ROHINI COLLEGE OF ENGINEERING AND TECHNOLOGY

Kanyakumari Main Road, Near Anjugramam, Palkulam, Tamil Nadu 629401

Department of Mechanical Engineering

VALUE ADDED COURSE ON COMPOSITE MATERIAL

SYLLABUS

UNIT-I Introduction:

Definitions, Composites, Reinforcements and matrices, Types of reinforcements, Types of matrices, Types of composites, Carbon Fiber composites, Properties of composites in comparison with standard materials, Applications of metal, ceramic and polymer matrix composites.

UNIT-II Manufacturing methods:

Hand and spray lay - up, injection molding, resin injection, filament winding, pultrusion, centrifugal casting and prepregs. Fiber/Matrix Interface, mechanical. Measurement of interface strength. Characterization of systems; carbon fiber/epoxy, glass fiber/polyester, etc.

UNIT-III Mechanical Properties -Stiffness and Strength:

Geometrical aspects – volume and weight fraction. Unidirectional continuous fiber, discontinuous fibers, Short fiber systems, woven reinforcements Mechanical Testing: Determination of stiffness and strengths of unidirectional composites; tension, compression, flexure and shear.

UNIT-IV Laminates:

Plate Stiffness and Compliance, Assumptions, Strains, Stress Resultants, Plate Stiffness and Compliance, Computation of Stresses, Types of Laminates, Symmetric Laminates, Antisymmetric Laminate, Balanced Laminate, Quasi-isotropic Laminates, Cross-ply Laminate, Angle-ply Laminate. Orthotropic Laminate, Laminate Moduli, Hygrothermal Stresses.

UNIT-V Joining Methods and Failure Theories:

Joining –Advantages and disadvantages of adhesive and mechanically fastened joints. Typical bond strengths and test procedures.

UNIT-I

INTRODUCTION

Introduction to Composite Materials

A composite material is a material that consists of one or more discontinuous components (particles/fibres/reinforcement) that are placed in a continuous medium (matrix).In a fibre composite the matrix binds together the fibres, transfers loads between the fibres and protects them from the environment and external damage.

Composite materials, or shortened to composites, are microscopic or macroscopic combinations of two or more distinct engineered materials (those with different physical and/or chemical properties) with a recognizable interface between them in the finished product. For structural applications, the definition can be restricted to include those materials that consist of a reinforcing phase such as fibers or particles supported by a binder or matrix phase. Wood composites are commonly seen examples of composite materials.

Other features of composites include the following: (1) The distribution of materials in the composite is controlled by mechanical means. (2) The term composite is usually reserved for materials in which distinct phases are separated on a scale larger than atomic,and in which the composite's mechanical properties are significantly altered from those of the constituent components. (3) The composite can be regarded as a combination of two or more materials that are used in combination to rectify a weakness in one material by a strength in another. (4) A recently developed concept of composites is that the composite should not only be a combination of two materials, but the combination should have its own distinctive properties. In terms of strength, heat resistance, or some other desired characteristic, the composite must be better than either component alone.

Composites were developed because no single, homogeneous structural material could be found that had all of the desired characteristics for a given application. Fiber-reinforced composites were first developed to replace aluminum alloys, which provide high strength and fairly high stiffness at low weight but are subject to corrosion and fatigue.

An example of a composite material is a glass-reinforced plastic fishing rod in which glass fibers are placed in an epoxy matrix. Fine individual glass fibers are characterized by their high tensile stiffnesses and very high tensile strengths, but because of theirsmall diameters, have very small bending stiffnesses. If the rod were made only of epoxy plastic, it would have good bending stiffness, but poor tensile properties. When the fibers are placed in the epoxy plastic, however, the resultant structure has high tensile stiffness, high tensile strength, and high bending stiffness.

The discontinuous filler phase in a composite is usually stiffer or stronger than the binder phase. There must be a substantial volume fraction of the reinforcing phase (about10%) present to provide reinforcement. Examples do exist, however, of composites where the discontinuous phase is more compliant and ductile than the matrix.

Natural composites include wood and bone. Wood is a composite of cellulose and lignin. Cellulose fibers are strong in tension and are flexible. Lignin cements these fibers together to make them stiff. Bone is a composite of strong but soft collagen (a protein) and hard but brittle apatite (a mineral).

Particulate composites

Various geometrical shapes (cubes, spheres, flakes, etc.)
Various materials (rubber, metal, plastics, etc.)
Have generally low strength.
Will not be treated further in this course.

Particle reinforced metal matrix composites are now being produced commerically, and in this paper the current status of these materials is reviewed. The different types of reinforcement being used, together with the alternative processing methods, are discussed. Depending on the initial processing method, different factors have to be taken into consideration to produce a high quality billet. With powder metallurgy processing, the composition of the matrix and the type of reinforcement are independent of one another. However, in molten metal processing they are intimately linked in terms of the different reactivities which occur between reinforcement and matrix in the molten state. The factors controlling the distribution of reinforcement are also dependent on the initial processing method. Secondary fabrication methods, such as extrusion and rolling, are essential in processing composites produced by powder metallurgy, since they are required to consolidate the composite fully. Other methods, such as spray casting, molten metal infiltration, and molten metal mixing give an essentially fully consolidated product directly, but extrusion, etc., can improve the properties by modifying the reinforcement distribution. The mechanical properties obtained in metal matrix composites are dependent on a wide range of factors, and the present understanding, and areas requiring further study, are discussed. The successful commercial production of metal matrix composites will finally depend on their cost effectiveness for different applications. This requires optimum methods of processing, machining, and recycling, and the routes being developed to achieve this are considered.

Fibre composites
Discontinuous or Continuous

Classification of Composites:

• Matrices:
Organic Matrix Composites (OMCs)
Polymer Matrix Composites (PMCs)
carbon-carbon composites
Metal Matrix Composites (MMCs)
Ceramic Matrix Composites (CMCs)
• Reinforcements:
Fibres reinforced composites
Laminar composites
Particulate composites

Carbon Fiber is a polymer and is sometimes known as graphite fiber. It is a very strong material that is also very lightweight. Carbon fiber is five-times stronger than steel and twice as stiff. Though carbon fiber is stronger and stiffer than steel, it is lighter than steel; making it the ideal manufacturing material for many parts. These are just a few reasons why carbon fiber is favored by engineers and designers for manufacturing.

Properties of Composite Materials:

1. **High Strength to Weight Ratio:**Fibre composites are extremely strong for their weight. By refining the laminate many characteristics can be enhanced. A common laminate of say 3mm Chopped strand mat, is quite flexible compared to say a 3 mm ply. However it will bend a long way more than the ply before yielding. Stiffness should not be confused with Strength. A carbon fibre laminate on the other hand, will have a stiffness of many times that of mild steel of the same thickness, increased ultimate strength, yet only be less than 1/4 of it's weight.
2. **Lightweight:**A standard Fibreglass laminate has a specific gravity in the region of 1.5, compared to Alloy of 2.7 or steel of 7.8. When you then start looking at Carbon laminates, strengths can be many times that of steel, but only a fraction of the weight. A DVD case lid was produced using carbon fibre to reduce the case's overall weight so that it could be carried as cabin baggage whilst traveling, and for improved security. It was used by support crew for the All Blacks during their 1999 Rugby World Cup campaign.
3. **Fire Resistance:**The ability for composites to withstand fire has been steadily improving over the years. There is two types of systems to be considered:
   Fire Retardant - Are self extinguishing laminates, usually made with chlorinated resins and additives such as Antimony trioxide. These release CO2 when burning so when the flame source is removed, the self extinguish.
   Fire Resistant - More difficult and made with the likes of Phenolic Resins. These are difficult to use, are cured with formaldehyde, and require a hi degree of post curing to achieve true fire resistance.
   Other materials are also becoming more readily available to be used as in tumescent layers, which expand and blanket the surface, preventing spread of flame. There is a paint on coating usually applied to the back of the product laminate, plus a thin fibre film to go under the Gelcoat giving the outer surface a blanketing coat as well.
   Fibreglass Developments Ltd produces a Fire Door as part of our **SteridorTM** range. Use of special Phenolic resin has allowed us to create the *only* fully tested Composite door in Australasia. Fire rated by BRANZ to 4 hours, this door is also approved by MAF as meeting all their Hygiene requirements.
4. **Electrical Properties:**Fibreglass Developments Ltd produced the Insulator Support straps for the Tranz Rail main trunk electrification. The straps, although only 4mm thick, meet the required loads of 22kN, as well as easily meeting insulation requirements.
5. **Chemical & Weathering Resistance:**Composite products have good weathering properties and resist the attack of a wide range of chemicals. This depends almost entirely on the resin used in manufacture, but by careful selection resistance to all but the most extreme conditions can be achieved. Because of this, composites are used in the manufacture of chemical storage tanks, pipes, chimneys and ducts, boat hulls and vehicle bodies.
   FDL manufactured architectural panels for the construction of the Auckland Marine Rescue Centre. Composite panels were chosen because of their ability to withstand salty sea side conditions without corrosion.
6. **Colour:**Almost any shade of any colour can be incorporated into the product during manufacture by pigmenting the gelcoat used. Costs are therefore reduced by no further finishing or painting. Soluble dyes can be used if a translucent product is desired.
   We do not however, recommend dark colours. These produce excessive heat on the

surface which can lead to the surface deteriorating and showing print through, where the Resin matrix cures more and shrinks, bringing the fibres to the surface. In extreme cases delamination can occur.

7 **Translucency:** Polyester resins are widely used to manufacture translucent mouldings and sheets. Light transmission of up to 85% can be achieved.

8 **Design Flexibility:** Because of the versatility of composites, product design is only limited by your imagination.

9 **Low Thermal Conductivity:** Fibreglass Developments has been involved in the development and production of specialized meat containers which maintain prime cuts of chilled meat at the correct temperature for Export markets. They are manufactured using the RTM process, with special reinforcing and foam inserts.

10 **Manufacturing Economy:** Fibreglass Developments produces several models of fuel pump covers for Fuel quip. Fibreglass is an ideal material for producing items of this type for many reasons, including being very economical. Because of its versatile properties, fibreglass can be used in many varied applications.

Advantages:

• Lower density (20 to 40%)
• Higher directional mechanical properties (specific tensile strength (ratio of material strength to density) 4 times greater than that of steel and aluminium.
• Higher Fatigue endurance .
• Higher toughness than ceramics and glasses.
• Versatility and tailoring by design.
• Easy to machine.
• Can combine other properties (damping, corrosion).
• Cost.

Applications:

Mulch: In nature we see plants and trees drop leaves that accumulate at their bases. Every year, a new layer is added while the old layers start to decompose. This is leaf mold, and it is a form of compost. What nature is doing is providing a protective layer over the roots of plants. This layer of vegetative material protects the bare soil during the summer months by reducing soil temperature, suppressing weed growth and reducing soil moisture loss. Our compost can do the same thing in our gardens and landscapes.To prepare any area for mulching, first clear away grass or weeds that might grow up through the mulch. Make sure to remove the roots of tough perennial weeds such as ground ivy. When using compost as a mulch in flower beds, vegetable gardens, landscape beds, or lawns, screen the finished compost. A simple screen can be made using ½-inch mesh hardware cloth and attaching it to a wooden frame. Place the screen over a wheel barrow or other container and sift the compost into it. The large pieces left behind can go into your next compost pile as an activator, introducing the necessary microorganisms. Cover the garden or bed area with screened compost to a depth of one to two inches. If you apply compost on a lawn, be sure it is finely ground or sifted. You have less of a chance of smothering the lawn. You may want to use ¼-inch mesh hardware cloth. One way to incorporate the compost is to aerate the sod, then apply a 1/8-inch to ¼-inch covering of fine compost. Use a rake to distribute the compost into the crevices. When mulching around trees and shrubs, screening may not be necessary. This is really a matter of aesthetic desire on your part.

Soil Amendment: We have already talked about how compost helps soil, especially sandy and clay soils. When starting a new garden soil amending is recommended before you plant. It is so much easier to add compost now than it is after the garden is planted. Cover the garden area with 3 to 4 inches of compost and till it into the upper six inches of the soil. If your garden is already established and you want to incorporate compost deeply into the soil, your options are limited.

With perennials, every time you add a new plant to the garden or divide an existing one, add compost. With annuals, you can add compost every spring. Loosen up the entire area where annuals will be planted and work in compost. Around trees and shrubs add at planting time, mixing no more than 25 percent of soil volume. Some references say not to use any at all for fear that the roots will remain in the planting hole area and not grow out into the surrounding soil. Keeping the compost level at one-quarter of the total soil volume will not lead to this problem. If you're concerned, use the compost as a mulch only.

Around existing trees it may be difficult to incorporate into the upper six inches of the soil. You can add compost by injecting nutrients the way professional arborists do. Drill 1-to 2-inch diameter holes 12 inches deep in the soil throughout the tree canopy and beyond at 18-inch spacing. Fill the bottom of each hole with recommended rates of dry fertilizer and then top off the holes with compost. For shrubs, the holes only need to be drilled 8 to 10 inches deep. This treatment should supply nutrients for two to three years.

Using Compost in Potting Mixes:

You can also blend fine-textured compost in potting mixtures. However, make sure the compost does not make up more than one quarter to one half of the potting mixture's volume. Plants growing in containers are entirely reliant on the water and nutrients provided in the potting mix. Compost is excellent for container growing mixes, because it stores moisture effectively and provides a variety of nutrients not typically supplied in commercial fertilizers or soil-free potting mixes. You still need to fertilize containers on a regular basis to provide the high volume of nutrients they need. Finely sifted compost can also be used in seed starting mixtures.

Compost Tea:

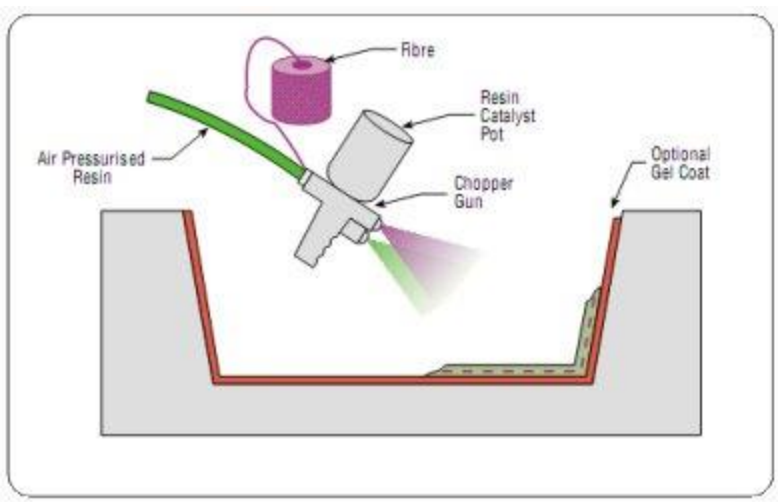
An old fashioned way of providing liquid fertilizer for plants is to brew compost tea. Similar to manure tea, compost tea gives your plants a good dose of nutrients. Compost tea works especially well for providing nutrients to new transplants and young seedlings. To make compost tea fill a burlap sack or an old pillow case with finished compost and secure the open end. Place in a tub, barrel, or watering can filled with water. Agitate for a few minutes and then let it steep for a few days. Water will leach out nutrients from the compost and the mixture will take on the color of tea. Spray or pour compost tea on and around plants. You can use the bag of compost for several batches. Afterwards, simply empty the bag's contents onto the garden.

UNIT-II

MANUFACTURING METHODS

Manufacturing of Composites

Spray Lay-Up



Description:

Fibre is chopped in a hand-held gun and fed into a spray of catalyzed resin directed at the mould. The deposited materials are left to cure under standard atmospheric conditions.

Material Options:

- Resins: Primarily polyester
- Fibres: Glass roving only
- Cores: None. These have to be incorporated separately

Typical Applications:

Simple enclosures, lightly loaded structural panels, e.g. caravan bodies, truck fairings, bathtubs, shower trays, some small dinghies.

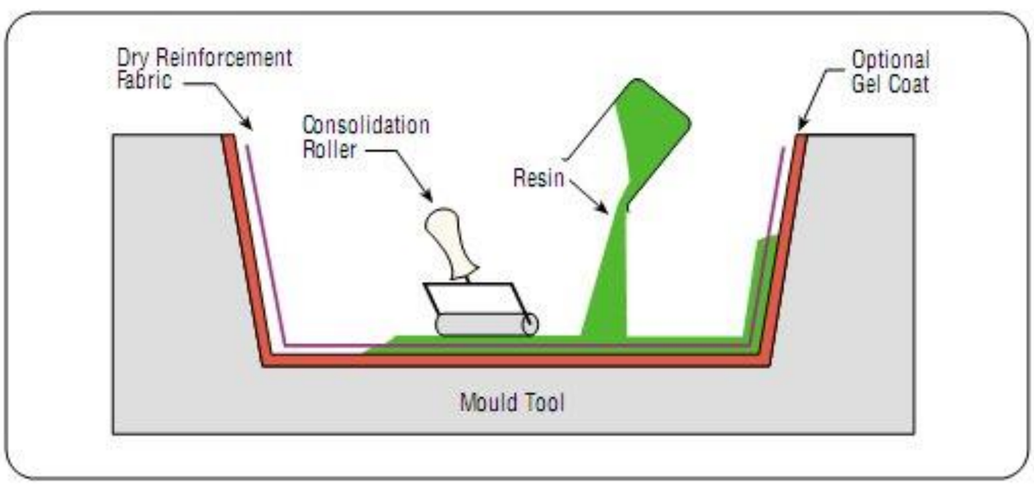
Advantages:

- Widely used for many years.
- Low cost way of quickly depositing fibre and resin.
- Low cost tooling.

Disadvantages:

- Laminates tend to be very resin-rich and therefore excessively heavy.
- Only short fibres are incorporated which severely limits the mechanical properties of the laminate.
- Resins need to be low in viscosity to be sprayable. This generally compromises their mechanical/thermal properties.
- The high styrene contents of spray lay-up resins generally mean that they have the potential to be more harmful and their lower viscosity means that they have an increased tendency to penetrate clothing.
- Limiting airborne styrene concentrations to legislated levels is becoming increasingly difficult.

Wet/Hand Lay-up



Description:

Resins are impregnated by hand into fibres which are in the form of woven, knitted, stitched or bonded fabrics. This is usually accomplished by rollers or brushes, with an increasing use of nip-roller type impregnators for forcing resin into the fabrics by means of rotating rollers and a bath of resin. Laminates are left to cure under standard atmospheric conditions.

Materials Options:

- Resins: Any, e.g. epoxy, polyester, vinylester, phenolic
- Fibres: Any, although heavy aramid fabrics can be hard to wet-out by hand.
- Cores: Any.

Typical Applications:

Standard wind-turbine blades, production boats, architectural mouldings.

