi}} \quad \ldots(4)$$

This equation (4) indicates the resultant field due to n element array at distant point P. The magnitude of the resultant field is given by,

The phase angle θ of the resultant field at point P is given by,

$$\therefore \quad \boxed{\theta = \frac{(n-1)}{2}\psi = \frac{(n-1)}{2}\ \beta d \cos\phi + \alpha} \qquad \dots (6)$$

Array of n elements with Equal Spacing and Currents Equal in Magnitude and Phase •

Broadside Array

Consider 'n' number of identical radiators carries currents which are equal in magnitude and in phase. The identical radiators are equispaced. Hence the maximum radiation occurs in the directions normal to the line of array. Hence such an array is known as Uniform broadside array. Consider a broadside array with n identical radiators as shown in the Fig. 10.

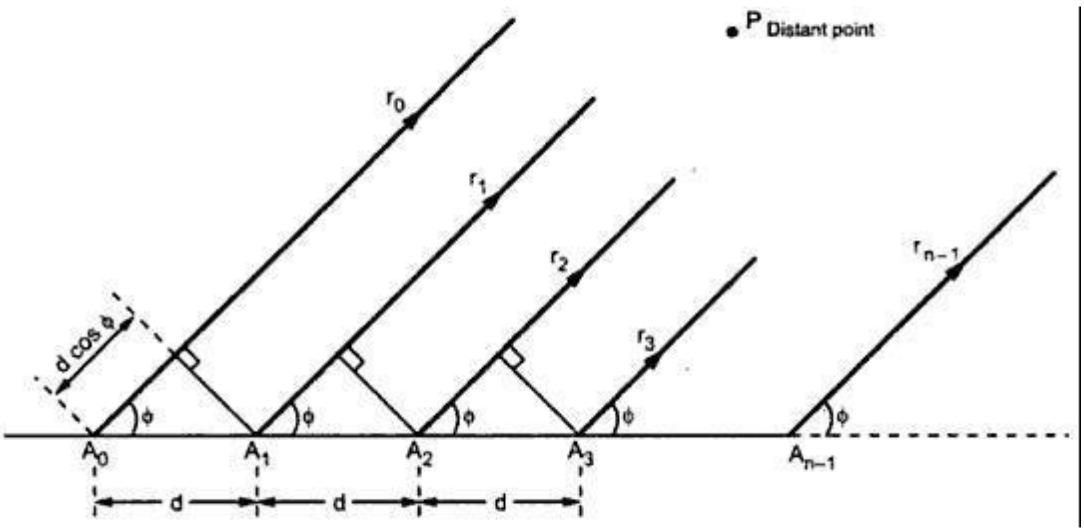


Fig 10 Array of n elements with Equal Spacing

The electric field produced at point P due to an element A0 is given by,

$$E_0 = \frac{I\, dL \sin\theta}{4\pi\omega\varepsilon_0}\left[j\frac{\beta^2}{r_0}\right] e^{-j\beta r_0} \quad \text{... (1)}$$

As the distance of separation d between any two array elements is very small as compared to the radial distances of point P from A_0, A_1, ...A_{n-1}, we can assume r_0, r_1, ...r_{n-1} are approximately same.

Now the electric field produced at point P due to an element A_1 will differ in phase as r_0 and r_1 are not actually same. Hence the electric field due to A_1 is given by,

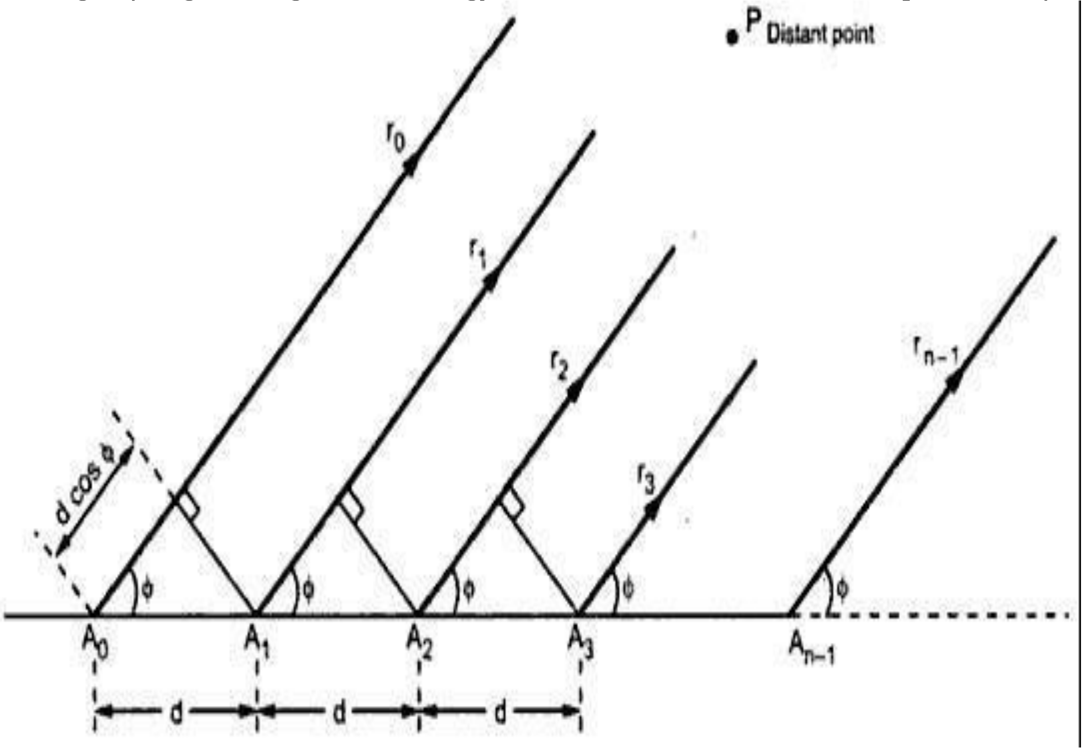
$$\therefore \quad \boxed{E_T = E_0\left[\frac{\sin n\frac{\psi}{2}}{\sin\frac{\psi}{2}}\right]} \quad \text{... (5)}$$

Exactly on the similar lines we can write the electric field produced at point P due to an element A_2 as,

$$E_2 = \frac{I\, dL \sin\theta}{4\pi\omega\varepsilon_0}\left[j\frac{\beta^2}{r_2}\right] e^{-j\beta r_2}$$

$$\therefore \quad E_2 = \frac{I\, dL \sin\theta}{4\pi\omega\varepsilon_0}\left[j\frac{\beta^2}{r_1}\right] e^{-j\beta(r_1 - d\cos\phi)} \quad \text{... } r_2 = r_1 - d\cos\phi$$

$$\therefore \quad E_2 = \left\{\frac{I\, dL \sin\theta}{4\pi\omega\varepsilon_0}\left[j\frac{\beta^2}{r_1}\right] e^{-j\beta r_1}\right\} e^{j\beta d\cos\phi}$$



But the term inside the bracket represent E1

$$\therefore \quad E_2 = E_1 \, e^{j\beta d\cos\phi}$$

From equation (2), substituting the value of E1, we get,

$$E_2 = \left[E_0 \, e^{j\beta d\cos\phi}\right] e^{j\beta d\cos\phi}$$

$$\therefore \quad E_2 = E_0 \cdot e^{j2\beta d\cos\phi} \qquad \text{... (3)}$$

Similarly, the electric field produced at point P due to element An-1 is given by,

$$E_{n-1} = E_0 \cdot e^{j(n-1)\beta d\cos\phi} \qquad \text{... (4)}$$

The total electric field at point P is given by,

$$E_T = E_0 + E_1 + E_2 + ... + E_{n-1}$$

$$\therefore \quad E_T = E_0 + E_0\, e^{j\beta d\cos\phi} + E_0\, e^{j2\beta d\cos\phi} + ... + E_0\, e^{j(n-1)\beta d\cos\phi}$$

Let βdcosυ = υ, then rewriting above equation,

Consider a series given by $s = 1 + r + r^2 + ... + r^{n-1}$ where $r = e^{j\upsilon}$ ... (i)

Multiplying both the sides of the equation (i) by r, $s \cdot r = r + r^2 + ... + r^n$ ... (ii)

Subtracting equation (ii) from (i), we get. $s(1-r) = 1-r^n$

$$E_T = E_0 + E_0\, e^{j\psi} + E_0\, e^{j2\psi} + ... + E_0\, e^{j(n-1)\psi}$$

$$\therefore \quad E_T = E_0\left[1 + e^{j\psi} + e^{j2\psi} + ... + e^{j(n-1)\psi}\right] \quad ...(5)$$

$$\therefore \quad s = \frac{1-r^n}{1-r} \quad ...(iii)$$

Using equation (iii), equation (5) can be modified as,

$$E_T = E_0\left[\frac{1-e^{jn\psi}}{1-e^{j\psi}}\right]$$

$$\therefore \quad \frac{E_T}{E_0} = \frac{e^{jn\frac{\psi}{2}}\left[e^{-jn\frac{\psi}{2}} - e^{jn\frac{\psi}{2}}\right]}{e^{j\frac{\psi}{2}}\left[e^{-j\frac{\psi}{2}} - e^{j\frac{\psi}{2}}\right]} \quad ...(6)$$

From the trigonometric identities,

$$e^{-j\theta} = \cos\theta - j\sin\theta$$

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$\text{and} \quad e^{-j\theta} - e^{j\theta} = -j\,2\sin\theta$$

$$\frac{E_T}{E_0} = \frac{e^{jn\frac{\psi}{2}}\left[-j\,2\sin\left(\frac{n\psi}{2}\right)\right]}{e^{j\frac{\psi}{2}}\left[-j\,2\sin\left(\frac{\psi}{2}\right)\right]}$$

$$\therefore \quad \frac{E_T}{E_0} = e^{j\frac{(n-1)\psi}{2}}\left[\frac{\sin\left(\frac{n\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)}\right] \qquad \text{... (7)}$$