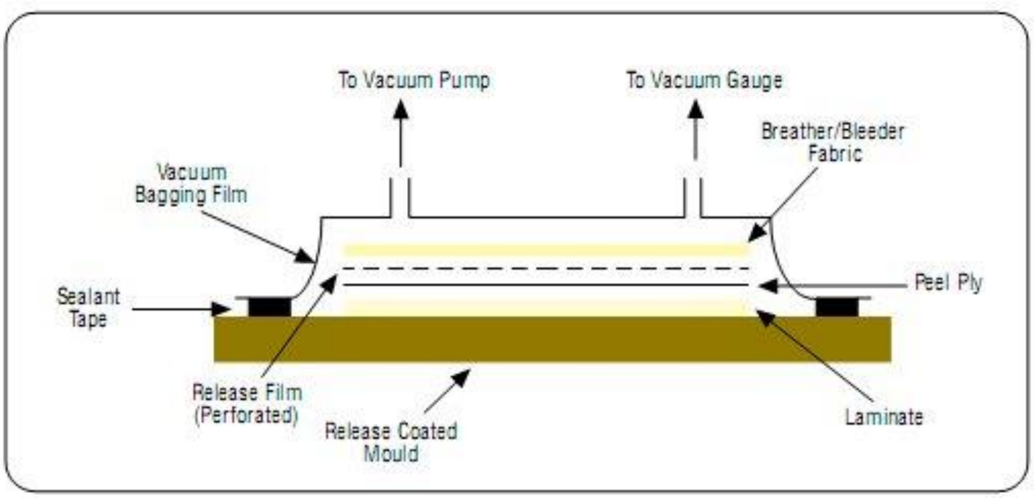
Advantages:

- Widely used for many years.
- Simple principles to teach.
- Low cost tooling, if room-temperature cure resins are used.
- Wide choice of suppliers and material types.
- Higher fibre contents and longer fibres than with spray lay-up.

Disadvantages:

- Resin mixing, laminate resin contents, and laminate quality are very dependent on the skills of laminators. Low resin content laminates cannot usually be achieved without the incorporation of excessive quantities of voids.
- Health and safety considerations of resins. The lower molecular weights of hand lay-up resins generally mean that they have the potential to be more harmful than higher molecular weight products. The lower viscosity of the resins also means that they have an increased tendency to penetrate clothing.
- Limiting airborne styrene concentrations to legislated levels from polyesters and vinylesters is becoming increasingly hard without expensive extraction systems.
- Resins need to be low in viscosity to be workable by hand. This generally compromises their mechanical/thermal properties due to the need for high diluent/styrene levels.

Vacuum Bagging



Description:

This is basically an extension of the wet lay-up process described above where pressure is applied to the laminate once laid-up in order to improve its consolidation. This is achieved by sealing a plastic film over the wet laid-up laminate and onto the tool. The air under the bag is extracted by a vacuum pump and thus up to one atmosphere of pressure can be applied to the laminate to consolidate it.

Materials Options:

- Resins: Primarily epoxy and phenolic. Polyesters and vinylesters may have problems due to excessive extraction of styrene from the resin by the vacuum pump.
- Fibres: The consolidation pressures mean that a variety of heavy fabrics can be wet-out.
- Cores: Any.

Typical Applications:

Large, one-off cruising boats, racecar components, core-bonding in production boats.

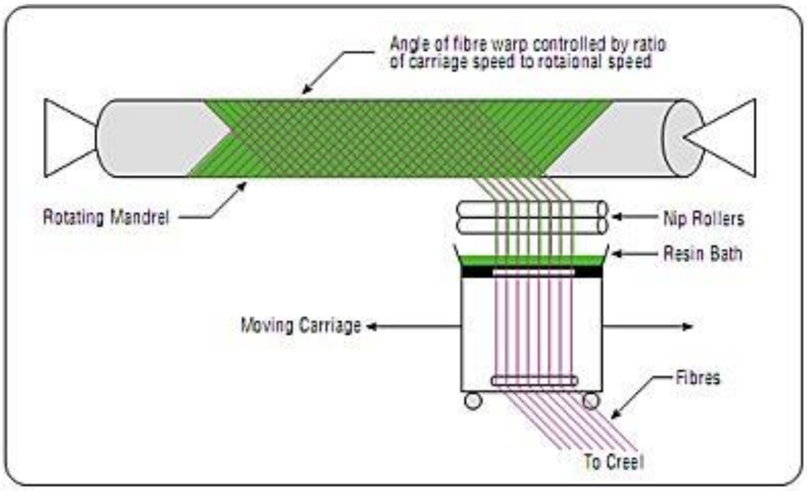
Advantages:

- Higher fibre content laminates can usually be achieved than with standard wet lay-up techniques.
- Lower void contents are achieved than with wet lay-up.
- Better fibre wet-out due to pressure and resin flow throughout structural fibres, with excess into bagging materials.
- Health and safety: The vacuum bag reduces the amount of volatiles emitted during cure.

Disadvantages:

- The extra process adds cost both in labour and in disposable bagging materials.
- A higher level of skill is required by the operators.
- Mixing and control of resin content still largely determined by operator skill.

Filament Winding:



Description:

This process is primarily used for hollow, generally circular or oval sectioned components, such as pipes and tanks. Fibre tows are passed through a resin bath before being wound onto a mandrel in a variety of orientations, controlled by the fibre feeding mechanism, and rate of rotation of the mandrel.

Materials Options:

- Resins: Any, e.g. epoxy, polyester, vinylester, phenolic
- Fibres: Any. The fibres are used straight from a creel and not woven or stitched into a fabric form
- Cores: Any, although components are usually single skin

  Typical Applications:

  Chemical storage tanks and pipelines, gas cylinders, fire-fighters breathing tanks

  Advantages:

  This can be a very fast and therefore economic method of laying material down.

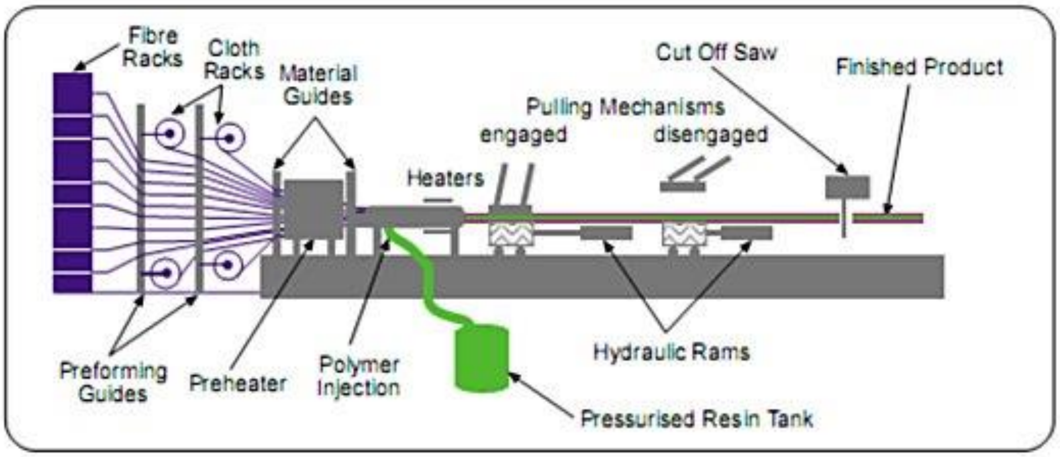
- Resin content can be controlled by metering the resin onto each fibre tow through nips or dies.
- Fibre cost is minimised since there is no secondary process to convert fibre into fabric prior to use.
- Structural properties of laminates can be very good since straight fibres can be laid in a complex pattern to match the applied loads.

Disadvantages:

- The process is limited to convex shaped components.
- Fibre cannot easily be laid exactly along the length of a component.
- Mandrel costs for large components can be high.
- The external surface of the component is unmoulded, and therefore cosmetically unattractive.

- Low viscosity resins usually need to be used with their attendant lower mechanical and health and safety properties.

11 Pultrusion



Description:

Fibres are pulled from a creel through a resin bath and then on through a heated die. The die completes the impregnation of the fibre, controls the resin content and cures the material into its final shape as it passes through the die. This cured profile is then automatically cut to length. Fabrics may also be introduced into the die to provide fibre direction other than at 0°. Although pultrusion is a continuous process, producing a profile of constant cross-section, a variant known as 'pulforming' allows for some variation to be introduced into the cross-section. The process pulls the materials through the die for impregnation, and then clamps them in a mould for curing. This makes the process non-continuous, but accommodating of small changes in cross-section.

Material Options:

- Resins: Generally epoxy, polyester, vinylester and phenolic
- Fibres: Any
- Cores: Not generally used

Typical Applications:

Beams and girders used in roof structures, bridges, ladders, frameworks

Advantages:

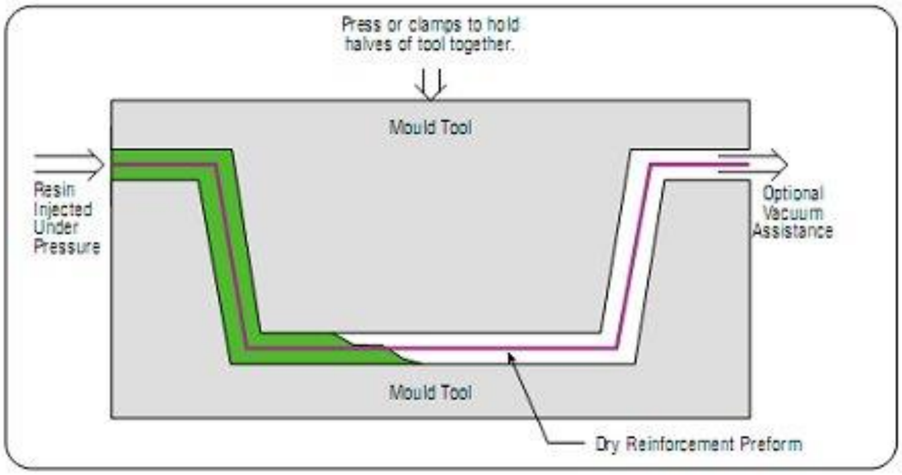
- This can be a very fast, and therefore economic, way of impregnating and curing materials.
- Resin content can be accurately controlled.
- Fibre cost is minimised since the majority is taken from a creel.

- Structural properties of laminates can be very good since the profiles have very straight fibres and high fibre volume fractions can be obtained.
- Resin impregnation area can be enclosed thus limiting volatile emissions.

Disadvantages:

- Limited to constant or near constant cross-section components.
- Heated die costs can be high.

Resin Transfer Moulding (RTM)



Description:

Fabrics are laid up as a dry stack of materials. These fabrics are sometimes pre-pressed to the mould shape, and held together by a binder. These 'preforms' are then more easily laid into the mould tool. A second mould tool is then clamped over the first, and resin is injected into the cavity. Vacuum can also be applied to the mould cavity to assist resin in being drawn into the fabrics. This is known as Vacuum Assisted Resin Injection (VARI). Once all the fabric is wet out, the resin inlets are closed, and the laminate is allowed to cure. Both injection and cure can take place at either ambient or elevated temperature.

Material Options:

- Resins: Generally epoxy, polyester, vinylester and phenolic, although high temperature resins such as bismaleimides can be used at elevated process temperatures.
- Fibres: Any. Stitched materials work well in this process since the gaps allow rapid resin transport. Some specially developed fabrics can assist with resin flow
- Cores: Not honeycombs, since cells would fill with resin, and pressures involved can crush some foams

Typical Applications:

Small complex aircraft and automotive components, train seats.

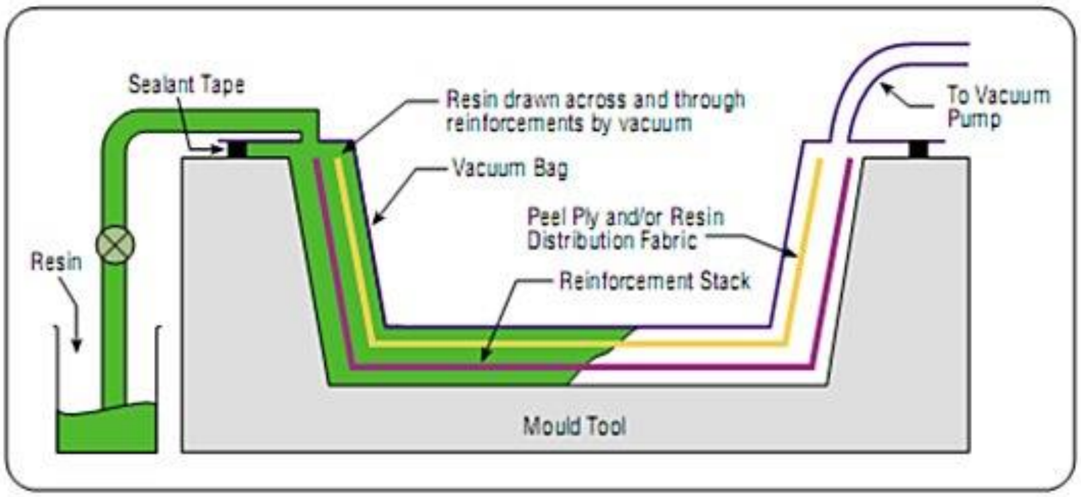
Advantages:

- High fibre volume laminates can be obtained with very low void contents.
- Good health and safety, and environmental control due to enclosure of resin.
- Possible labour reductions.
- Both sides of the component have a moulded surface.

Disadvantages:

- Matched tooling is expensive and heavy in order to withstand pressures.
- Generally limited to smaller components.
- Unimpregnated areas can occur resulting in very expensive scrap parts.

Other Infusion Processes



Description:

Fabrics are laid up as a dry stack of materials as in RTM. The fibre stack is then covered with peel ply and a knitted type of non-structural fabric. The whole dry stack is then vacuum bagged, and once bag leaks have been eliminated, resin is allowed to flow into the laminate. The resin distribution over the whole laminate is aided by resin flowing easily through the non-structural fabric, and wetting the fabric out from above.

Materials Options:

- Resins: Generally epoxy, polyester and vinylester.
- Fibres: Any conventional fabrics. Stitched materials work well in this process since the gaps allow rapid resin transport.
- Cores: Any except honeycombs.

Typical Applications:

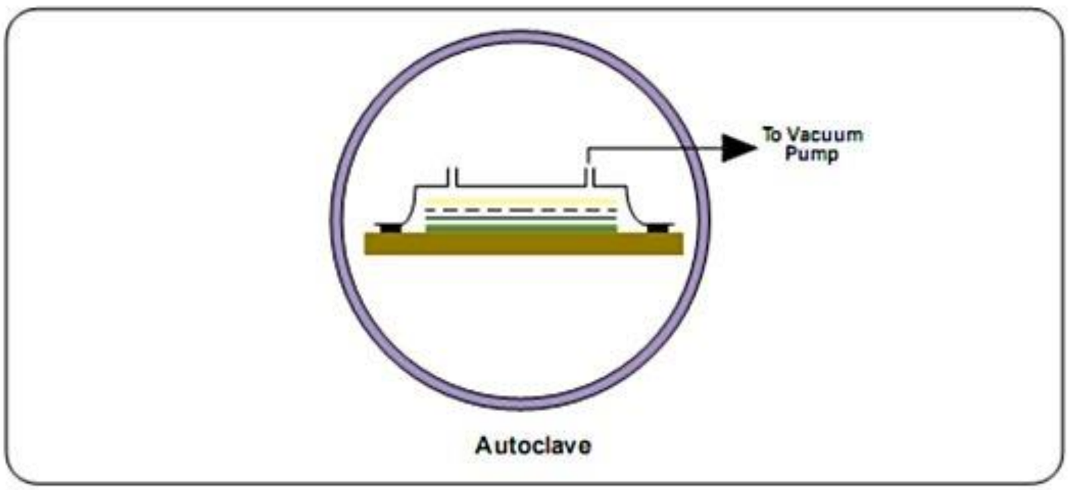
Semi-production small yachts, train and truck body panels

Advantages:

- As RTM above, except only one side of the component has a moulded finish.
- Much lower tooling cost due to one half of the tool being a vacuum bag, and less strength being required in the main tool.
- Large components can be fabricated.
- Standard wet lay-up tools may be able to be modified for this process.
- Cored structures can be produced in one operation.

Disadvantages:

- Relatively complex process to perform well.
- Resins must be very low in viscosity, thus comprising mechanical properties.
- Unimpregnated areas can occur resulting in very expensive scrap parts.



Description:

Fabrics and fibres are pre-impregnated by the materials manufacturer, under heat and pressure or with solvent, with a pre-catalyzed resin. The catalyst is largely latent at ambient temperatures giving the materials several weeks, or sometimes months, of useful life when defrosted. However to prolong storage life the materials are stored frozen. The resin is usually a near-solid at ambient temperatures, and so the pre-impregnated materials (prepregs) have a light sticky feel to them, such as that of adhesive tape. Unidirectional materials take fibre direct from a creel, and are held together by the resin alone. The prepregs are laid up by hand or machine onto a mould surface, vacuum bagged and then heated to typically 120-180°C. This allows the resin to initially reflow and eventually to cure. Additional pressure for the moulding is usually provided by an autoclave (effectively a pressurized oven) which can apply up to 5 atmospheres to the laminate.

Materials Options:

- Resins: Generally epoxy, polyester, phenolic and high temperature resins such as polyimides, cyanate esters and bismaleimides.
- Fibres: Any. Used either direct from a creel or as any type of fabric.
- Cores: Any, although special types of foam need to be used due to the elevated temperatures involved in the process.

Typical Applications:
Aircraft structural components (e.g. wings and tail sections), F1 racing cars, sporting goods such as tennis racquets and skis.

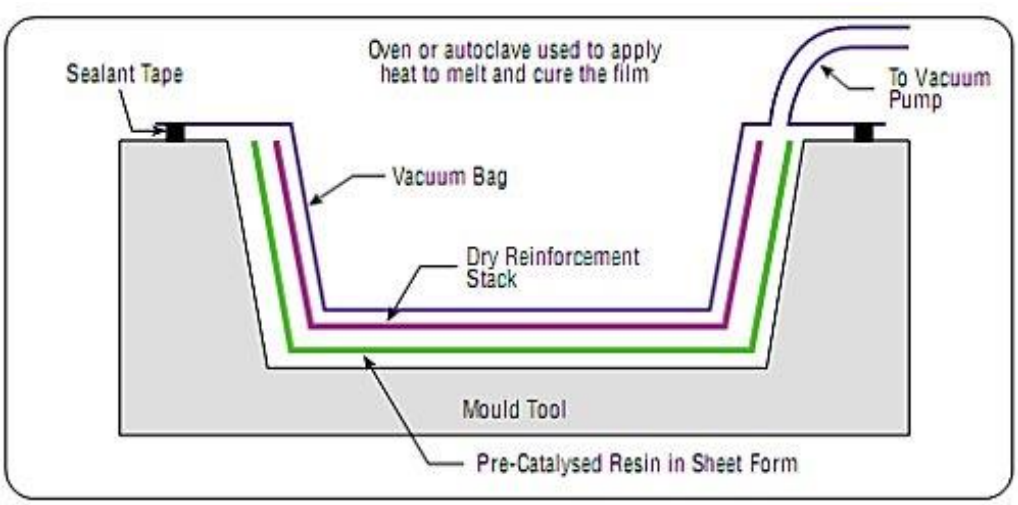
Advantages:

- Resin/catalyst levels and the resin content in the fibre are accurately set by the materials manufacturer. High fibre contents can be safely achieved.
- The materials have excellent health and safety characteristics and are clean to work with.
- Fibre cost is minimized in unidirectional tapes since there is no secondary process to convert fibre into fabric prior to use.
- Resin chemistry can be optimized for mechanical and thermal performance, with the high viscosity resins being impregnable due to the manufacturing process.
- The extended working times (of up to several months at room temperatures) means that structurally optimized, complex lay-ups can be readily achieved.
- Potential for automation and labour saving.

Disadvantages:

- Materials cost is higher for preimpregnated fabrics.
- Autoclaves are usually required to cure the component. These are expensive, slow to operate and limited in size.
- Tooling needs to be able to withstand the process temperatures involved.
- Core materials need to be able to withstand the process temperatures and pressures.

Resin Film Infusion (RFI)



Description:

Dry fabrics are laid up interleaved with layers of semi-solid resin film supplied on a release paper. The lay-up is vacuum bagged to remove air through the dry fabrics, and then heated to allow the resin to first melt and flow into the air-free fabrics, and then after a certain time, to cure.

Materials Options:

- Resins: Generally epoxy only.
- Fibres: Any
- Cores: Most, although PVC foam needs special procedures due to the elevated temperatures involved in the process

Typical Applications:

Aircraft radomes and submarine sonar domes.

Main Advantages:

- High fibre volumes can be accurately achieved with low void contents.
- Good health and safety and a clean lay-up, like prepreg.
- High resin mechanical properties due to solid state of initial polymer material and elevated temperature cure.
- Potentially lower cost than prepreg, with most of the advantages.
- Less likelihood of dry areas than SCRIMP process due to resin traveling through fabric thickness only.

Disadvantages:

- Not widely proven outside the aerospace industry.
- An oven and vacuum bagging system is required to cure the component as for prepreg, although the autoclave systems used by the aerospace industry are not always required.
- Tooling needs to be able to withstand the process temperatures of the resin film (which if using similar resin to those in low-temperature curing prepregs, is typically 60-100°C).
- Core materials need to be able to withstand the process temperatures and pressures.

UNIT-III

MECHANICAL PROPERTIES STIFNESS AND STRENGTH

Mechanical Properties Stiffness and Strength

Volume Fractions

Consider a composite consisting of fiber and matrix. Take the following symbol notations:

$v_{c,f,m}$ = volume of composite, fiber, and matrix, respectively

$\rho_{c,f,m}$ = density of composite, fiber, and matrix, respectively.

Now define the fiber volume fraction

Vf and the matrix volume fraction Vm as

$$V_f = \frac{v_f}{v_c}, \qquad V_m = \frac{v_m}{v_c}.$$

Note that the sum of volume fractions is

$$V_f + V_m = 1, \quad v_f + v_m = v_c.$$

Mass Fractions

Consider a composite consisting of fiber and matrix and take the following

symbol notation:

wc,f,m= mass of composite, fiber, and matrix, respectively. The mass fraction (weight fraction) of the fibers (*Wf*) and the matrix (*Wm*) are defined as

$$W_f = \frac{w_f}{w_c}, \text{ and}$$

$$W_m = \frac{w_m}{w_c}.$$

Note that the sum of mass fractions is

$$W_f + W_m = 1, \quad w_f + w_m = w_c.$$

From the definition of the density of a single material,

$$w_c = r_c v_c,$$

$$w_f = r_f v_f, \text{ and}$$

$$w_m = r_m v_m.$$

Substituting above equations, the mass fractions and volume fractions are related as

$$W_f = \frac{\rho_f}{\rho_c} V_f \text{, and}$$

$$W_m = \frac{\rho_m}{\rho_c} V_m,$$

in terms of the fiber and matrix volume fractions. In terms of individual constituent properties, the mass fractions and volume fractions are related by

$$W_f = \frac{\frac{\rho_f}{\rho_m}}{\frac{\rho_f}{\rho_m} V_f + V_m} V_f,$$

$$W_m = \frac{1}{\frac{\rho_f}{\rho_m}(1 - V_m) + V_m} V_m .$$

One should always state the basis of calculating the fiber content of a composite. It is given in terms of mass or volume. Based on above Equation, it is evident that volume and mass fractions are not equal and that the mismatch between the mass and volume fractions increases as the ratio between the density of fiber and matrix differs from one.

Density

The derivation of the density of the composite in terms of volume fractions is found as follows. The mass of composite *wc* is the sum of the mass of the fibers *wf* and the mass of the matrix *wm* as

$$w_c = w_f + w_m.$$

$$\rho_c v_c = \rho_f v_f + \rho_m v_m, \quad \rho_c = \rho_f \frac{v_f}{v_c} + \rho_m \frac{v_m}{v_c} .$$

Using the definitions of fiber and matrix volume fractions from Equation

$$\rho_c = \rho_f V_f + \rho_m V_m.$$

Now, consider that the volume of a composite *vc* is the sum of the volumes of the fiber *vf* and matrix (*vm*):

$$v_c = v_f + v_m .$$

The density of the composite in terms of mass fractions can be found as

$$\frac{1}{\rho_c} = \frac{W_f}{\rho_f} + \frac{W_m}{\rho_m}.$$

Typical Properties of Fibers (SI System of Units)

Property	Units	Graphite	Glass	Aramid
Axial modulus	GPa	230	85	124
Transverse modulus	GPa	22	85	8
Axial Poisson's ratio	—	0.30	0.20	0.36
Transverse Poisson's ratio	—	0.35	0.20	0.37
Axial shear modulus	GPa	22	35.42	3
Axial coefficient of thermal expansion	µm/m/°C	–1.3	5	–5.0
Transverse coefficient of thermal expansion	µm/m/°C	7.0	5	4.1
Axial tensile strength	MPa	2067	1550	1379
Axial compressive strength	MPa	1999	1550	276
Transverse tensile strength	MPa	77	1550	7
Transverse compressive strength	MPa	42	1550	7
Shear strength	MPa	36	35	21
Specific gravity	—	1.8	2.5	1.4

Typical Properties of Matrices (SI System of Units)

Property	Units	Epoxy	Aluminum	Polyamide
Axial modulus	GPa	3.4	71	3.5
Transverse modulus	GPa	3.4	71	3.5
Axial Poisson's ratio	—	0.30	0.30	0.35
Transverse Poisson's ratio	—	0.30	0.30	0.35
Axial shear modulus	GPa	1.308	27	1.3
Coefficient of thermal expansion	µm/m/°C	63	23	90
Coefficient of moisture expansion	m/m/kg/kg	0.33	0.00	0.33
Axial tensile strength	MPa	72	276	54
Axial compressive strength	MPa	102	276	108
Transverse tensile strength	MPa	72	276	54
Transverse compressive strength	MPa	102	276	108
Shear strength	MPa	34	138	54
Specific gravity	—	1.2	2.7	1.2

Idealization of Microstructure of Fibrous Composite:

As mentioned earlier, the micromechanics is a study at fibre and matrix level. Thus, the geometry of arrangement of the fibres and matrix in a composite is an essential requirement to develop a model for the study. Some of the methods do not use the geometry of arrangement. Most of the methods developed for micromechanical analysis assume that:

1. The fibers and matrix are perfectly bonded and there is no slip between them.
2. The fibres are continuous and parallel.
3. The fibres are assumed to be circular in cross section with a uniform diameter along its length.
4. The space between the fibres is uniform throughout the composite.
5. The elastic, thermal and hygral properties of fibre and matrix are known and uniform.
6. The fibres and matrix obey Hooke's law.
7. The fibres and the matrix are only two phases in the composite.
8. There are no voids in the composite.

There are many ways to idealize the cross section of a lamina. In Figure 1 are shown two popular idealizations. The most commonly preferred arrangements are square packed and hexagonal packed arrays of fibres in matrix. The square and hexagonal packed arrays can be as shown in Figure 1(a), and (b), respectively.

In these idealizations it is seen that due to symmetry and periodicity of these arrays one can consider only one array to analyze the lamina at micro scale. Further, if this one array represents the general arrangement of fibres with respect to matrix and the interactions of fibre and matrix phases, then such array is called ***Representative Volume Element*** **(RVE)**. Further, this RVE as a volume of material statistically represents a homogeneous material. In the analysis of an RVE the boundary conditions are chosen such that they reflect the periodicity. Thus, the arrays shown in Figure 1 are various RVEs. One should be able to see that the RVE also reflects the volume fractions. The term RVE was first coined by Hill in 1963.
For example, the square RVE represents a lower fibre volume fraction than a hexagonal RVE. Note that RVE is also called as ***Unit Cell.***

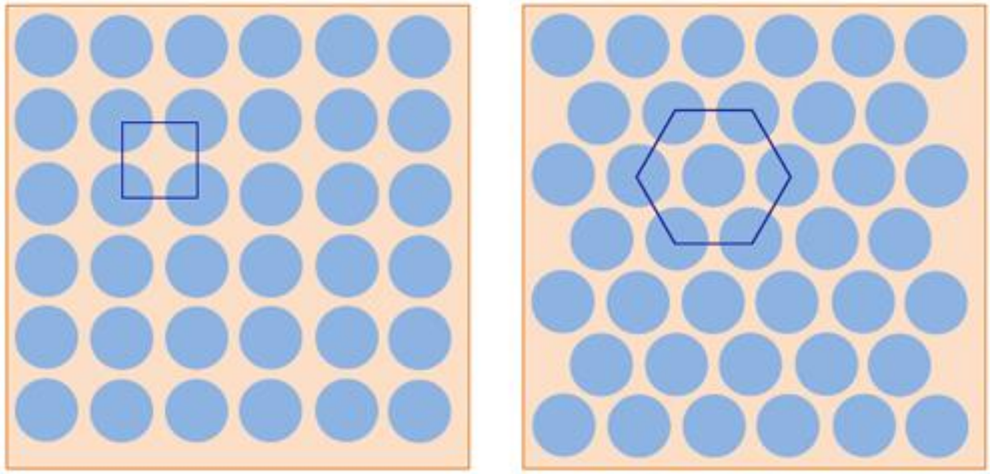


Figure 1.: Idealization of cross section of lamina

StrengthofMaterialApproximations:

In general, the laminates made are thin. Hence, for such laminates the analysis done using Kirchhoff and plane stress assumptions is reasonably good. For such analysis, one needs the engineering constants that occur in defining planar constitutive equations. These engineering constants are:

1. E_1^* - **the axial modulus**
2. $E_2^* = E_3^*$ - **transverse modulus**
3. $\nu_{12}^* = \nu_{13}^*$ - **axial Poisson's ratio (for loading in** x_1**- direction)**
4. $G_{12}^* = G_{13}^*$- **axial shear modulus (shear stress parallel to the fibers)**

Further, it is seen that for transversely isotropic composite, four out of five (the fifth one is G_{23}^*) properties can be developed from this approach. For the planar hygro-thermal analysis of such laminates, one can also obtain the in-plane coefficients of thermal expansions α_1^* and α_2^* and hygroscopic expansion β_1^* and β_2^* as well.

It is important to note that this approach involves assumptions which do not necessarily satisfy the requirements of an exact elasticity solution. In this approach the effective properties will be expressed in terms of the elastic properties and volume fractions of the fiber and matrix. The lamina is considered to be an alternate arrangement of fibres and matrix. The RVE chosen in these derivations is shown in Figure 2. The RVE here does not take into account the cross sectional arrangement of fibres and matrix, rather it represents volume of the material through the cross sectional area of fibre and matrix.

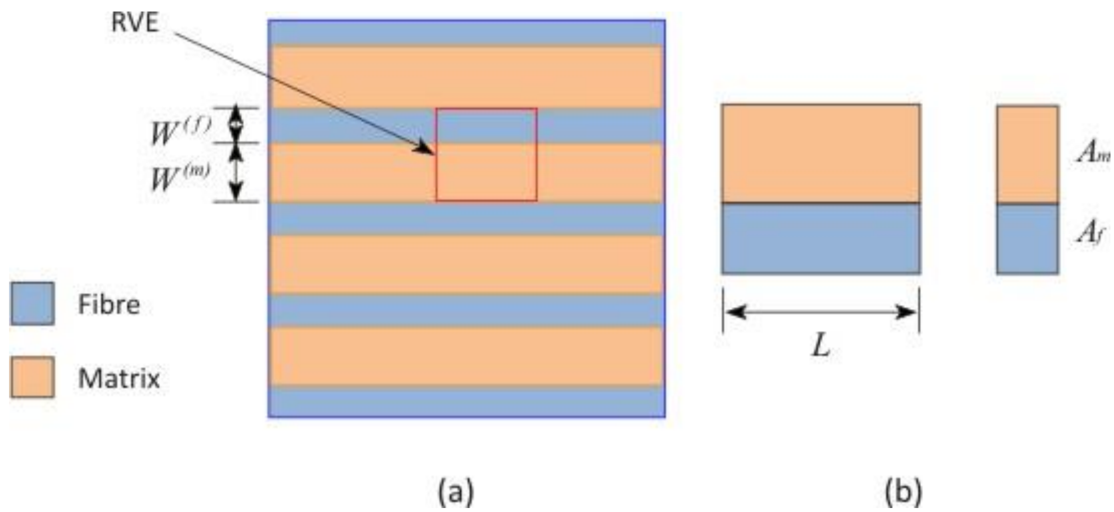


Figure 2: (a) Unidirectional lamina, (b) RVE for unidirectional composite for prediction of elastic properties

Let, A_f and A_m represent fibre area and matrix area, respectively. $W^{(f)}$ and $W^{(m)}$ represent fibre and matrix widths, respectively. L be the length of the RVE.

Effective Axial Modulus E_1^***:**

The unit cell as shown in Figure 2 is used to compute the effective axial modulus E_1^*. It should be noted that the thickness of the unit cell is not important in this computation. Further, the cross sectional shapes are not considered in this calculation. However, the cross sectional areas are important in this calculation. The thicknesses of the fibre and matrix constituents are same in the unit cell. Hence, the areas of the constituents represent the volume fractions of the constituents.

In the calculation of effective axial modulus, it is assumed that the axial strain in the composite is uniform such that the axial strains in the fibers and matrix are identical. This assumption is justified by the fact that the fibre and the matrix in the unit cell are perfectly bonded. Hence, the elongation in the axial direction of the fibre and matrix will also be identical. Thus, the strains in the fibre and matrix can be given as

$$\bar{\varepsilon}_1 = \varepsilon_1^{(f)} = \varepsilon_1^{(m)} = \frac{\Delta L}{L} \qquad (1)$$

where, $\bar{\varepsilon}_1$ is the axial strain in the composite and $\varepsilon_1^{(f)}$ and $\varepsilon_1^{(m)}$ are the axial strains in fibre and matrix, respectively. Now, let $E_1^{(f)}$ and $E^{(m)}$ be the axial Young's moduli of the fibre and matrix, respectively. We can give the axial stress in the fibre, $\sigma_1^{(f)}$ and matrix, $\sigma_1^{(m)}$ as

$$\sigma_1^{(f)} = E_1^{(f)} \varepsilon_1^{(f)} \quad \text{and} \quad \sigma_1^{(m)} = E^{(m)} \varepsilon_1^{(m)}$$

Using the above equation and the cross section areas of the respective constituent in the unit cell, we can calculate the forces in them as

$$F_1^{(f)} = \sigma_1^{(f)} A_f \quad \text{and} \quad F_1^{(m)} = \sigma_1^{(m)} A_m$$

The total axial force in the composite is sum of the axial forces in fibre and matrix. Thus, the total axial force in the composite substituting the expressions for axial strains in fibre and matrix from Equation (1) in above equation, can be given as

$$F_1 = F_1^{(f)} + F_1^{(m)} = \sigma_1^{(f)} A_f + \sigma_1^{(m)} A_m = \left(E_1^{(f)} A_f + E^{(m)} A_m \right) \frac{\Delta L}{L} \quad (2)$$

Now $\bar{\sigma}_1$ be the average axial stress in composite. The total cross sectional area of the composite is $A = A_f + A_m$. Thus, using the average axial stress and cross sectional area of the composite, the axial force is

$$F_1 = \bar{\sigma}_1 A \quad (3)$$

Thus, combining Equation (2) and Equation (3) and rearranging, we get

$$\bar{\sigma}_1 = \left(E_1^{(f)} \frac{A_f}{A} + E^{(m)} \frac{A_m}{A} \right) \frac{\Delta L}{L} \quad (4)$$

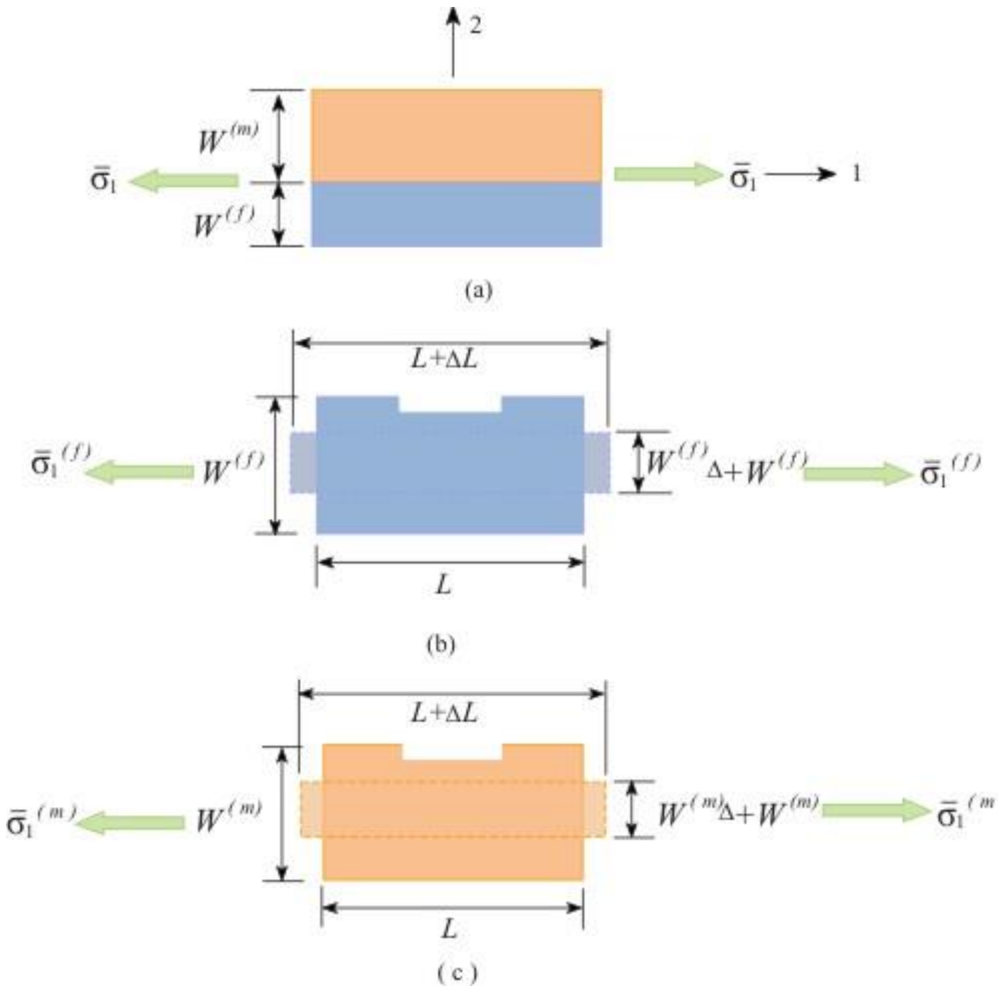


Figure 3: (a) Undeformed unit cell under $\bar{\sigma}_1$ (b) and (c) deformed individual constituents of the unit cell

Let us define

$$\bar{\sigma}_1 = E_1^* \bar{\varepsilon}_1 = E_1^* \frac{\Delta L}{L} \qquad (5)$$

Further, noting that the ratios $\frac{A_f}{A}$ and $\frac{A_m}{A}$ for same length of fibre and matrix represent the fibre and matrix volume fractions, respectively. Thus, combining Equations (4) and (5), we get

$$E_1^* = E_1^{(f)} V_f + E^{(m)} V_m = E_1^{(f)} V_f + E^{(m)} \left(1 - V_f\right) \quad (6)$$

The above equation relates the axial modulus of the composite to the axial moduli of the fibre and matrix through their volume fractions. Thus, the effective axial modulus is a linear function of the fiber volume fraction. This equation is known as rule of mixtures equation. It should be noted that the effective properties are functions of the fiber volume fractions; hence it should always be quoted in reporting the effective properties of a composite.