Equation (6) can be written as,

The exponential term in equation (7) represents the phase shift. Now considering magnitudes of the electric fields, we can write,

$$\left|\frac{E_T}{E_0}\right| = \frac{\sin\frac{n\psi}{2}}{\sin\frac{\psi}{2}} \qquad \text{... (8)}$$

Properties of Broadside Array 1. Major lobe

In case of broadside array, the field is maximum in the direction normal to the axis of the array. Thus the condition for the maximum field at point P is given by,

$$\psi = 0 \text{ i.e. } \beta d \cos\phi = 0 \qquad \text{...(9)}$$

$$\text{i.e. } \cos\phi = 0$$

$$\text{i.e. } \boxed{\phi = 90^\circ \text{ or } 270^\circ} \qquad \text{...(10)}$$

Thus $\upsilon = 90^0$ and 270^0 are called directions of principle maxima.

2. Magnitude of major lobe

The maximum radiation occurs when υ=0. Hence we can write,

$$|\text{Major lobe}| = \left|\frac{E_T}{E_0}\right| = \lim_{\psi \to 0}\left\{\frac{\frac{d}{d\psi}\left(\sin n\frac{\psi}{2}\right)}{\frac{d}{d\psi}\left(\sin\frac{\psi}{2}\right)}\right\}$$

$$= \lim_{\psi \to 0}\left\{\frac{\left(\cos n\frac{\psi}{2}\right)\left(n\frac{\psi}{2}\right)}{\left(\cos\frac{\psi}{2}\right)\left(\frac{\psi}{2}\right)}\right\}$$

$$\therefore \quad \boxed{|\text{Major lobe}| = n} \quad \ldots(11)$$

where, n is the number of elements in the array.

Thus from equation (10) and (11) it is clear that, all the field components add up together to give total field which is 'n' times the individual field when $\phi = 90^0$ and 270^0.

3. Nulls

The ratio of total electric field to an individual electric field is given by,

$$\left|\frac{E_T}{E_0}\right| = \frac{\sin n\frac{\psi}{2}}{\sin\frac{\psi}{2}}$$

Equating ratio of magnitudes of the fields to zero,

$$\therefore \quad \left|\frac{E_T}{E_0}\right| = \frac{\sin n\frac{\psi}{2}}{\sin\frac{\psi}{2}} = 0$$

$$\therefore \quad \boxed{\sin n\frac{\psi}{2} = 0; \quad \text{but } \sin\frac{\psi}{2} \neq 0} \quad \ldots(12)$$

The condition of minima is given by,

Hence we can write,

$$\sin n\frac{\psi}{2} = 0$$

$$\text{i.e. } n\frac{\psi}{2} = \sin^{-1}(0) = \pm m\pi, \text{ where } m = 1, 2, 3, \ldots\ldots$$

$$\text{Now } \psi = \beta d \cos\phi = \frac{2\pi}{\lambda}(d)\cos\phi$$

$$\therefore \quad \frac{n}{2}\left(\frac{2\pi}{\lambda}d\right)\cos\phi_{min} = \pm m\pi$$

$$\text{i.e.} \quad \frac{nd}{\lambda}\cos\phi_{min} = \pm m$$

$$\therefore \quad \boxed{\phi_{min} = \cos^{-1}\left(\pm\frac{m\lambda}{nd}\right)} \quad ...(13)$$

where, n= number of elements in the array d= spacing

between elements in meter

λ= wavelength in meter

m= constant= 1, 2 , 3....

Thus equation (13) gives direction of nulls

4. Side Lobes Maxima

The directions of the subsidary maxima or side lobes maxima can be obtained if in equation (8),

$$\sin\left(n\frac{\psi}{2}\right) = \pm 1$$

$$\therefore \quad \boxed{n\frac{\psi}{2} = \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}, ...} \quad ...(14)$$

Hence sin(nυ/2), is not considered. Because if nυ/2=π/2 then sin nυ/2 =1 which is the direction of principle maxima.

Hence we can skip sin nυ/2 = ±π/2 value Thus, we get

$$\psi = \pm\frac{3\pi}{n}, \pm\frac{5\pi}{n}, \pm\frac{7\pi}{n}, ...$$

$$\text{Now} \quad \psi = \beta d\cos\phi = \left(\frac{2\pi}{\lambda}\right)d\cos\phi$$

Now equation for υ can be written as,

$$\frac{2\pi}{\lambda}d\cos\phi = \pm\frac{3\pi}{n}, \pm\frac{5\pi}{n}, \pm\frac{7\pi}{n}, ...$$

$$\therefore \quad \cos\phi = \frac{\lambda}{2\pi d}\left[\pm\frac{(2m+1)}{n}\pi\right] \quad \text{where } m = 1, 2, 3,........$$

$$\therefore \quad \boxed{\phi = \cos^{-1}\left[\pm\frac{\lambda(2m+1)}{2nd}\right]} \quad ...(15)$$

The equation (15) represents directions of subsidary maxima or side lobes maxima.

5. Beamwidth of Major Lobe

Beamwidth is defined as the angle between first nulls. Alternatively beamwidth is the angle equal to twice the angle between first null and the major lobe maximum direction. Hence beamwidth between first nulls is given by,

$$\therefore \quad \boxed{BWFN = 2 \times \gamma, \quad \text{where } \gamma = 90 - \phi} \qquad \ldots(16)$$

But $\quad \phi_{min} = \cos^{-1}\left(\pm \frac{m\lambda}{nd}\right), \quad$ where m = 1, 2, 3,......

Also $\quad 90 - \phi_{min} = \gamma \quad$ i.e. $90 - \gamma = \theta_{min}$

Hence $\quad 90 - \gamma = \cos^{-1}\left(\pm \frac{m\lambda}{nd}\right)$

Taking cosine of angle on both sides, we get

$$\cos(90 - \gamma) = \cos\left[\cos^{-1}\left(\pm \frac{m\lambda}{nd}\right)\right]$$

$$\therefore \quad \sin\gamma = \pm \frac{m\lambda}{nd} \qquad \ldots(17)$$

If γ is very small, then $\sin \gamma \approx \gamma$. Substituting n above equation we get,

$$\gamma = \pm \frac{m\lambda}{nd} \qquad \ldots(18)$$

For first null i.e. m=1,

$$\gamma = + \frac{\lambda}{nd}$$

$$\therefore \quad BWFN = 2\gamma = \frac{2\lambda}{nd}$$

But nd≈ (n-1)d if n is very large. This L= (nd) indicates total length of the array.

$$\therefore \quad \boxed{BWFN = \frac{2\lambda}{L} \text{ rad} = \frac{2}{\left(\frac{L}{\lambda}\right)} \text{ rad}} \qquad \ldots(19)$$

BWFN in degree is written as,

$$\boxed{\text{BWFN} = \frac{114.6\lambda}{L} = \frac{114.6}{\left(\frac{L}{\lambda}\right)} \text{ degrees}} \qquad \ldots(20)$$

Now HPBW is given by,

$$\boxed{\text{HPBW} = \frac{\text{BWFN}}{2} = \frac{1}{\left(\frac{L}{\lambda}\right)} \text{ rad}} \qquad \ldots(21)$$

HPBW in degree is written as,

$$\therefore \quad \boxed{\text{HPBW} = \frac{57.3}{\left(\frac{L}{\lambda}\right)} \text{ degrees}} \qquad \ldots(22)$$

6. Directivity

The directivity in case of broadside array is defined as,

$$G_{Dmax} = \frac{\text{Maximum radiation intensity}}{\text{Average radiation intensity}} = \frac{U_{max}}{U_{avg}} = \frac{U_{max}}{U_0} \qquad \ldots(23)$$

where, U_0 is average radiation intensity which is given by,

$$U_0 = \frac{P_{rad}}{4\pi} = \frac{1}{4\pi}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} |E(\theta,\phi)|^2 \sin\theta \; d\theta \, d\phi \qquad \ldots(24)$$

From the expression of ratio of magnitudes we can write,

$$\left|\frac{E_T}{E_0}\right| = n$$

or $\quad |E_T| = n|E_0|$

For the normalized condition let us assume $E_0 = 1$, then

$$|E_T| = n$$

Thus field from array is maximum in any direction θ when υ = 0. Hence normalized field pattern is given by,

$$E_{Normalized} = \left|\frac{E_T}{E_{Tmax}}\right| = \frac{|E_0|}{n|E_0|} = \frac{1}{n}$$

Hence the field is given by,

$$\therefore \quad E_{Normalized} = \frac{\sin n\frac{\psi}{2}}{n\left(\sin\frac{\psi}{2}\right)} \qquad \ldots(25)$$

where υ = βdcosυ

Equation (23) indicated array factor, hence we can write electric field due to n array as

$$E = \frac{1}{n}\left[\frac{\sin\frac{n\beta d\cos\phi}{2}}{\sin\frac{\beta d\cos\phi}{2}}\right]$$

Assuming d is very small as compared to length of an array,

$$\sin\frac{\beta d\cos\phi}{2} \approx \frac{\beta d\cos\phi}{2}.$$

Then,

$$E = \frac{1}{n}\left[\frac{\sin\frac{n\beta d\cos\phi}{2}}{\sin\frac{\beta d\cos\phi}{2}}\right] \qquad \ldots(26)$$