Effective Axial (Major) Poison's Ratio ν_{12}^{*}**:**

To determine the effective axial Poisson's ratio we consider the loading as in the case applied for determining the effective axial modulus. Here, for this loading we have $\bar{\sigma}_1 \neq 0$ and other stresses are zero. We define the effective axial Poisson's ratio as

$$\nu_{12}^{*} = -\frac{\bar{\varepsilon}_2}{\bar{\varepsilon}_1}$$

The effective strain in direction 2 from Figure 3(b) and (c) can be given as

$$\bar{\varepsilon}_2 = \frac{\Delta W}{W} = \frac{\Delta W^{(f)} + \Delta W^{(m)}}{W^{(f)} + W^{(m)}}$$

Now, the changes in $W^{(f)}$ and $W^{(m)}$ can be obtained using the Poisson's ratio of individual constituents. The axial Poisson's ratios for fibre and matrix are given as

$$\nu_{12}^{(f)} = -\frac{\varepsilon_2^{(f)}}{\varepsilon_1^{(f)}} = -\frac{\Delta W^{(f)} / W^{(f)}}{\Delta L / L} \quad \text{and} \quad \nu^{(m)} = -\frac{\varepsilon_2^{(m)}}{\varepsilon_1^{(m)}} = -\frac{\Delta W^{(m)} / W^{(m)}}{\Delta L / L} \quad (7.24)$$

Thus, the changes in $W^{(f)}$ and $W^{(m)}$ are given as

$$\Delta W^{(f)} = -\nu_{12}^{(f)} W^{(f)} \frac{\Delta L}{L} \quad \text{and} \quad \Delta W^{(m)} = -\nu^{(m)} W^{(m)} \frac{\Delta L}{L} \quad (7.25)$$

The total change in W is given as

$$\Delta W = \Delta W^{(f)} + \Delta W^{(m)} \qquad (7)$$

The strain in direction 2 for the composite can be given using Equation (6) and Equation (7) as

$$\bar{\varepsilon}_2 = \frac{\Delta W}{W} = \frac{\Delta W^{(f)} + \Delta W^{(m)}}{W} = -\left(\nu_{12}^{(f)} \frac{W^{(f)}}{W} + \nu^{(m)} \frac{W^{(m)}}{W} \right) \frac{\Delta L}{L} \quad (8)$$

Here, $\frac{W^{(f)}}{W}$ and $\frac{W^{(m)}}{W}$ denote the fibre and matrix volume fractions for same length of fibre and matrix. Note that $\frac{\Delta L}{L}$ denotes the effective axial strain $\bar{\varepsilon}_1$. Thus, from Eq. (8) the effective axial Poisson's ratio is written as

$$\nu_{12}^{(*)} = \nu_{12}^{(f)} V_f + \nu^{(m)} V_m$$

The above equation is the rule of mixtures expression for composite axial Poisson's ratio.

Effective Transverse Modulus E_2^*:

Here, we are going to derive the effective transverse modulus by loading the RVE in direction 2 as shown in Figure 4(a). There are two considerations while deriving this effective modulus. The first approach considers that the deformation of the each constituent is independent of each other as shown in Figure 4(b) and (c) and the deformation in direction 1 is not considered. The second approach considers that deformations of the fibre and matrix in direction 1 are identical as they are perfectly bonded.

To calculate the effective modulus in direction 2, a stress $\bar{\sigma}_2$ is applied to the RVE as shown in Figure 4(a).

***First Approach*:**

As mentioned, the fibre and matrix deform independently of each other. The resulting deformation in direction 1 is not considered here. This assumption is simplistic and was used by early researchers.
The fibre and matrix are subjected to same state of stress. The state of stress is unidirectional, that is, $\sigma_2^{(f)} = \sigma_2^{(m)} = \bar{\sigma}_2$. Now, using the individual moduli and deformations in direction 2, these stresses can be given as

$$\sigma_2^{(f)} = E_2^{(f)} \varepsilon_2^{(f)} = E_2^{(f)} \frac{\Delta W^{(f)}}{W^{(f)}}$$

$$\sigma_2^{(m)} = E^{(m)} \varepsilon_2^{(m)} = E^{(m)} \frac{\Delta W^{(m)}}{W^{(m)}}$$

From this equation we can write the individual deformations, which give the total deformation in direction 2 as

$$\Delta W = \Delta W^{(f)} + \Delta W^{(m)} = \left(\frac{W^{(f)}}{E_2^{(f)}} + \frac{W^{(m)}}{E^{(m)}} \right) \bar{\sigma}_2$$

Now, the composite strain in direction 2 can be calculated from the definition as

$$\bar{\varepsilon}_2 = \frac{\Delta W}{W} = \left(\frac{W^{(f)}}{W} \frac{1}{E_2^{(f)}} + \frac{W^{(m)}}{W} \frac{1}{E^{(m)}} \right) \bar{\sigma}_2$$

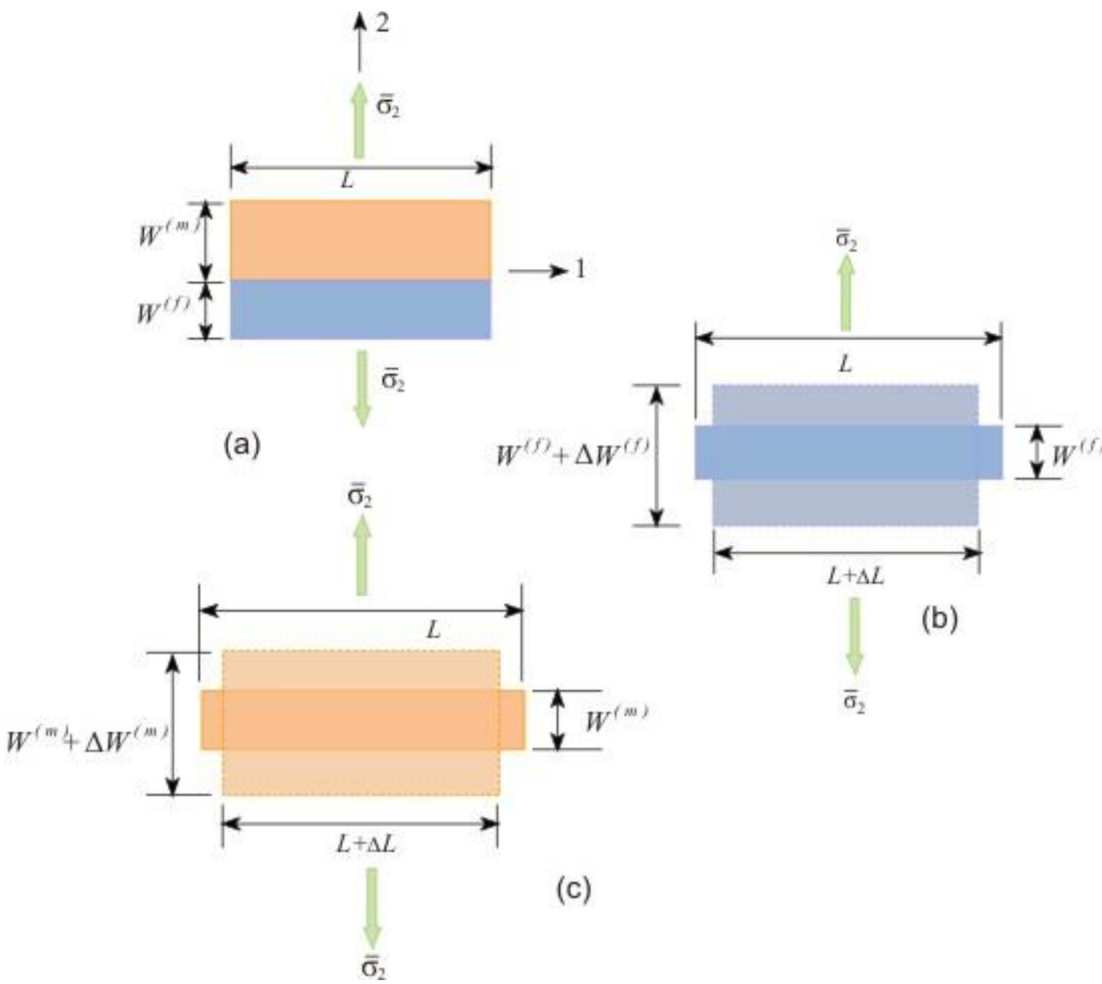


Figure 4: (a) Undeformed unit cell under uniform $\bar{\sigma}_2$ stress (b) and (c) deformed individual constituents of the unit cell

Introducing the volume fractions in the above equation,

$$\bar{\varepsilon}_2 = \left(V_f \frac{1}{E_2^{(f)}} + V_m \frac{1}{E^{(m)}} \right) \bar{\sigma}_2$$

Noting that $\frac{\bar{\sigma}_2}{\bar{\varepsilon}_2} = E_2^*$, from the above equation, we get

$$\frac{1}{E_2^*} = \frac{V_f}{E_2^{(f)}} + \frac{V_m}{E^{(m)}} = \frac{V_f}{E_2^{(f)}} + \frac{\left(1 - V_f\right)}{E^{(m)}}$$

This equation is the rule of mixtures equation for effective modulus E_2^*.

Background to Mechanical Testing of Composites:

Objectives of Mechanical Testing:

The development of the mechanical testing of the materials depends upon other scientific factors. These factors help in better understanding and facilitate the progress in evaluating the various processes. These processes include:

1. quality control of a process
2. quality assurance for the material developed and structure fabricated from thereof
3. better material selection
4. comparisons between available materials
5. can be used as indicators in materials development programmes
6. design analysis
7. predictions of performance under conditions other than test conditions
8. starting points in the formulation of new theories

It should be noted that these processes are dependent upon each other. However, if they are considered individually then the data required can be different for the evaluation. For example, some tests are carried out as multipurpose tests using various processes. A conventional tensile test carried out under fixed conditions may serve quality control function whereas one carried out varying factors like temperature, strain rate, humidity etc. may provide information on load bearing capacity of the material.

The properties evaluated for materials like composite is very sensitive to various internal structure factors. However, these factors depend mainly upon the fabrication process or other factors. The internal structure factors that affect the properties are, in general, at atomic or molecular level. These factors mostly affect the matrix and fibre-matrix interface structure.

The mechanical properties of the fibrous composite depend on several factors of the composition. These factors are listed below again for the sake of completeness.

1. properties of the fibre
2. surface character of the fibre
3. properties of the matrix material
4. properties of any other phase
5. volume fraction of the second phase (and of any other phase)
6. spatial distribution and alignment of the second phase (including fabric weave)
7. nature of the interfaces

Another important factor is processing of the composites. There are many parameters that control the processing of composites that access the quality of adhesion between fibre and matrix, physical integrity and the overall quality of the final structure.

In case of composite the spatial distribution and alignment of fibres are the most dominating factor which causes the variation of properties. The spatial distribution and alignment of the fibres can change during the same fabrication process. Thus, for a given fabrication process the property evaluated from the composite material may show a large variation.

Tensile Testing

The well known purpose of the tensile testing is to measure the ultimate tensile strength and modulus of the composite. However, one can measure the axial Poison's ratio with additional instrumentations. The standard specimen used for tensile testing of continuous fiber composites is a flat, straight-sided coupon. A flat coupons in ASTM standard D 3039/D 3039M-93 for 0° and 90° have been shown.

The specimen, as mentioned above is flat rectangular coupon. The tabs are recommended for gripping the specimen. It protects the specimen from load being directly applied to the specimen causing the damage. Thus, the load is applied to the specimen through the grips. Further, it protects the outer fibres of the materials. The tabs can be fabricated from a variety of materials, including fiberglass, copper, aluminum or the material and laminate being tested. When the tabs of composite material are used then according to ASTM specifications the inner plies of the tabs should match with the outer plies of the composite. This avoids the unwanted shear stresses at the interface of the specimen and tabs. However, the recent versions of the ASTM standards allow the use of tabs with reinforcement at $\pm 45^\circ$. Further, end-tabs can also facilitate accurate alignment of the specimen in the test machine, provided that they are symmetrical and properly positioned on the specimen. The tabs are pasted to the specimen firmly with adhesive.

This specimen can provide data on:

1. The axial modulus E_x,
2. In-plane and through thickness Poisson's ratio γ_{xy}, γ_{xz}
3. Tensile ultimate stress σ_x^{ult},
4. Tensile ultimate strain ε_x^{ult} ,
5. Any nonlinear, inelastic response

In general, the tensile tests are done on coupons with 0° laminae/laminate for corresponding axial properties and coupons with 90° laminae/laminate for corresponding transverse properties. The off axis laminae specimen also provides data on coefficient of mutual influence and the in-plane shear response.

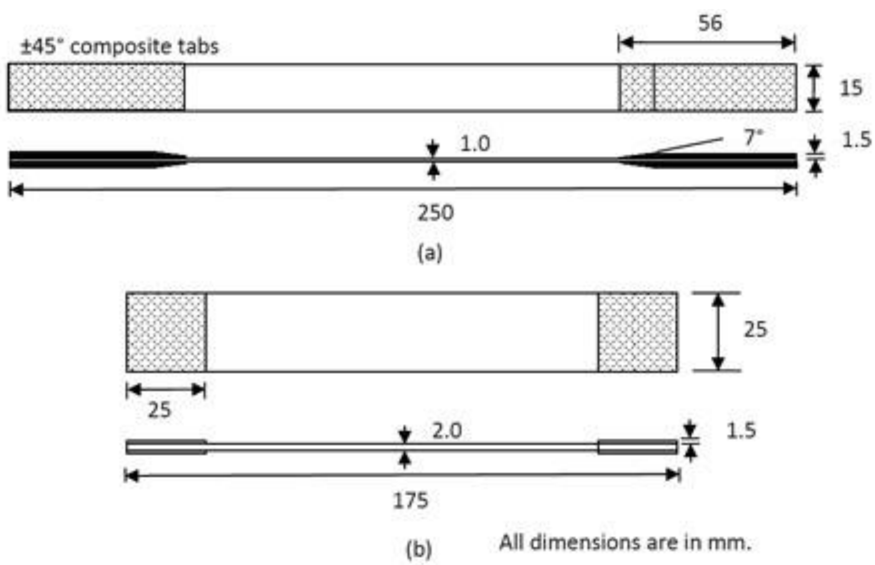


Figure 4:Composite tensile test specimens
(a) ASTM D 3039 for 0° and
(b) ASTM D 3039 for 90°.

***Orthotropic Laminae and Laminate*:**

For orthotropic, symmetric laminates with 0° and 90° laminae, the effective axial modulus and Poisson's ratio is given as

$$E_x = \frac{1}{a_{11}^*}, v_{xv} = \frac{-a_{12}^*}{a_{11}^*}$$

where, the quantities with asterisk are for laminate as mentioned in Chapter on Laminate Theory. These properties can be measured directly from a tensile test on a specimen of thickness t under axial force per unit length N_x as follows:

$$E_x = \frac{\sigma_{xx}}{\varepsilon_{xx}} = \frac{N_{xx}}{t\varepsilon_{xx}}, v_{xy} = -\frac{\varepsilon_{yy}}{\varepsilon_{xx}}$$

The tensile strength is defined as the average stress at failure. Thus, the tensile strength can be given using the maximum applied force per unit length N_x and thickness t as

$$\bar{\sigma}_x^{ult} = \frac{N_x^{\max}}{t}$$

It should be noted that the failure of laminates is often influenced by inter laminar stresses along the free edge effects of the coupon. These factors will be explained in brief in one of the lecture.

The measurement of tensile strength by experiments can also provide information on the comparison of laminate theory with experiments.

Off-Axis Laminae

One can measure the tensile properties by conducting experiments on off-axis laminae. However, there are certain issues associated with this kind of experiments. For example, the presence of axial-shear coupling is associated with the nonzero a_{16}^* . Alternately, one can say that this term is associated with coefficient of mutual influence $\eta_{xy,x}$. Hence, these tests are not straight forward as in case of symmetric laminates with 0° and 90° laminae. Therefore, sometimes these tests are called as specialized tests.

When the experiments are conducted to measure the properties like E_x, v_{xy} and v_{xz} one can get the other properties along with these tests. For example, the coefficient of mutual influence $\eta_{xy,x}$, the nonlinear response and strength of an off-axis lamina for given fibre orientation can also be obtained.

There is an important issue associated with these tests is that what boundary conditions one should impose on the specimen? If a pure, uniform state of axial stress $\sigma_{xx} \neq 0, \sigma_{yy} = \tau_{xy} = 0$ can be applied to the ends and sides of a specimen and the specimen is free to assume any desired deformation pattern, the state of stress will be uniform and constant through-out the specimen. The deformation pattern is shown in Figure 8.4(a).