Substituting value of E in equation (24) we get

$$U_0 = \frac{1}{4\pi}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi}\left[\frac{\sin\frac{n\beta d\cos\phi}{2}}{\sin\frac{\beta d\cos\phi}{2}}\right]^2 \sin\theta\ d\theta\, d\phi$$

$$= \frac{1}{4\pi}\int_{\phi=0}^{2\pi} d\phi \cdot \int_{\theta=0}^{\pi}\left[\frac{\sin\frac{n}{2}\beta d\cos\phi}{\frac{n}{2}\beta d\cos\phi}\right]^2 \sin\theta\ d\theta$$

$$= \frac{1}{4\pi}[2\pi]\cdot\int_{\theta=0}^{\pi}\left[\frac{\sin z}{z}\right]^2 \sin\theta\ d\theta \qquad \ldots(27)$$

Let

$$z = \frac{n}{2}\beta d \cos\theta$$

$$\therefore \quad dz = -\frac{n}{2}\beta d \sin\theta \, d\theta$$

$$\therefore \quad \sin\theta \, d\theta = -\frac{dz}{\frac{n}{2}\beta d}$$

Also when $\theta = \pi$, $z = -\frac{n}{2}\beta d$, and

when $\theta = 0$, $z = +\frac{n}{2}\beta d$

Rewritting above equation we get,

$$U_0 = \frac{1}{2}\int_{+\frac{n}{2}\beta d}^{-\frac{n}{2}\beta d}\left[\frac{\sin z}{z}\right]^2 \cdot \frac{dz}{-\frac{n}{2}\beta d}$$

$$\therefore \quad U_0 = -\frac{1}{n\beta d}\int_{\frac{n}{2}\beta d}^{-\frac{n}{2}\beta d}\left[\frac{\sin z}{z}\right]^2 dz$$

For large array, n is large hence $n\beta d$ is also very large (assuming tending to infinity). Hence rewriting above equation.

$$U_0 = -\frac{1}{n\beta d}\int_{\infty}^{-\infty}\left[\frac{\sin z}{z}\right]^2 dz$$

Interchanging limits of integration, we get

$$U_0 = +\frac{1}{n\beta d}\int_{-\infty}^{\infty}\left[\frac{\sin z}{z}\right]^2 dz$$

By integration formula,

$$\int_{-\infty}^{\infty}\left[\frac{\sin z}{z}\right]^2 dz = \pi.$$

Using above property in above equation we can write,

$$\boxed{U_0 = \frac{1}{n\beta d}[\pi] = \frac{\pi}{n\beta d}} \quad \ldots(28)$$

From equation (23), the directivity is given by,

$$G_{Dmax} = \frac{U_{max}}{U_0}$$

But U_{max} = 1 at υ = 90° and substituting value of U_0 from equation (28), we get,

$$G_{Dmax} = \frac{1}{\left(\frac{\pi}{n\beta d}\right)} = \frac{n\beta d}{\pi} \qquad \ldots(29)$$

But $\beta = 2\pi/\lambda$

Hence

The total length of the array is given by, L = (n - 1) d ≈ nd, if n is very large. Hence the directivity can be expressed in terms of the total length of the array as,

$$G_{Dmax} = 2\left(\frac{L}{\lambda}\right)$$

Array of n Elements with Equal Spacing and Currents Equal in Magnitude but with Progressive Phase Shift - End Fire Array

Consider n number of identical radiators supplied with equal current which are not in phase as shown in the Fig. 11. Assume that there is progressive phase lag of βd radians in each radiator.

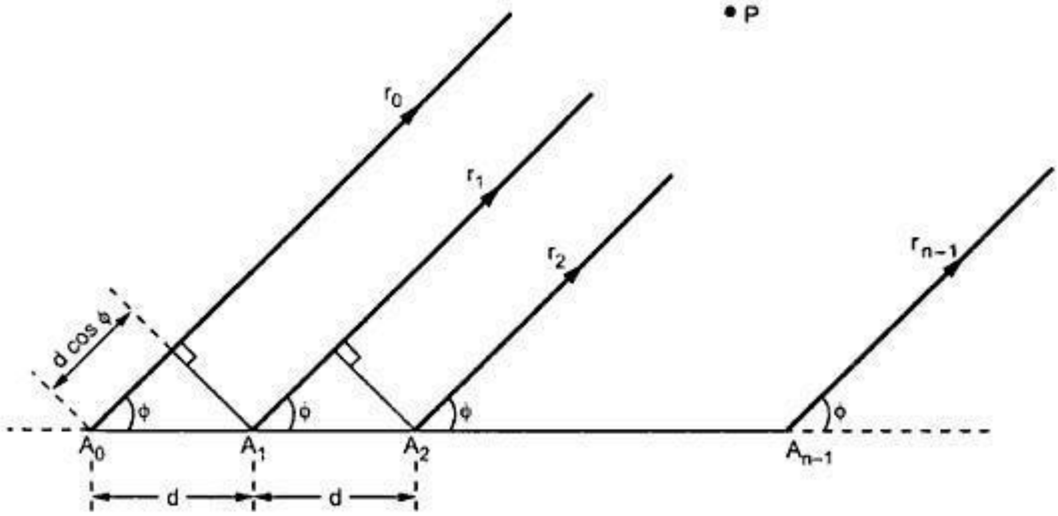


Fig.11 End fire array

Consider that the current supplied to first element A0 be I0. Then the current supplied to A1 is given by,

$$I_1 = I_0 \cdot e^{-j\beta d}$$

Similarly the current supplied to A2 is given by,

$$I_2 = I_1 \cdot e^{-j\beta d} = \left[I_0 \cdot e^{-j\beta d}\right] e^{-j\beta d} = I_0 \cdot e^{-j2\beta d}$$

Thus the current supplied to last element is

$$I_{n-1} = I_0 \, e^{-j(n-1)\beta d}$$

The electric field produced at point P, due to A0 is given by,

$$E_0 = \frac{I \, dL \sin\theta}{4\pi\omega\varepsilon_0}\left[j\frac{\beta^2}{r_0}\right] e^{-j\beta r_0} \quad \ldots (1)$$

The electric field produced at point P, due to A1 is given by,

$$E_1 = \frac{I \, dL \sin\theta}{4\pi\omega\varepsilon_0}\left[j\frac{\beta^2}{r_1}\right] e^{-j\beta r_1} \cdot e^{-j\beta d}$$

But $r_1 = r_0 - d\cos\upsilon$

$$\therefore \quad E_1 = \frac{I\,dL\sin\theta}{4\pi\omega\varepsilon_0}\left[j\frac{\beta^2}{r_0}\right]e^{-j\beta(r_0-d\cos\phi)}\cdot e^{-j\beta d}$$

$$\therefore \quad E_1 = \left[\frac{I\,dL\sin\theta}{4\pi\omega\varepsilon_0}\left[j\frac{\beta^2}{r_0}\right]e^{-j\beta r_0}\right]e^{j\beta d\cos\phi}\cdot e^{-j\beta d}$$

$$\therefore \quad E_1 = E_0\cdot e^{j\beta d(\cos\phi-1)} \qquad \ldots (2)$$

Let υ = βd(cosυ -1)

$$\therefore \quad E_1 = E_0\,e^{j\psi} \qquad \ldots (3)$$

The electric field produced at point P, due to A2 is given by,

$$E_2 = E_0\cdot e^{j2\psi} \qquad \ldots (4)$$

Similarly electric field produced at point P, due to An-1 is given by,

$$E_{n-1} = E_0\,e^{j(n-1)\psi} \qquad \ldots (5)$$

The resultant field at point p is given by,

$$E_T = E_0 + E_1 + E_2 + \ldots + E_{n-1}$$

$$\therefore \quad E_T = E_0 + E_0\,e^{j\psi} + E_0\,e^{j2\psi} + \ldots + E_0\,e^{j(n-1)\psi}$$

$$\therefore \quad E_T = E_0\left[1 + e^{j\psi} + e^{j2\psi} + \ldots + e^{j(n-1)\psi}\right] \qquad \ldots (6)$$

$$E_T = E_0\cdot\frac{1-e^{jn\psi}}{1-e^{j\psi}}$$

$$\therefore \quad \frac{E_T}{E_0} = \frac{\sin\frac{n\psi}{2}}{\sin\frac{\psi}{2}}\cdot e^{j\frac{(n-1)}{2}\psi} \qquad \ldots (7)$$

Considering only magnitude we get,

$$\therefore \quad \boxed{\left|\frac{E_T}{E_0}\right| = \frac{\sin\frac{n\psi}{2}}{\sin\frac{\psi}{2}}} \qquad \ldots (8)$$

Properties of End Fire Array 1. Major lobe

For the end fire array where currents supplied to the antennas are equal in amplitude but the phase changes progressively through array, the phase angle is given by,

υ = βd(cosυ -1) **...(9)**

In case of the end fire array, the condition of principle maxima is given by, υ = = 0 i.e.

$$\boxed{\beta d(\cos\phi - 1) = 0} \quad \ldots(10)$$

i.e. cosυ

= 1

0

i.e. υ = 0 **...(11)**

0

Thus υ = 0 indicates the direction of principle maxima.

2. Magnitude of the major lobe

The maximum radiation occurs when υ= 0. Thus we can write,

$$|\text{Major lobe}| = \lim_{\psi \to 0} \left\{ \frac{\frac{d}{d\psi}\left(\sin n\frac{\psi}{2}\right)}{\frac{d}{d\psi}\left(\sin\frac{\psi}{2}\right)} \right\} = \lim_{\psi \to 0} \left\{ \frac{\left(\cos n\frac{\psi}{2}\right)\left(n\frac{\psi}{2}\right)}{\left(\cos\frac{\psi}{2}\right)\left(\frac{\psi}{2}\right)} \right\}$$

$$\therefore \quad \boxed{|\text{Major lobe}| = n} \quad \ldots(12)$$

where, n is the number of elements in the array.