For uniform, far-field axial stress loading, that is $\sigma_{xx} \neq 0$, the stresses in principal material directions can be given as

$$\sigma_{11} = m^2\sigma_{xx}, \sigma_{22} = n^2\sigma_{xx}, \tau_{12} = -mn\sigma_{xx}$$

Further, the global elastic constants associated with axial stress loading are measured as

$$E_x = \frac{\sigma_{xx}}{\varepsilon_{xx}}, v_{xy} = -\frac{\varepsilon_{yy}}{\varepsilon_{xx}}, \eta_{xy,x} = \frac{\gamma_{xy}}{\varepsilon_{xx}}$$

Combining above two equations, we get

$$\begin{aligned}
\varepsilon_{11} &= \frac{\sigma_{11}}{E_1} - \frac{v_{21}\sigma_{22}}{E_2} = (m^2 - n^2 v_{12})\frac{\sigma_{xx}}{E_1} \\
\varepsilon_{22} &= -\frac{v_{12}\ \sigma_{11}}{E_1} + \frac{\sigma_{22}}{E_2} = \left(-\frac{v_{12} m^2}{E_1} + \frac{n^2}{E_2}\right)\sigma_{xx} \\
\gamma_{12} &= \frac{\tau_{12}}{G_{12}} = \frac{-mn\ \sigma_{xx}}{G_{12}}
\end{aligned}$$

From the above equation all three strain components can be obtained for non zero value of axial stress. Thus, from the third of the above equation we can find the shear modulus.

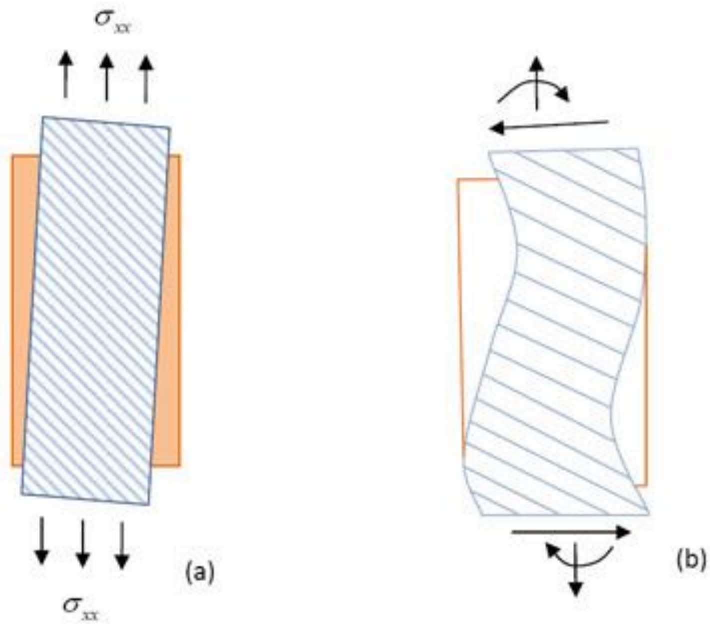


Figure 5: Axial load on off-axis laminae with effect of end constraint
(a) unconstrained displacement and
(b) constrained displacement

It is not easy to apply pure, uniform tensile stress to an off-axis coupon. The specimens are gripped in such a manner that the ends of the specimen are constrained and boundary condition is actually a specification of the axial end displacement. Further, there are more issues with these tests like the constrained displacement induces a doubly curved displacement field in the specimen. The deformed shape of the coupon with restrictions on the ends is depicted in Figure 5(b). We will not deal the complete analysis for the measurements of the properties with tests on off-axis laminae.

The bone shaped specimens for chopper-fiber, metal matrix composite tensile tests. More details can be seen in ASTM D3552-77(1989). Further, for the tensile testing for transverse properties of hoop-wound polymer matrix composite cylinders are used. The details of this testing can be seen in ASTM D5450/D5450M-93.

Measurement of modulus

It should be noted that due to progressive damage the stiffness of the lamina or laminae/laminate changes causing the stress strain curve to be non-linear. The measurement of modulus in a tensile testing from a non-linear loading curve can be done by three methods.

In the first method the modulus is taken as a tangent to the initial part of the curve. In the second method a tangent is constructed at a specified strain level. For example, in the Figure 5 the modulus is measured at 0.25% strain or 0.0025 strain (Point B). In the third method, a secant is constructed between two points. For example in Figure 8.5 a secant is constructed between points A and B.

Typically, the strain values at these points are 0.0005 and 0.0025. In ASTM standards the secant is called as *chord*. The modulus measured by these methods is known as **'*initial tangent modulus*', '*B% modulus*' and '*A%-B% secant (chord) modulus*',** respectively.

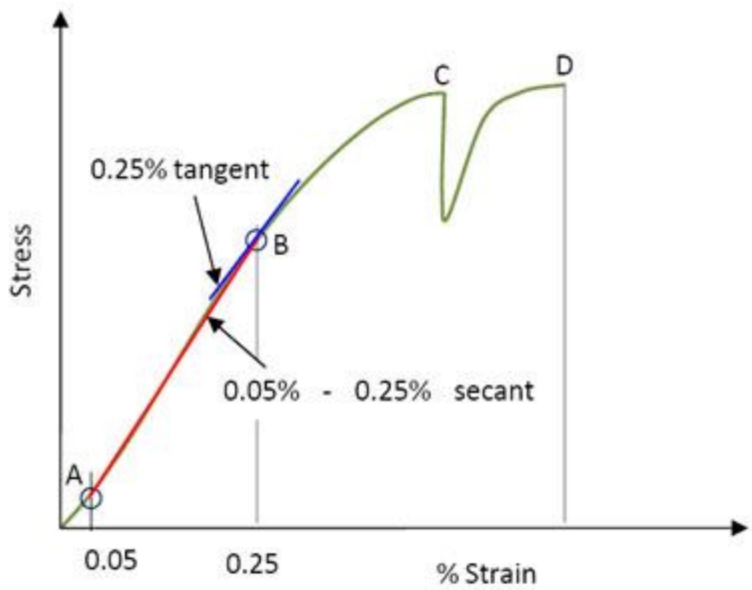


Figure 6: Typical tensile stress-strain curve with details

Compression Testing

Most of the structural members include the compression members. Such members can be loaded directly in compression or under a combination of flexural and compression loading. The axial stiffness of such members depends upon the cross-sectional area. Thus, it is proportional to the weight of the structure. One can alter the stiffness by changing the geometry of the cross section within limits. However, some of the composites have low compressive strength and this fact limits the full potential application of these composites.

The compression testing of the composites is very challenging due to various reasons. The application of compressive load on the cross section can be done in three ways: directly apply the compressive load on the ends of a specimen, loading the edges in shear and mixed shear and direct loading. These three ways of imposing the loads for compression testing are shown .

During compression loading the buckling of the specimen should be avoided. This demands a special requirement on the holding of the specimen for loading purpose. Further, it demands for special geometry of the specimen. These specimens are smaller in size as compared to the tensile testing specimens. A compression test specimen according to ASTM D695 (modified) standard is shown.

The compression testing of composites is a vast topic. Additional reading on this topic from other literature is suggested to readers.

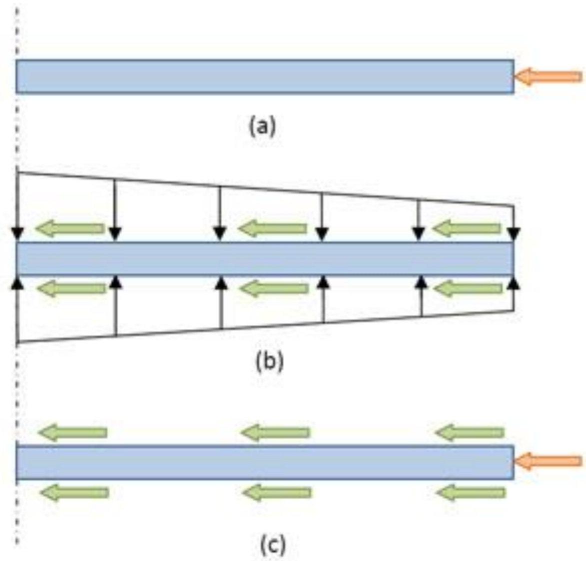


Figure 7: Load imposition methods for compression testing. (a) Direct end loading (b) Shear loading and (c) Mixed shear and direct loading

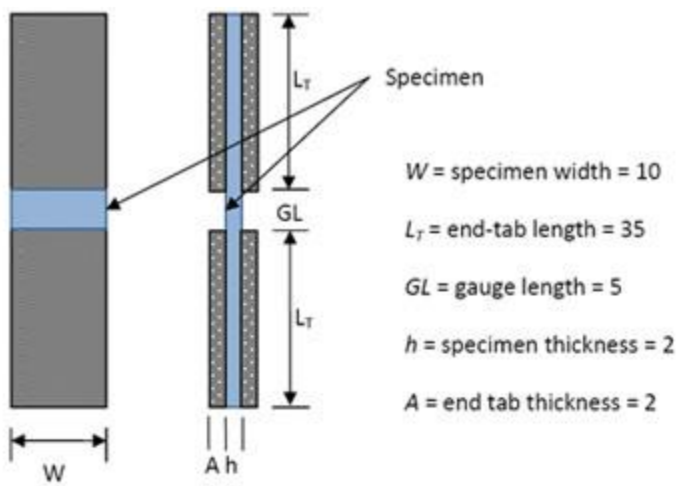


Figure 8: Composite compression test specimen according to ASTM D695 (modified) standard.

Shear Testing

Here we will see measurement of in-plane shear modulus G_{12} only. The methods are listed below:

1. Tension of a $[\pm 45]_s$ laminate

2. Tension of an off-axis lamina
3. Torsion of a unidirectional tube
4. Iosipescu shear of unidirectional laminae and cross ply laminates
5. Rail shear of unidirectional laminae
6. Picture frame test

1. $[\pm 45]_s$***Tensile Test*:**

A tension test on $[\pm 45]_s$ laminate is popularly used test for the measurement of in-plane shear modulus G_{12}. The more details of this test are available in ATSM standard D3518/D3518/M–91. According to ASTM standard the method uses a 250 mm long rectangular specimen with width 25 mm and thickness 2 mm. Further, it is recommended that for materials constructed with layers thicker than 0.125mm, the laminate should consist of 16 layers, that is, $[\pm 45]_{4s}$.The specimen is shown . The dimensions in this figure are in mm.

When a $[\pm 45]_{4s}$ is subjected to axial tensile stress $\bar{\sigma}_{xx}$ then stresses in principal material coordinates developed in each of the $+45^\circ$ and -45° lamina are given as

$$\sigma_{11} = B\bar{\sigma}_{xx}$$
$$\sigma_{22} = (1-B)\bar{\sigma}_{xx}$$
$$\tau_{12} = \frac{-1}{2mn}[B(1-2m^2)+m^2]\bar{\sigma}_{xx}$$

where,

$$B = \left[\frac{m^2(2m^2-1)+4m^2n^2\frac{G_{12}}{E_2}\left(\frac{E_2}{E_1}\,v_{12}+1\right)}{4m^2n^2\,\frac{G_{12}}{E_2}\left(\frac{E_2}{E_1}+2\frac{E_2}{E_1}\,v_{12}+1\right)+(2m^2-1)(m^2-n^2)}\right]$$

and other quantities as defined in earlier chapters. For a special case with $\theta = 45^\circ$ we get the shear stress as

$$\tau_{12} = (\pm\theta) = \mp\frac{\bar{\sigma}_{xx}}{2}$$

Thus, from this equation one can see that the shear stress in principal material directions is statically determinate, that is, it is independent of material properties of the specimen and only depends upon the magnitude of the applied stress. The magnitude of this stress is half of the applied stress.

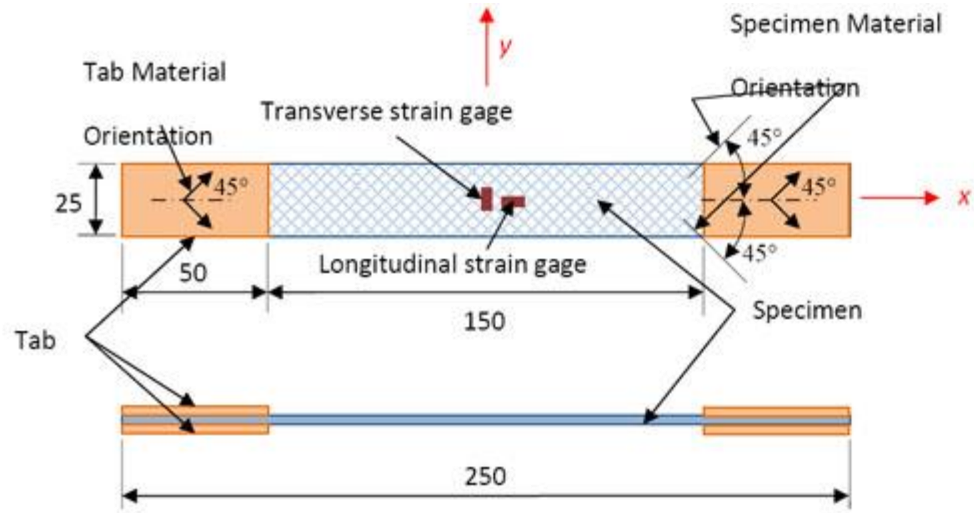


Figure 9: Specimen geometry and strain gage positioning for $[\pm 45]_{4s}$ tensile testing

From the knowledge of linear elastic behaviour of the orthotropic materials it is clear that the shear response is uncoupled from the normal response. Hence, in-plane shear modulus G_{12} can be determined directly from a tensile test on a $[\pm 45]_{4s}$ laminate.

Now the shear strain γ_{12} in principal material coordinates can be found by transformation of the measured axial and transverse strains ε_{xx} and ε_{yy}. It should be noted that the shear strain γ_{xy} is zero for orthotropic laminates under tension and γ_{12} is independent of γ_{xy} for $\theta = \pm 45^{\circ}$ (see the strain transformation relations). Thus, from the strain transformation relations, we can get the shear strain in principal material directions as

$$\gamma_{12} = -(\varepsilon_{xx} - \varepsilon_{yy})$$

Thus, from the definition of the shear modulus we get

$$G_{12} = \frac{\bar{\sigma}_{xx}}{2(\varepsilon_{xx} - \varepsilon_{yy})}$$

The above equation can be rearranged in the following manner to express the shear modulus in terms of effective properties of $[\pm 45]_{4s}$ laminate.

$$G_{12} = \frac{\frac{\bar{\sigma}_{xx}}{\varepsilon_{xx}}}{2\left(\frac{\varepsilon_{xx}}{\varepsilon_{xx}} - \frac{\varepsilon_{yy}}{\varepsilon_{xx}}\right)} = \frac{E_x}{2(1+v_{xy})}$$

Here, E_x is the effective modulus of the $[\pm 45]_{4s}$ laminate.

The measurement of in-plane shear modulus from shear stress-strain curve is done as follows. The shear stress-strain curve for $\pm 45^\circ$ specimen is obtained first. A typical shear stress-strain curve for such a specimen is shown. The shear modulus is obtained from the initial slope of the this curve in the range of 0.1-0.5% strain as

$$G_{12} = \frac{\tau_{12}^{'} - \tau_{12}^{'}}{\gamma_{12}^{'} - \gamma_{12}^{'}}$$

The tensile test on $\pm 45^\circ$ specimen provides an acceptable method for the measurement of in-plane shear modulus. However, one should be careful while interpreting the ultimate shear strength and strain. It should be noted that the laminae are subjected to a biaxial state of stress and not a pure shear. The normal stresses act along the shear planes causing the onset of mixed mode fracture. Other kind of failure like multiple ply cracking, fibre rotation and edge or internal delaminations occur prior to final failure. Therefore, the true failure is very difficult to determine. The shear strength is specified by different standards corresponding either to the ultimate load generated during the test or to a specified strain level. It is recommended in ISO standard that the test be terminated at $\gamma_{12} = 5\%$. The shear strength is taken as the peak load at or before 5% strain.

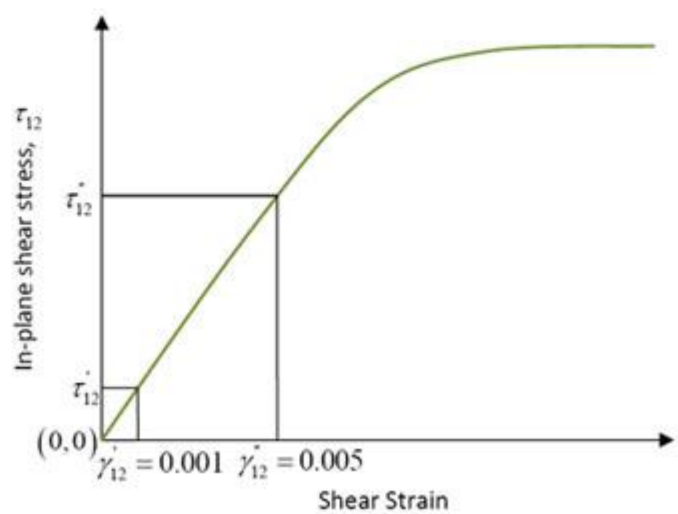


Figure 10: Typical shear stress-strain curve for specimen

***2. Shear Of an Off-Axis Lamina*:**

In similar way to the tensile testing of a $[\pm 45]_{4s}$ laminate one can use a unidirectional off-axis tensile coupon to determine the shear response of a composite in the principal material coordinates. A tensile test on 10° off-axis lamina is a commonly used. Specimen has same geometry . The state of stress in principal material coordinate directions can be obtained from transformation relations. Since, the shear response in the principal material coordinates is uncoupled from the normal response we can write the shear modulus as

$$G_{12} = \frac{\tau_{12}}{\gamma_{12}}$$

The shear stress in the principal material directions due to axial tensile stress can be given using transformation relations as

$$\bar{\tau}_{12} = -mn\bar{\sigma}_{xx}$$

The shear strain is measured from the strains $\varepsilon_{xx}, \varepsilon_{yy}$ and γ_{xy} with the help of strain transformation relations. Then the apparent shear modulus $\bar{G}_{12}$ can be given as

$$\bar{G}_{12} = \frac{-mn\bar{\sigma}_{xx}}{\gamma_{12}}$$

***1. Rail Shear Test*:**

This is a very popular method used to measure in-plane shear properties. This method is extensively used in aerospace industry. The shear loads are imposed on the edges of the laminate using specialized fixtures. There are two types of such fixtures: Two rail and three rail fixture. The ASTM D4255 standard covers the specification for two and three rail specimens for both continuous and discontinuous (0° and 90° fibre alignment), symmetric laminates and randomly oriented fibrous laminates.

a. Two Rail Shear Test

The two rail shear test fixture along with a laminate to be tested is shown . The Figure shows the specimen geometry according to ASTM D4255 standard. The two rail shear test fixture has two rigid parallel steel rails for loading purpose. The rails are aligned to the loading direction as shown . Thus, it induces the shear load in the specimen which is bolted to these rails. A strain gage is bonded at 45° to the longitudinal axis of the specimen.

The Shear strength is obtained as

$$\tau_{xy}^{ult} = \frac{P_{\max}}{Lh}$$

where, $P_{\max}$ is ultimate failure load, L is the specimen length along the rails and h is the specimen thickness.

The shear modulus is given as

$$G_{xy} = \frac{\Delta\tau_{xy}}{\Delta\gamma_{xy}} = \frac{\Delta P}{2Lh\Delta\varepsilon_{45}}$$

where, ΔP is the change in applied load and $\Delta\varepsilon_{45}$ is the change in strain for $+45^{\circ}$ or -45° strain gage in the initial linear stress-strain regime. It is suggested that the change in the strain is taken as the average of the change in strains on the both sides of the specimen.

Various modes of failure are seen. The modes are highly dependent upon the microstructure of the material.

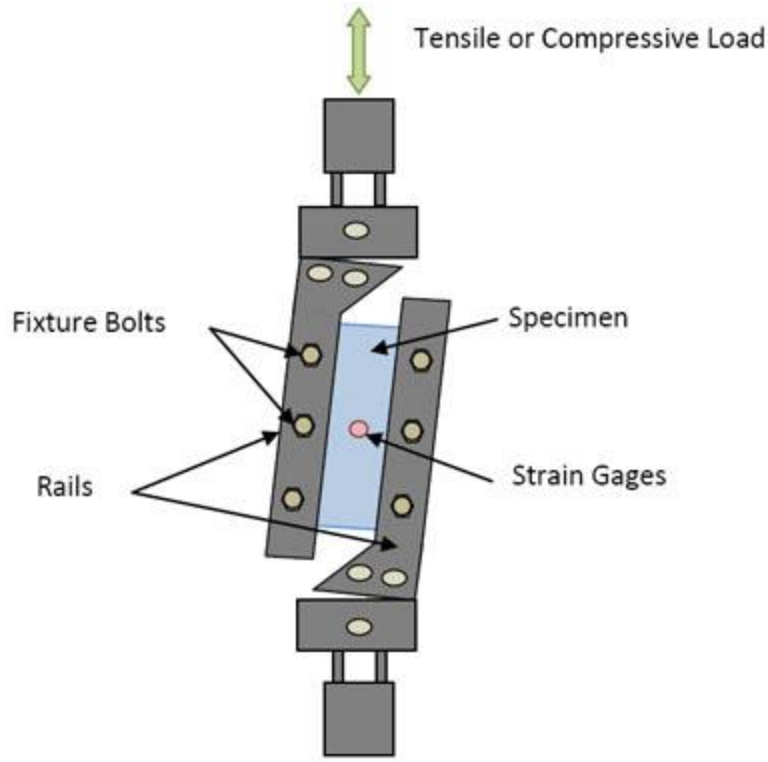


Figure 11: Two rail shear fixture for shear testing

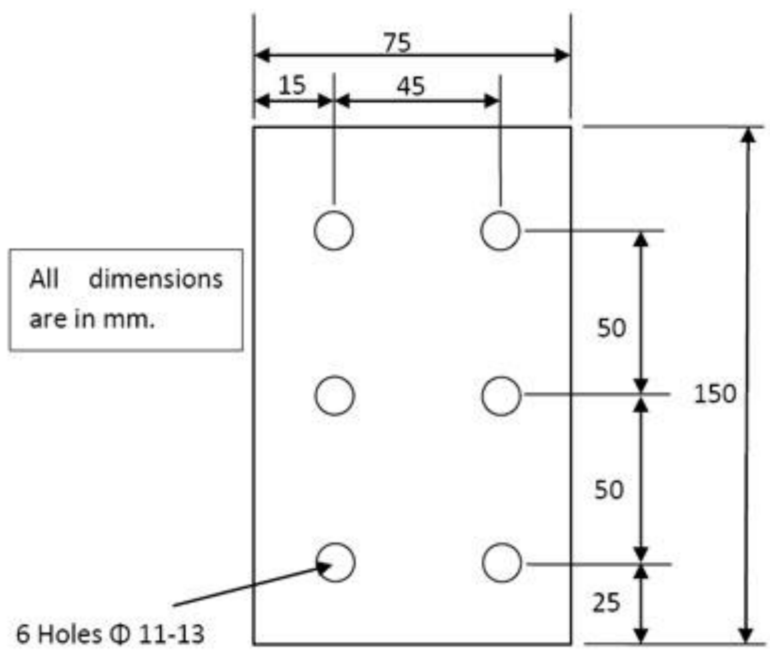


Figure 12: Two rail shear test specimen

***b. Three Rail (Symmetric) Shear Test*:**

The three rail shear test is an improved version of the rail shear test. Using one more rail in two rail shear test fixture it can produce a closer approximation to pure shear. The fixture consists of 3 pairs of rails clamped to the test specimen as shown . The outside pairs are attached to a base plate which rests on the test machine. Another pair (third middle) pair of rails is guided through a slot in the top of the base fixture. The middle pair loaded in compression. The shear force in laminate is generated via friction between rail and specimen. The strain gages bonded to the specimen at 45° to the specimen's longitudinal axis. The specimen geometry is shown.
The shear strength is given as

$$\tau_{xy}^{ult} = \frac{P_{\max}}{2Lh}$$

And the shear modulus is given as

$$G_{xy} = \frac{\Delta\tau_{xy}}{\Delta\gamma_{xy}} = \frac{\Delta P}{4Lh\Delta\varepsilon_{45}}$$

where, all variables in these two equations are given previously.

It should be noted that the holes in the specimen are slightly oversized than the bolts used for clamping. Further, the bolts are tightened in such a manner to ensure that there is no bearing contact between the bolt and specimen in the loading direction. It is recommended that each bolt is tightened with a 100 Nm torque.

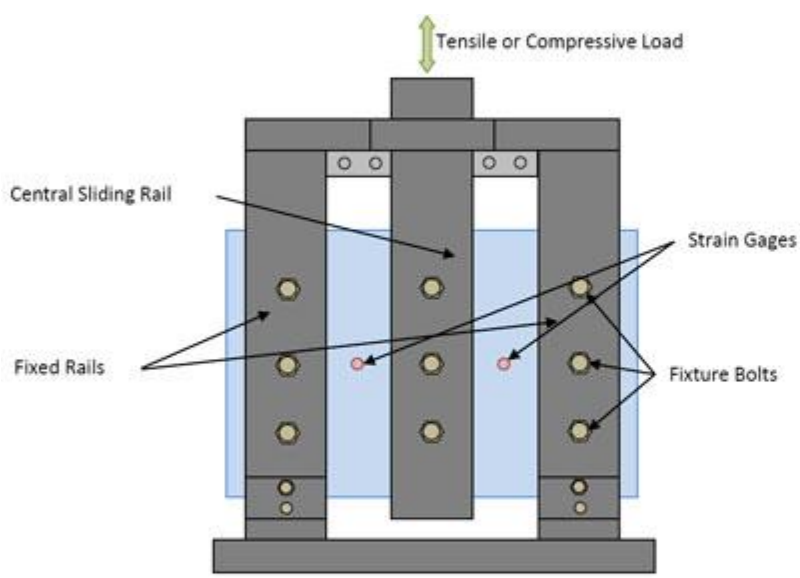


Figure 12: Three rail shear fixture for shear testing

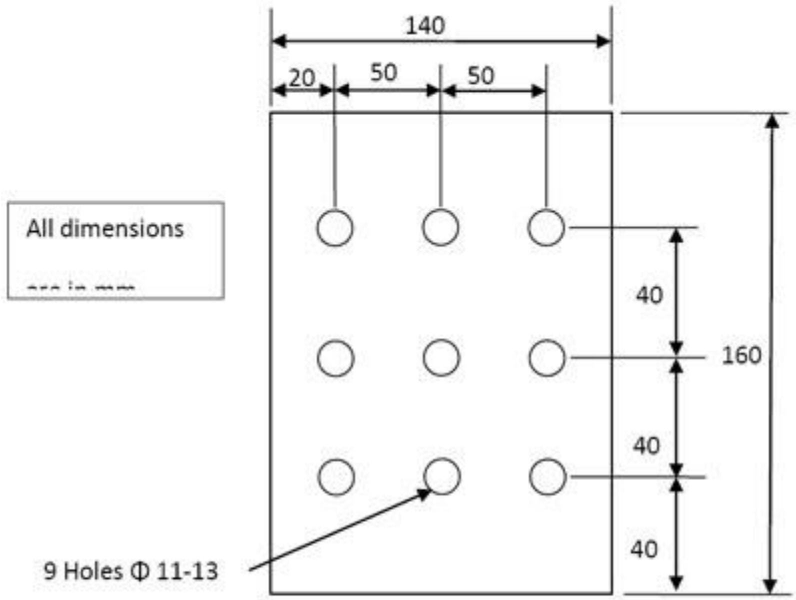


Figure 13: Specimen dimensions for three-rail shear test

Flexural Tests:

The flexural tests are conducted to determine the mechanical properties of resin and laminated fiber composite materials. Further, these tests are used to determine the interlaminar shear strength of a laminate, shear modulus, shear strength, tensile and compression moduli along with flexural and shear stiffness. These tests are not only used for composites but also for sandwich beams.

These tests are simple one. Further, they need simple instrumentation and equipment required. These tests conducted on beams of uniform cross section. These beam specimens do not require the end tabs.

There are two methods to carry out these tests. The beam is a flat rectangular specimen and is simply supported close to its ends. In the first method the beam is centrally loaded. Thus gives three point bending. Since there are three important points (two end supports and one central loading point) along the span of the beam this method is called as *three-point bending* test. In the second method the beam is loaded by two loads placed symmetrically between the supports. In this method there are four important points (two end supports and two loading points) along the span of the beam. Thus, it gives four-point bending. Hence, this method is called *four point bending*. These methods are shown schematically. Also shown in this figure are the shear force diagram (SFD) and bending moment diagrams (BMD) related to the particular loading regimes.

From the shear force and bending moment diagrams it is clear that there is a stress concentration at the point of loading. However, for four point bending there is uniform bending moment and both shear force and interlaminar shear stress are zero between the loading points. Thus, it leads to the pure bending loading. Such a state of stress is desirable in testing.

The properties are assumed to be uniform through the thickness as composite as it is a unidirectional composite or isotropic material. For such a material the normal stress varies linearly across the thickness. The maximum in compression is on one side and an equal maximum in tension on other side of the thickness and passes through zero at the mid-plane. The maximum normal stress is given as

$$|\sigma_C| = |\sigma_T| = \frac{6\mathrm{M}}{bh^2}$$

where, M is the bending moment, b is width and h is the thickness of the specimen. Further, σ_C and σ_T denote compressive and tensile normal stresses, respectively.

The shear stress varies parabolic through the thickness with maximum at mid plane and zero at the outer surface. The maximum shear stress at the mid plane is given as

$$\tau = \frac{3Fs}{2bh}$$

where Fs is the shear force on the specimen cross section. The normal stress and shear force variation through the thickness is shown.The flexural response of the beam in three or four point bending test is obtained by recording the load applied and the resulting strain. The resulting strains are measured using the strain gages bonded on the beam in the gage length. It is clear from the distribution of the shear force and bending moment that the state of stress in specimens subjected to three and four-point bending tests are somewhat different. Thus, it may lead to differences in the results.

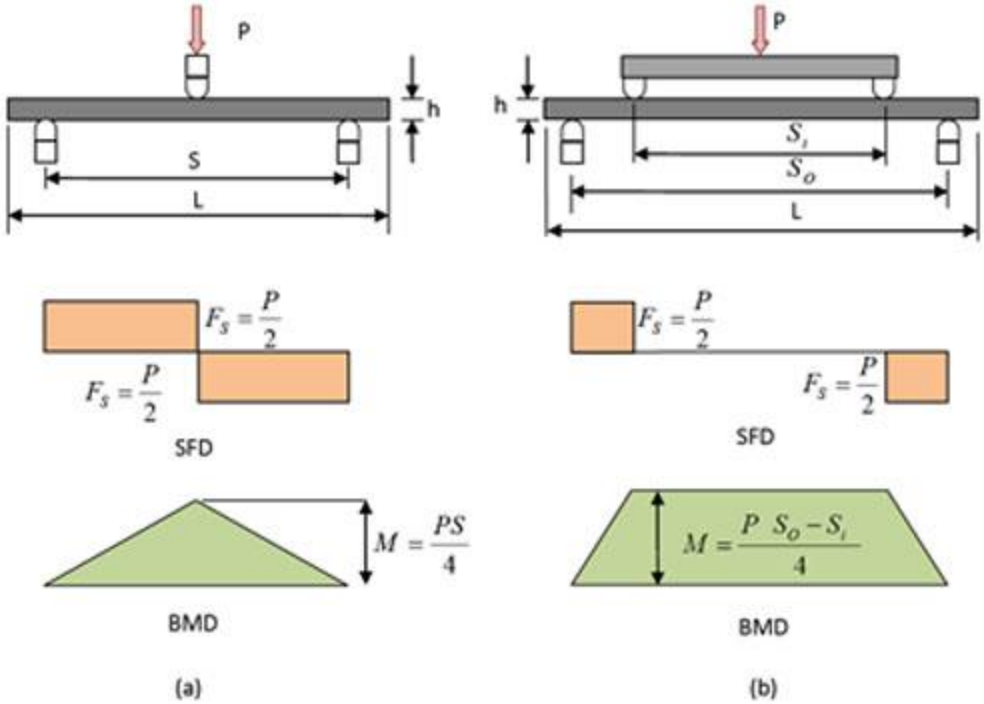


Figure 13: Shear force and bending moment diagrams for (a) three point and (b) four point bending test

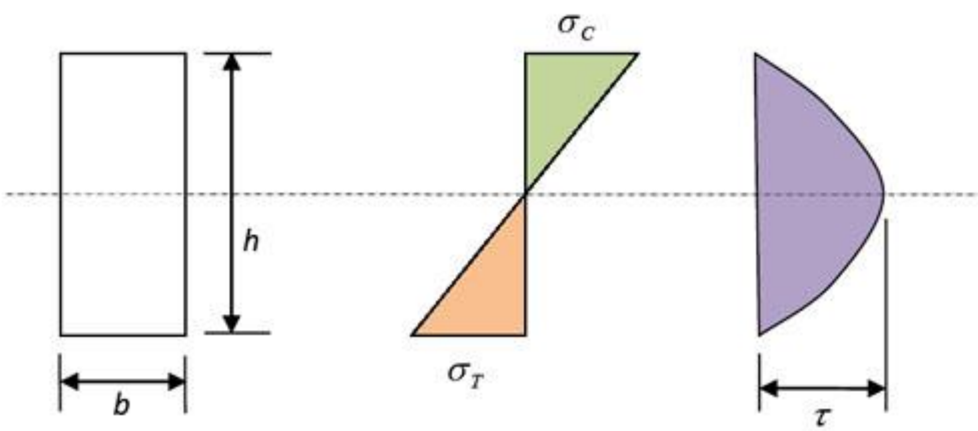


Figure 14: Bending and shearing stresses in the thickness direction

In the following we will see the measurement of flexural modulus and maximum stress on the outer surface of the beam.

***Flexural strength*:** This is the stress on the surface of the specimen at failure, which should be accompanied by the breaking of fibers, rather than inter laminar shear.

In the three point bending method the flexural modulus E_f is given as

$$E_f = \frac{S^3 m}{4b\, h^3}$$

where, E_f is flexural modulus, S is the support span, m is the slope of the load-deflection curve, b and h are the width and thickness of the specimen, respectively.

In case of four point bending there are two options according to ASTM D790 standard. In the first option the loading span is one third of the support span. For this case the flexural modulus is given as

$$F_f = 0.21 \frac{S^3 m}{b h^3}$$

In the second option the loading span is half of the support span. The flexural modulus for this case is given as

$$F_f = 0.17 \frac{S^3 m}{b h^3}$$

where, the parameters in these two equations are as defined earlier.

The maximum stress on outer surface of the beam is given below for all the cases.

$$\sigma = \frac{3PS}{2b\, h^2} \quad \text{3 point bending}$$
$$\sigma = \frac{PS}{b\, h^2} \quad \text{4 point bending with loading span equal to one third support span}$$
$$\sigma = \frac{3PS}{4b\, h^2} \quad \text{4 point bending with loading span equal to one third support span}$$

It is important to note that the measurement of width and thickness of the beam is important for accurate measurement of flexural modulus and maximum stresses.

For more details on these tests one can refer to ASTM D790-92 and ASTM D790M-93.

UNIT-IV

LAMINATES

Plate stiffness and compliance

Assumptions

1. The laminate consists of perfectly bonded layers. There is no slip between the adjacent layers. In other words, it is equivalent to saying that the displacement components are continuous through the thickness.
2. Each lamina is considered to be a homogeneous layer such that its effective properties are known.
3. Each lamina is in a state of plane stress.
4. The individual lamina can be isotropic, orthotropic or transversely isotropic.
5. The laminate deforms according to the Kirchhoff - Love assumptions for bending and stretching of thin plates (as assumed in classical plate theory). The assumptions are:
    a. The normals to the mid-plane remain straight and normal to the midplane even after deformation.
    b. The normals to the mid-plane do not change their lengths.
6. The classical laminate theory is abbreviated as CLT. This theory is known as the classical laminated plate theory and abbreviated as CLPT.