3. Nulls

The ratio of total electric field to an individual electric field is given by,

$$\left|\frac{E_T}{E_0}\right| = \frac{\sin n\frac{\psi}{2}}{\sin\frac{\psi}{2}}$$

Equating ratio of magnitudes of the fields to zero,

$$\therefore \quad \left|\frac{E_T}{E_0}\right| = \frac{\sin n\frac{\psi}{2}}{\sin\frac{\psi}{2}} = 0$$

The condition of minima is given by,

$$\boxed{\sin n\frac{\psi}{2} = 0, \quad \text{but } \sin\frac{\psi}{2} \neq 0} \quad \ldots(13)$$

Henc

e we can write,

$$\sin n\frac{\psi}{2} = 0$$

$$\text{i.e. } n\frac{\psi}{2} = \sin^{-1}(0) = \pm m\pi, \text{ where } m = 1, 2, 3, \ldots\ldots$$

Substituting value of υ from equation (9), we get,

$$\therefore \frac{n\beta d(\cos\phi - 1)}{2} = \pm m\pi$$

But β= 2π/λ

$$\therefore \frac{nd}{\lambda}(\cos\phi - 1) = \pm m \quad \ldots(14)$$

Note that value of (cosυ-1) is always less than 1. Hence it is always negative.

Hence only considering -ve values, R.H.S., we get

$$\frac{nd}{\lambda}(\cos\phi - 1) = -m$$

$$\text{i.e. } \cos\phi - 1 = -\frac{m\lambda}{nd}$$

$$\boxed{\phi_{min} = \cos^{-1}\left[1 - \frac{m\lambda}{nd}\right]} \quad \ldots(15)$$

where, n= number of elements in the array d= spacing

between elements in meter

λ= wavelength in meter

m= constant= 1, 2 , 3....

Thus equation (15) gives direction of nulls

Consider equation(14),

$$\cos\phi_{min} - 1 = \pm\frac{m\lambda}{nd}$$

Expressing term on L.H.S. in terms of halfangles, we get,

$$2\sin^2\frac{\phi_{min}}{2} = \pm\frac{m\lambda}{nd} \quad \ldots\left(\cos\theta - 1 = 2\sin^2\frac{\theta}{2}\right)$$

$$\therefore \quad \sin^2\frac{\phi_{min}}{2} = \pm\frac{m\lambda}{2nd}$$

$$\therefore \quad \boxed{\phi_{min} = 2\sin^{-1}\left[\pm\sqrt{\frac{m\lambda}{2nd}}\right]} \quad \ldots(16)$$

4. Side Lobes Maxima

The directions of the subsidary maxima or side lobes maxima can be obtained if in equation (8),

$$\sin\left(n\frac{\psi}{2}\right) = \pm 1$$

$$\therefore \quad \boxed{n\frac{\psi}{2} = \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}, \ldots\ldots} \qquad \ldots(17)$$

Hence sin(nυ/2), is not considered. Because if nυ/2=±π/2 then sin nυ/2 =1 which is the direction of principle maxima.

Hence we can skip sin nυ/2 = ±π/2 value Thus, we get

$$\frac{n\psi}{2} = \pm(2m+1)\frac{\pi}{2}, \text{ where } m = 1, 2, 3, \ldots\ldots$$

Putting value of υ from equation (9) we get

$$\frac{n\beta d(\cos\phi - 1)}{2} = \pm(2m+1)\frac{\pi}{2}$$

$$\therefore n\beta d(\cos\phi - 1) = \pm(2m+1)\pi$$

Now equation for υ can be written as, But β = 2π/λ

$$n\left(\frac{2\pi}{\lambda}\right)d(\cos\phi - 1) = \pm(2m+1)\pi$$

$$\text{i.e. } \cos\phi - 1 = \pm(2m+1)\frac{\lambda}{2nd}$$

Note that value of (cosυ-1) is always less than 1. Hence it is always negative. Hence only considering -ve values, R.H.S., we get

$$\cos\phi - 1 = -(2m+1)\frac{\lambda}{2nd}$$

i.e. $$\cos\phi = 1-(2m+1)\frac{\lambda}{2nd}$$

i.e. $$\boxed{\phi = \cos^{-1}\left[1-\frac{(2m+1)\lambda}{2nd}\right]} \quad \ldots(18)$$

5. Beamwidth of Major Lobe

Beamwidth is defined as the angle between first nulls. Alternatively beamwidth is the angle equal to twice the angle between first null and the major lobe maximum direction.