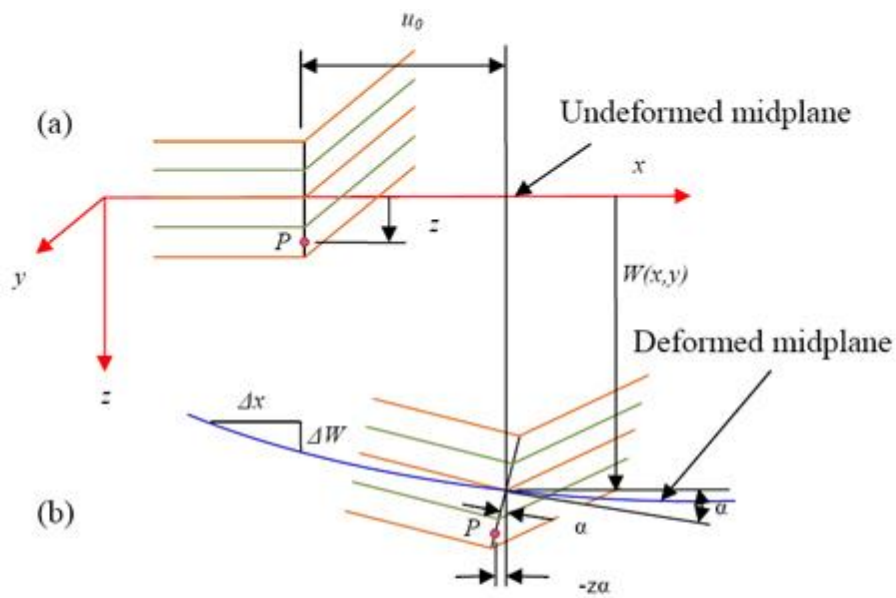


Fig: Plate deformation

$$u(x,y,z) = u_0(x,y) - z\tan\alpha = u_0(x,y) - z\,\alpha = u_0(x,y) - z\,\frac{\partial w}{\partial x}$$

Similarly, for the deformation in *yz* plane we can express the slope of the deformed mid-plane as. Thus, the displacement of a generic point along *y* axis can be given as

$$v(x,y,z) = v_0(x,y) - z\,\frac{\partial w}{\partial y}$$

Thus, the complete displacement field for a generic point in the laminate according to the classical laminate theory is given below:

$$u(x,y,z) = u_0(x,y) - z\,\frac{\partial w}{\partial x}$$
$$v(x,y,z) = v_0(x,y) - z\,\frac{\partial w}{\partial y}$$
$$w(x,y,z) = w_0(x,y)$$

From the first assumption of the Kirchhoff-Love theory that the normals remain straight and normal to mid-plane even after deformation, results into zero transverse shear strains. Thus,

$$\gamma_{xz} = \gamma_{yz} = 0$$

Using the definitions of small strain, we can write the above equation as

$$\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = 0$$
$$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = 0$$

From the first of the above equation we can write

$$\frac{\partial u}{\partial z} = -\frac{\partial w}{\partial x}$$

Integrating this with respect to *z*, we get

$$u(x,y,z) = -z\frac{\partial w}{\partial x} + u_0(x,y)$$

Where $u_0(x,y)$ is a constant of integration which is function of *x* and *y* alone. Similarly, from the second of Equation (5.9), we can get

$$v(x,y,z) = -z\frac{\partial w}{\partial y} + v_0(x,y)$$

Strain Displacements Relations:

The strain displacement relations for infinitesimal strains using the displacement field can be given as

$$\epsilon_{xx} = \frac{\partial u}{\partial x} = \frac{\partial u_0}{\partial x} - z\frac{\partial^2 w}{\partial x^2}$$
$$\epsilon_{yy} = \frac{\partial v}{\partial y} = \frac{\partial v_0}{\partial y} - z\frac{\partial^2 w}{\partial y^2}$$
$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} - 2z\frac{\partial^2 w}{\partial x \partial y}$$

The above equation can be written as

$$\begin{Bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \epsilon_{xx}^{(0)} \\ \epsilon_{yy}^{(0)} \\ \gamma_{xy}^{(0)} \end{Bmatrix} + z \begin{Bmatrix} \kappa_{xx} \\ \kappa_{yy} \\ \kappa_{xy} \end{Bmatrix}$$

or

$$\{\epsilon\}_{xy} = \left\{\varepsilon^{(0)}\right\}_{xy} + z\,\{\kappa\}_{xy}$$

$$\left\{\epsilon^{(0)}\right\}_{xy} = \left\{\epsilon_{xx}^{(0)} \quad \epsilon_{yy}^{(0)} \quad \gamma_{xy}^{(0)}\right\}^T = \left\{\frac{\partial u_0}{\partial x} \quad \frac{\partial v_0}{\partial y} \quad \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x}\right\}^T$$

$$\{\kappa\}_{xy} = \{\kappa_{xx} \quad \kappa_{yy} \quad \kappa_{xy}\}^T = \left\{-\frac{\partial^2 w}{\partial x^2} \quad -\frac{\partial^2 w}{\partial y^2} \quad -2\frac{\partial^2 w}{\partial x \partial y}\right\}^T$$

The terms κ_{xx} and κ_{yy} are the bending moment curvatures and κ_{xy} is the twisting moment curvature.

Strain Displacements Relations:

The strain displacement relations for infinitesimal strains using the displacement field can be given as

$$\epsilon_{xx} = \frac{\partial u}{\partial x} = \frac{\partial u_0}{\partial x} - z\frac{\partial^2 w}{\partial x^2}$$
$$\epsilon_{yy} = \frac{\partial v}{\partial y} = \frac{\partial v_0}{\partial y} - z\frac{\partial^2 w}{\partial y^2}$$
$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} - 2z\frac{\partial^2 w}{\partial x \partial y}$$

The above equation can be written as

$$\begin{Bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \epsilon_{xx}^{(0)} \\ \epsilon_{yy}^{(0)} \\ \gamma_{xy}^{(0)} \end{Bmatrix} + z \begin{Bmatrix} \kappa_{xx} \\ \kappa_{yy} \\ \kappa_{xy} \end{Bmatrix}$$

or

$$\{\epsilon\}_{xy} = \left\{\varepsilon^{(0)}\right\}_{xy} + z\,\{\kappa\}_{xy}$$

where $\left\{\epsilon^{(0)}\right\}_{xy} = \left\{\epsilon_{xx}^{(0)} \quad \epsilon_{yy}^{(0)} \quad \gamma_{xy}^{(0)}\right\}^T = \left\{\frac{\partial u_0}{\partial x} \quad \frac{\partial v_0}{\partial y} \quad \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x}\right\}^T$ are the midplane strains and $\{\kappa\}_{xy} = \{\kappa_{xx} \quad \kappa_{yy} \quad \kappa_{xy}\}^T = \left\{-\frac{\partial^2 w}{\partial x^2} \quad -\frac{\partial^2 w}{\partial y^2} \quad -2\frac{\partial^2 w}{\partial x \partial y}\right\}^T$ represents the midplane curvatures. The terms κ_{xx} and κ_{yy} are the bending moment curvatures and κ_{xy} is the twisting moment curvature.

State of Stress in a Laminate:

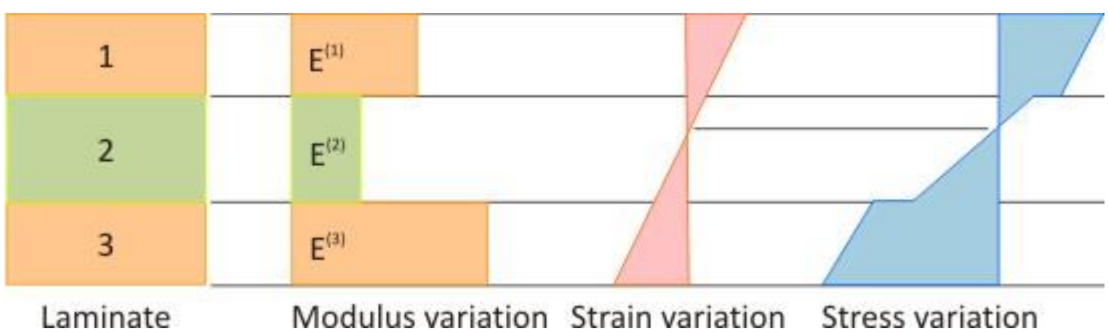
The stresses at any location can be calculated from the strains and lamina constitutive relations. It is assumed that the lamina properties are known. Hence, the constitutive equation for a *kth* lamina is known, that is, the reduced stiffness matrices (in principal material directions and global directions) are known. Thus, the stresses in k^{th} lamina can be given as

$$\{\sigma\}_{xy}^k = [\bar{Q}]^k \{\epsilon\}_{xy}^k$$

Now, using Equation (5.13), we can write the stresses as

$$\{\sigma\}_{xy}^k = [\bar{Q}]^k \left\{\epsilon^{(0)}\right\}_{xy} + [\bar{Q}]^k z \{\kappa\}_{xy}$$

. In these equations, the strains are given at a *z* location where the stresses are required. It should be noted that the strains are continuous and vary linearly through the thickness. If we look at the stress distribution through the thickness it is clear that the stresses are not continuous through the thickness, because the stiffness is different for different laminae in thickness direction. In a lamina the stress varies linearly. The slope of this variation in a lamina depends upon its moduli. However, at the interface of two adjacent laminae there is a discontinuity in the stresses. The same thing is depicted in below Figure with three layers.



Elucidation of stress discontinuity at lamina interfaces in a laminate

Inplane Resultant Forces:

The inplane forces per unit length are defined as

$$N_{xx} = \int_{-H}^{H} \sigma_{xx}\, dz, \qquad N_{yy} = \int_{-H}^{H} \sigma_{yy}\, dz, \qquad N_{xy} = \int_{-H}^{H} \tau_{xy}\, dz \quad (5.16)$$

Or these can be written as

$$\{N\}_{xy} = \int_{-H}^{H} \{\sigma\}_{xy} \, dz$$

$$\{N\}_{xy} = \sum_{k=1}^{N_{Lay}} \int_{-z_{k-1}}^{z_k} \left[\overline{Q}\right]^k \left\{\epsilon^{(0)}\right\}_{xy} dz + \sum_{k=1}^{N_{Lay}} \int_{-z_{k-1}}^{z_k} \left[\overline{Q}\right]^k \{\kappa\}_{xy} \, z \, dz$$

Now recall that the midplane strains $\left\{\epsilon^{(0)}\right\}_{xy}$ and the curvatures $\{\kappa\}_{xy}$ are independent of *z* location. The reduced transformed stiffness matrix $\left[\overline{Q}\right]$ is function of thickness and constant over a given lamina thickness. Now we can replace the integration over the laminate thickness as sum of the integrations over individual lamina thicknesses. Thus, Equation can be written as

$$\{N\}_{xy} = \sum_{k=1}^{N_{Lay}} \int_{-z_{k-1}}^{z_k} \left[\overline{Q}\right]^k \left\{\epsilon^{(0)}\right\}_{xy} dz + \sum_{k=1}^{N_{Lay}} \int_{-z_{k-1}}^{z_k} \left[\overline{Q}\right]^k \{\kappa\}_{xy} \, z \, dz$$

Here, N_{Lay} is the total number of layers in the laminate. This equation can be written as

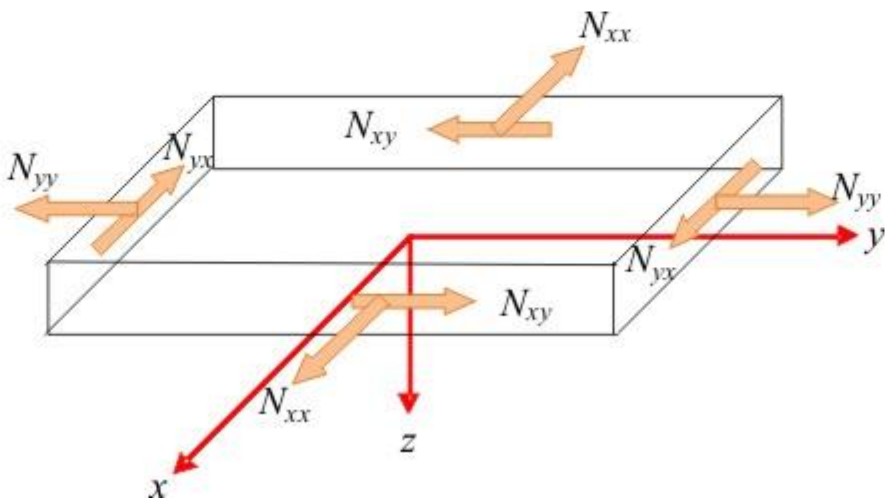
$$\{N\}_{xy} = [A]\left\{\epsilon^{(0)}\right\}_{xy} + [B]\{\kappa\}_{xy}$$

where

$$[A] = \sum_{k=1}^{N_{Lay}} \left[\overline{Q}\right]^k (z_k - z_{k-1}) \quad \text{and} \quad [B] = \frac{1}{2} \sum_{k=1}^{N_{Lay}} \left[\overline{Q}\right]^k (z_k^2 - z_{k-1}^2)$$

The matrix $[A]$ represents the in-plane stiffness, that is, it relates the in-plane forces with mid-plane strains and the matrix $[B]$ represents the bending stiffness coupling, that is, it relates the in-plane forces with mid-plane curvatures.

It should be noted that the matrices $[A]$ and $[B]$ are symmetric as the matrix $\left[\overline{Q}\right]$ is also symmetric for each lamina in the laminate.
The resultant in-plane forces are shown.



In plane resultant forces per unit length on a laminate

Resultant Moments:

The resultant moments per unit length are defined as

$$M_{xx} = \int_{-H}^{H} \sigma_{xx}\, z\, dz, \qquad M_{yy} = \int_{-H}^{H} \sigma_{yy}\, z\, dz, \qquad M_{xy} = \int_{-H}^{H} \tau_{xy}\, z\, dz$$

Or these can be written as

$$\{M\}_{xy} = \int_{-H}^{H} \{\sigma\}_{xy}\, z\, dz$$

Now, using Equation (5.15) we can write,

$$\{M\}_{xy} = \int_{-H}^{H} [\bar{Q}]^{k} \{\epsilon^{(0)}\}_{xy}\, z\, dz + \int_{-H}^{H} [\bar{Q}]^{k} \{\kappa\}_{xy}\, z^{2}\, dz$$

Now, with the same justification as given, we can write the above equation as

$$\{M\}_{xy} = \sum_{k=1}^{N_{Lay}} \int_{-z_{k-1}}^{z_k} [\bar{Q}]^{k} \{\epsilon^{(0)}\}_{xy}\, z\, dz + \sum_{k=1}^{N_{Lay}} \int_{-z_{k-1}}^{z_k} [\bar{Q}]^{k} \{\kappa\}_{xy}\, z^{2}\, dz$$

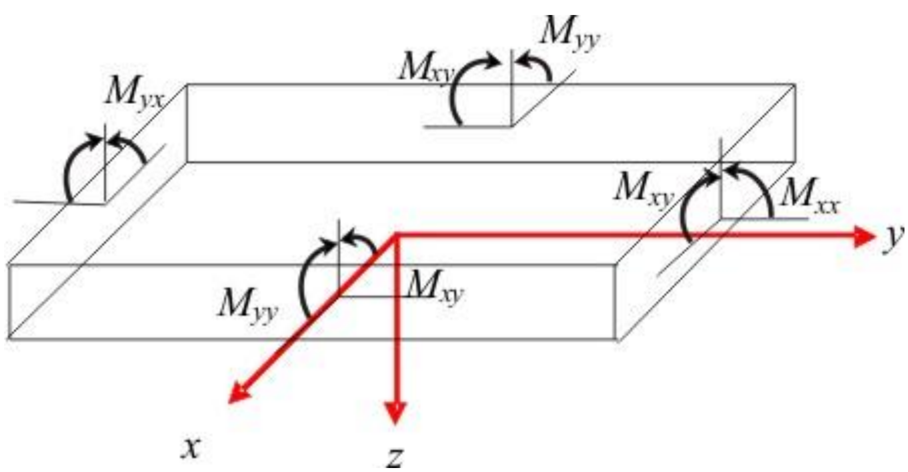
This can be written as

$$\{M\}_{xy} = [B]\{\epsilon^{(0)}\}_{xy} + [D]\{\kappa\}_{xy} \qquad (5.26)$$

where

$$[D] = \frac{1}{3}\sum_{k=1}^{N_{Lay}} \left[\overline{Q}\right]^k (z_k^3 - z_{k-1}^3) \qquad (5.27)$$

The matrix $[D]$ represents the bending stiffness, that is, it relates resultant moments with mid-plane curvatures. Again, the matrix $[D]$ is also symmetric. Further, it is important to note that the matrix $[B]$ relates the resultant moments with mid-plane curvatures as well.



Resultant moments per unit length on a laminate

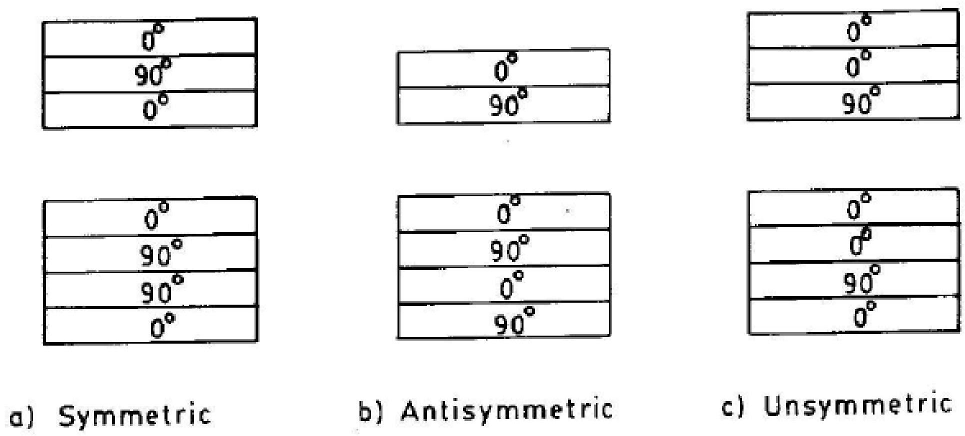
Types of Laminates:-

1. Based on Layer angle orientation

- Crossply Laminate
- Angle-ply Laminate

2. Based on layer orientation about midplane

- Symmetric Laminate
- Anti symmetric Laminate
- Un symmetric Laminate



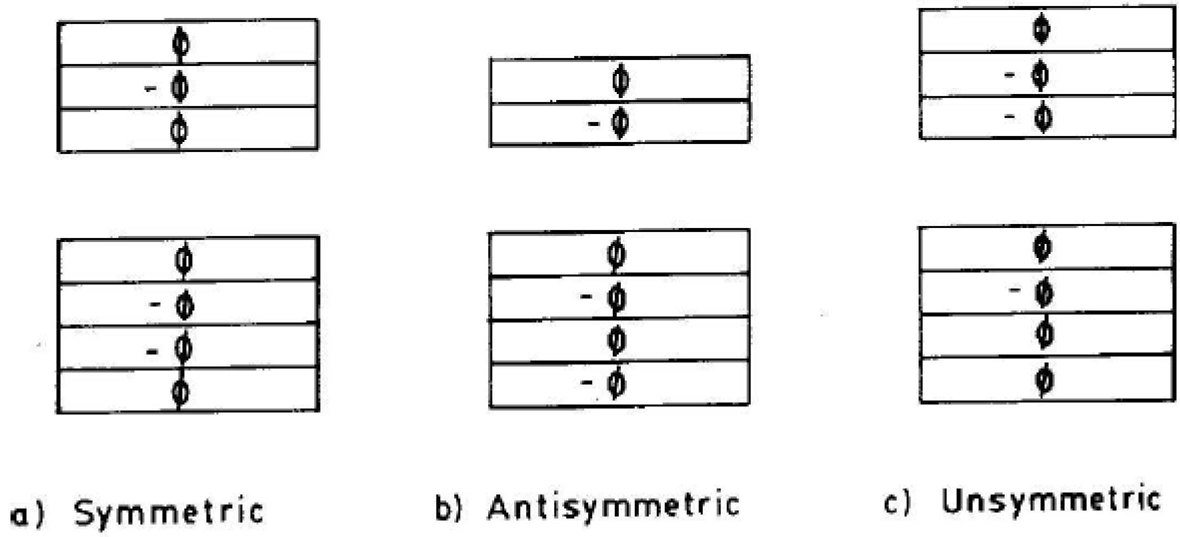
Cross-ply laminates

A laminate is called cross-ply laminate if all the plies used to fabricate the laminate are only 0° and 90°

For a cross ply laminate the terms $A_{16} = A_{26} = B_{16} = B_{26} = D_{16} = D_{26} = 0$. This is because these terms involve the terms $\bar{Q}_{16}$ and $\bar{Q}_{26}$ which have the products of *mn* terms. This product is zero for any cross-ply. Thus, the terms $\bar{Q}_{16}$ and $\bar{Q}_{26}$ are identically zero for each ply.