From equation (16) we get,

$$\phi_{min} = 2\sin^{-1}\left[\pm\sqrt{\frac{m\lambda}{2nd}}\right] \quad \ldots(19)$$

$$\therefore \quad \sin\frac{\phi_{min}}{2} = \pm\sqrt{\frac{m\lambda}{2nd}}$$

υminis very low

Hence sin υmin/2 ≈ υmin/2

$$\frac{\phi_{min}}{2} = \pm\sqrt{\frac{m\lambda}{2nd}}$$

$$\phi_{min} = \pm\sqrt{\frac{4m\lambda}{2nd}} = \pm\sqrt{\frac{2m\lambda}{nd}} \quad \ldots(20)$$

But nd≈ (n-1)d if n is very large. This L= (nd) indicates total length of the array. So equation (20) becomes,

$$\phi_{min} = \pm\sqrt{\frac{2m\lambda}{L}} = \pm\sqrt{\frac{2m}{L/\lambda}} \quad \ldots(21)$$

BWFN is given by,

$$\boxed{BWFN = 2\phi_{min} = \pm 2\sqrt{\frac{2m}{L/\lambda}}} \quad \ldots(22)$$

BWFN in degree is expressed as

$$BWFN = \pm 2\sqrt{\frac{2m}{L/\lambda}}\times 57.3 = \pm 114.6\sqrt{\frac{2m}{L/\lambda}} \text{ degree}$$

For m=1,

$$\text{BWFN} = \pm 2\sqrt{\frac{2}{L/\lambda}}\ \text{rad} = 114.6\sqrt{\frac{2}{L/\lambda}}\ \text{degree} \quad \ldots(23)$$

6. Directivity

The directivity in case of endfire array is defined as,

$$G_{Dmax} = \frac{\text{Maximum radiation intensity}}{\text{Average radiation intensity}} = \frac{U_{max}}{U_{avg}} = \frac{U_{max}}{U_0} \quad \ldots(23)$$

where, U_0 is average radiation intensity which is given by, For endfire array, U_{max} = 1and $U_0 = \frac{\pi}{2n\beta d}$

$$\therefore \quad G_{Dmax} = \frac{1}{\frac{\pi}{2n\beta d}} = \frac{2n\beta d}{\pi}$$

$$\therefore \quad G_{Dmax} = 2n\left(\frac{2\pi}{\lambda}\right)\cdot\frac{d}{\pi}$$

$$\therefore \quad G_{Dmax} = 4\left(\frac{nd}{\lambda}\right) \quad \ldots(24)$$

The total length of the array is given by, L = (n - 1) d ≈ nd, if n is very large. Hence the directivity can be expressed in terms of the total length of the array as,

$$\therefore \quad G_{Dmax} = 4\left(\frac{L}{\lambda}\right) \quad \ldots(25)$$

Multiplication of patterns

In the previous sections we have discussed the arrays of two isotropic point sources radiating field of constant magnitude. In this section the concept of array is extended to non-isotropic sources. The sources identical to point source and having field patterns of definite shape and orientation. However, it is not necessary that amplitude of individual sources is equal. The simplest case of non-isotropic sources is when two short dipoles are superimposed over the two isotopic point sources separated by a finite distance. If the field pattern of each source is given by

$$E_0 = E_1 = E_2 = E' \sin\theta$$

Then the total far-field pattern at point P becomes

$$E_T = 2E_0 \cos\left(\frac{\psi}{2}\right) = 2E' \sin\theta \cos\left(\frac{\psi}{2}\right) \Rightarrow E_{Tn} = \sin\theta \cos\left(\frac{\psi}{2}\right) \quad ...(1)$$

$$E_{Tn} = E(\theta) \times \cos\left(\frac{\psi}{2}\right)$$

where

$$\psi = \left(\frac{2\pi d}{\lambda}\cos\theta + \alpha\right)$$

Equation (1) shows that the field pattern of two non-isotropic point sources (short dipoles) is equal to product of patterns of individual sources and of array of point sources. The pattern of array of two isotropic point sources, i.e., cos υ/2 is widely referred as an array factor. That is

E_T= E (Due to reference source) x Array factor

This leads to the principle of pattern multiplication for the array of identical elements. In general, the principle of pattern multiplication can he stated as follows:

The resultant field of an array of non-isotropic hut similar sources is the product of the fields of individual source and the field of an array of isotropic point sources, each located at the phase centre of individual source and hating the relative amplitude and phase. The total phase is addition of the phases of the individual source and that of isotropic point sources. The same is true for their respective patterns also.

The normalized total field (i.e., E_{Tn}), given in Eq. (1), can re-written as

$$E = E_1(\theta) \times E_2(\theta)$$

where $E_1(\theta) = \sin\theta$ = Primary pattern of array

$$E_2(\theta) = \cos\left(\frac{2\pi d}{\lambda}\cos\theta + \alpha\right)$$ = Secondary pattern of array.

Thus the principle of pattern multiplication is a speedy method of sketching the field pattern of complicated array. It also plays an important role in designing an array. There is no restriction on the number of elements in an array; the method is valid to any number of identical elements which need not have identical magnitudes, phase and spacing between then). However, the array factor varies with the number of elements and their arrangement, relative magnitudes, relative phases and element spacing. The array of elements having identical amplitudes, phases and spacing provides a simple array factor. The array factor does not depend on the directional characteristic of the array elements; hence it can be formulated by using pattern multiplication techniques. The proper selection of the individual radiating element and their excitation are also important for the performance of array. Once the array factor is derived using the point-source array, the total field of the actual array can be obtained using Eq. (2).