***Note*:** For a cross-ply following relations hold true. The readers should verify these relations from earlier lectures on planar constitutive relations.

$$\bar{Q}_{11}(0) = \bar{Q}_{22}(90), \quad \bar{Q}_{22}(0) = \bar{Q}_{11}(90)$$
$$\bar{Q}_{12}(0) = \bar{Q}_{12}(90), \quad \bar{Q}_{66}(0) = \bar{Q}_{66}(90)$$



Angle ply laminates

***Angle-Ply Laminates*:**

A laminate is called angle-ply laminate if it has plies of the same thickness and material and are oriented at $+\theta$ and $-\theta$.

For example $[45/-45/-30/30]$ is shown. For angle-ply laminates the terms $A_{16} = A_{26}$ are zero. This can be justified by that fact that $\bar{Q}_{16}$ and $\bar{Q}_{26}$ have the term *mn*. Due to this term $\bar{Q}_{16}$ and $\bar{Q}_{26}$ have opposite signs for layers with $+\theta$ and $-\theta$ fibre orientation. Since the thicknesses and materials of these layers are same, by the definition the terms $A_{16} = A_{26}$ are zero for the laminate.

***Note*:** For angle-ply laminates the following relations are very useful in computing $[A], [B]$ and $[D]$.

$$\bar{Q}_{11}(+\theta) = \bar{Q}_{11}(-\theta), \quad \bar{Q}_{22}(+\theta) = \bar{Q}_{22}(-\theta)$$
$$\bar{Q}_{12}(+\theta) = \bar{Q}_{12}(-\theta), \quad \bar{Q}_{66}(+\theta) = \bar{Q}_{66}(-\theta)$$
$$\bar{Q}_{16}(+\theta) = -\bar{Q}_{16}(-\theta), \quad \bar{Q}_{26}(+\theta) = -\bar{Q}_{26}(-\theta)$$

***Anti-symmetricLaminates*:**

A laminate is called anti-symmetric when the material and thickness of the plies are same above and below the mid-plane but the orientation of the plies at same distance above and below the mid-plane have opposite signs.

For example, $[45/-30/30/-45]$ is shown in Figure.
For anti-symmetric laminates the terms $A_{16} = A_{26} = D_{16} = D_{26} = 0$. The proof is left to the readers as an exercise.

***BalancedLaminates*:**

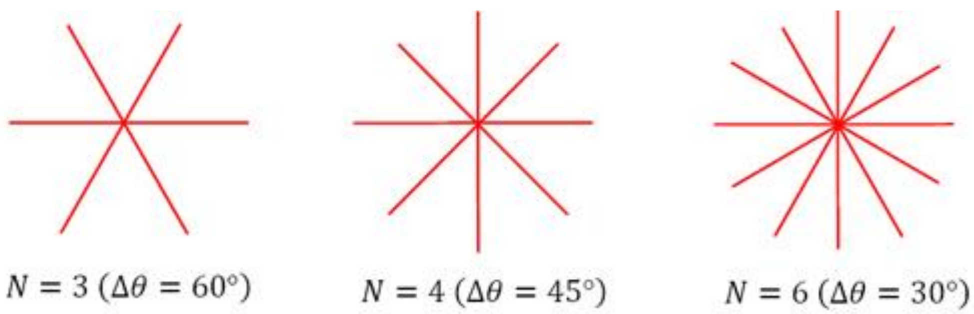
A laminate is called balanced laminate when it has pairs of plies with same thickness and material and the angles of plies are $+\theta$ and $-\theta$. However, the balanced laminate can also have layers oriented at 0° and 90°. For this laminate also $A_{16} = A_{26}$ are zero. It should be noted that angle-ply laminates are balanced laminates. For example, $[30/60/-45/-30/-60/45]$ is shown .

***Quasi-Isotropic Laminates*:**

A laminate is called quasi-isotropic when its extensional stiffness matrix behaves like an isotropic material. This requires that $A_{11} = A_{22}$, $A_{16} = A_{26} = 0$ and $A_{66} = (A_{11} - A_{12})/2$. Further, this extensional stiffness matrix is independent of orientation of layers in laminate. This requires a laminate with $N \geq 3$ equal thickness layers and *N* equal angles between adjacent fibre orientations. The *N*equal angles, $\Delta\theta$ between the fibre orientations in this case can be given as

$$\Delta\theta = \frac{\pi}{N}$$

The quasi-isotropic laminate with this construction for *N*=3, 4 and 6 will have fibre orientations as shown .



Fibre orientations in a typical quasi-isotropic laminates

It should be noted that the isotropy in these laminates is in-plane only. The matrices *B* and *D* may not behave like an isotropic material. Hence, such laminates are quasi-isotropic in nature.

Some examples of quasi-isotropic laminate are: $[0/\pm60]_S$, $[0/\pm45/90]_S$

Example 1 Consider Example 5.3. Let this laminate be subjected to the forces $N_{xx} = 1000\ N/mm$, $N_{yy} = 500\ N/mm$ and $N_{xy} = 100\ N/mm$.

Calculate global strains and stresses in each ply.

Solution**:** The laminate in this example is a symmetric laminate. Hence, *B* matrix is zero. It means that there is no coupling between extension and bending actions. Thus, the applied stresses will produce only in-plane and shear strains and it will not produce any curvatures. Thus, it is easy to understand that the mid-plane strains will be the strains in each ply.

We can find the mid-plane strains as follows:

$$\{N\} = [A]\{\epsilon^{(0)}\} + [B]\{\kappa\}$$
$$= [A]\{\epsilon^{(0)}\}$$

This gives

$$\{\epsilon^{(0)}\} = [A]^{-1}\{N\}$$

Thus,

$$\begin{Bmatrix} \epsilon_{xx}^{(0)} \\ \epsilon_{yy}^{(0)} \\ \gamma_{xy}^{(0)} \end{Bmatrix} = 10^{-3}\begin{bmatrix} 0.01120 & -0.00773 & 0 \\ -0.00773 & 0.01120 & 0 \\ 0 & 0 & 0.00759 \end{bmatrix}\begin{Bmatrix} 1000 \\ 500 \\ 100 \end{Bmatrix} = 10^{-3}\begin{Bmatrix} 7.335 \\ -2.130 \\ 0.759 \end{Bmatrix}$$

The strains are same in all layers. However, the stresses in each layer will be different as their stiffnesses are different.

Stresses in $+45°$ layer are

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix}_{(+45)} = 10^{-3}\begin{bmatrix} 42.63 & 29.43 & 28.94 \\ 29.43 & 42.63 & 28.94 \\ 28.94 & 28.94 & 32.93 \end{bmatrix}\begin{Bmatrix} 7.335 \\ -2.130 \\ 0.759 \end{Bmatrix} = \begin{Bmatrix} 0.2719 \\ 0.1471 \\ 0.1756 \end{Bmatrix} GPa$$

And stresses in $-45°$ layer are

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix}_{(-45)} = 10^{-3}\begin{bmatrix} 42.63 & 29.43 & -28.94 \\ 29.43 & 42.63 & -28.94 \\ -28.94 & -28.94 & 32.93 \end{bmatrix}\begin{Bmatrix} 7.335 \\ -2.130 \\ 0.759 \end{Bmatrix} = \begin{Bmatrix} 0.2281 \\ 0.1031 \\ -0.1256 \end{Bmatrix} GPa$$

Now, let us find the strains and stresses in principal material directions as well for these laminae.

Let us transform the strains in $+45°$ layer as

$$\{\epsilon\}_{12} = [T_2(+45)]\{\epsilon\}_{xy}$$

$$\begin{Bmatrix}\epsilon_{11}\\ \epsilon_{22}\\ \gamma_{12}\end{Bmatrix} = \begin{bmatrix}0.5 & 0.5 & 0.5\\ 0.5 & 0.5 & -0.5\\ -1.0 & 1.0 & 0.0\end{bmatrix}\begin{Bmatrix}\epsilon_{xx}^{(0)}\\ \epsilon_{yy}^{(0)}\\ \gamma_{xy}^{(0)}\end{Bmatrix}$$

$$\begin{Bmatrix}\epsilon_{11}\\ \epsilon_{22}\\ \gamma_{12}\end{Bmatrix} = \begin{Bmatrix}0.00298\\ 0.00222\\ -0.00946\end{Bmatrix}$$

Similarly, the strains in $-45°$ layer in principal directions are

$$\begin{Bmatrix}\epsilon_{11}\\ \epsilon_{22}\\ \gamma_{12}\end{Bmatrix} = \begin{Bmatrix}0.00222\\ 0.00298\\ 0.00946\end{Bmatrix}$$

Now, stresses in principal directions in $+45°$ layer are

$$\{\sigma\}_{12} = [T_1(+45)]\{\sigma\}_{xy}$$

$$\begin{Bmatrix}\sigma_{11}\\ \sigma_{22}\\ \tau_{12}\end{Bmatrix} = \begin{Bmatrix}0.29728\\ 0.12166\\ -0.12500\end{Bmatrix} GPa$$

And stresses in principal material directions for $-45°$ layer are

$$\{\sigma\}_{12} = [T_1(-45)]\{\sigma\}_{xy}$$

$$\begin{Bmatrix}\sigma_{11}\\ \sigma_{22}\\ \tau_{12}\end{Bmatrix} = \begin{Bmatrix}0.29114\\ 0.03990\\ 0.06250\end{Bmatrix} GPa$$

Hygrothermal Stresses

$$\sigma_{Al} = E_{Al}\varepsilon_{Al}^{M}$$

$$\sigma_{S} = E_{S}\varepsilon_{S}^{M}$$

$$\sigma_{S} = 2\sigma_{Al}$$

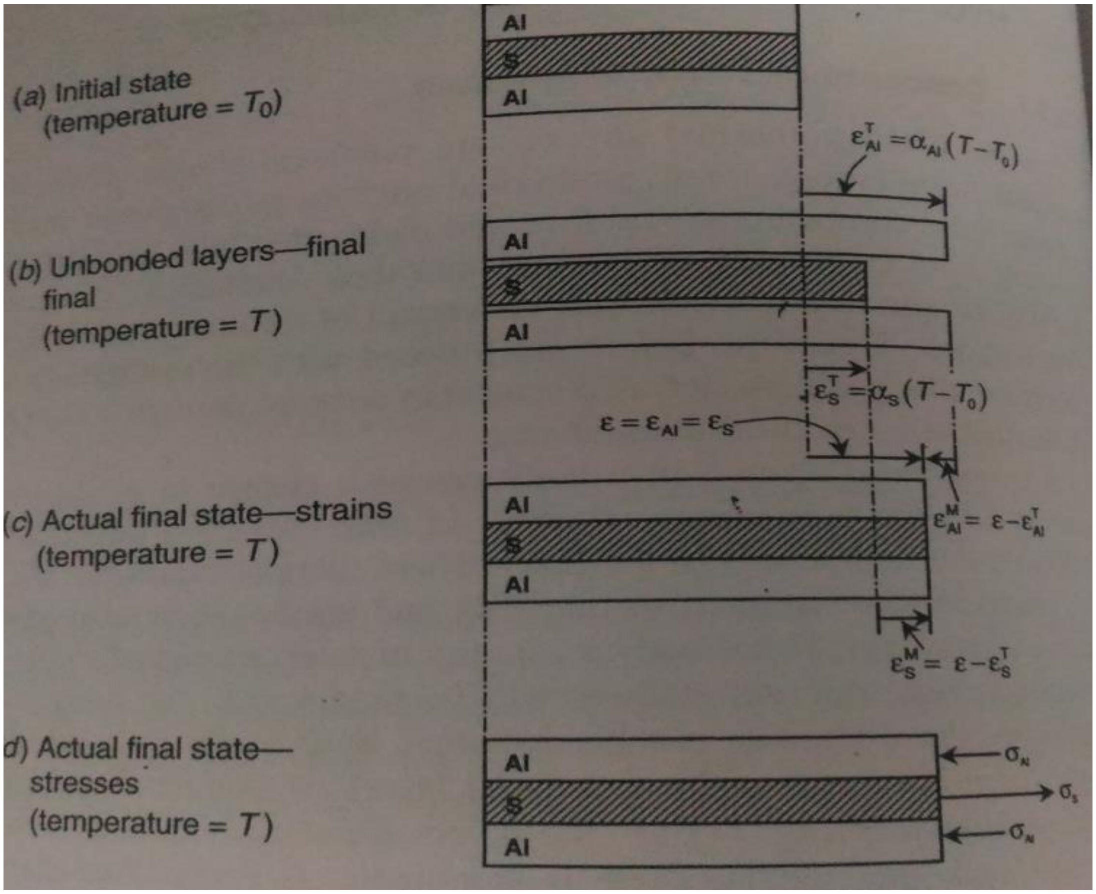


Fig: Thermal strain and stresses in a three-layered symmetric laminate

The strain due to change in temperature and moisture can be represented as

$$\varepsilon^{T} = \alpha\, \Delta T$$

$$\varepsilon^{H} = \beta\, \Delta C$$

Hygrothermal changes in longitudinal and transverse directions are

$$\varepsilon_L^T = \alpha_L\, \Delta T$$
$$\varepsilon_T^T = \alpha_T\, \Delta T$$

$$\varepsilon_L^H = \beta_L\, \Delta C$$
$$\varepsilon_T^H = \beta_T\, \Delta C$$

$$\begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix} = [T_2]^{-1} \begin{Bmatrix} \alpha_L \\ \alpha_T \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{Bmatrix} = [T_2]^{-1} \begin{Bmatrix} \beta_L \\ \beta_T \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} \varepsilon_x^T \\ \varepsilon_y^T \\ \gamma_{xy}^T \end{Bmatrix} = \begin{Bmatrix} \alpha_x \, \Delta T \\ \alpha_y \, \Delta T \\ \alpha_{xy} \, \Delta T \end{Bmatrix}$$

$$\begin{Bmatrix} \varepsilon_x^H \\ \varepsilon_y^H \\ \gamma_{xy}^H \end{Bmatrix} = \begin{Bmatrix} \beta_x \, \Delta C \\ \beta_y \, \Delta C \\ \beta_{xy} \, \Delta C \end{Bmatrix}$$

The mechanical strains then are given as

$$\begin{Bmatrix} \varepsilon_x^M \\ \varepsilon_y^M \\ \gamma_{xy}^M \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} - \begin{Bmatrix} \varepsilon_x^T \\ \varepsilon_y^T \\ \gamma_{xy}^T \end{Bmatrix} - \begin{Bmatrix} \varepsilon_x^H \\ \varepsilon_y^H \\ \gamma_{xy}^H \end{Bmatrix}$$

Using the above equations the mechanical strain can be rewritten as

$$\begin{Bmatrix} \varepsilon_x^M \\ \varepsilon_y^M \\ \gamma_{xy}^M \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 + zk_x \\ \varepsilon_y^0 + zk_y \\ \gamma_{xy}^0 + zk_{xy} \end{Bmatrix} - \begin{Bmatrix} \alpha_x \, \Delta T \\ \alpha_y \, \Delta T \\ \alpha_{xy} \, \Delta T \end{Bmatrix} - \begin{Bmatrix} \beta_x \, \Delta C \\ \beta_y \, \Delta C \\ \beta_{xy} \, \Delta C \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_x^T \\ \sigma_y^T \\ \tau_{xy}^T \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 + zk_x - \alpha_x \, \Delta T - \beta_x \, \Delta C \\ \varepsilon_y^0 + zk_y - \alpha_y \, \Delta T - \beta_y \, \Delta C \\ \gamma_{xy}^0 + zk_{xy} - \alpha_{xy} \, \Delta T - \beta_{xy} \, \Delta C \end{Bmatrix}$$

Performing integration the resultant forces and moments are represented as below

$$\begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix} = \begin{Bmatrix} N_x^{\mathrm{T}} \\ N_y^{\mathrm{T}} \\ N_{xy}^{\mathrm{T}} \end{Bmatrix} + \begin{Bmatrix} N_x^{\mathrm{H}} \\ N_y^{\mathrm{H}} \\ N_{xy}^{\mathrm{H}} \end{Bmatrix} \tag{6.63}$$

$$\begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix} = \begin{Bmatrix} M_x^{\mathrm{T}} \\ M_y^{\mathrm{T}} \\ M_{xy}^{\mathrm{T}} \end{Bmatrix} + \begin{Bmatrix} M_x^{\mathrm{H}} \\ M_y^{\mathrm{H}} \\ M_{xy}^{\mathrm{H}} \end{Bmatrix}$$

where $\{N^{\mathrm{T}}\}$, $\{M^{\mathrm{T}}\}$, $\{N^{\mathrm{H}}\}$, and $\{M^{\mathrm{H}}\}$ are

$$\begin{Bmatrix} N_x^{\mathrm{T}} \\ N_y^{\mathrm{T}} \\ N_{xy}^{\mathrm{T}} \end{Bmatrix} = \Delta T \sum_{k=1}^{n} \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix}_k (h_k - h_{k-1})$$

$$\begin{Bmatrix} M_x^{\mathrm{T}} \\ M_y^{\mathrm{T}} \\ M_{xy}^{\mathrm{T}} \end{Bmatrix} = \frac{1}{2} \Delta T \sum_{k=1}^{n} \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix}_k (h_k^2 - h_{k-1}^2)$$

$$\begin{Bmatrix} N_x^{\mathrm{H}} \\ N_y^{\mathrm{H}} \\ N_{xy}^{\mathrm{H}} \end{Bmatrix} = \Delta C \sum_{k=1}^{n} \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \begin{Bmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{Bmatrix}_k (h_k - h_{k-1})$$

$$\begin{Bmatrix} M_x^{\mathrm{H}} \\ M_y^{\mathrm{H}} \\ M_{xy}^{\mathrm{H}} \end{Bmatrix} = \frac{1}{2} \Delta C \sum_{k=1}^{n} \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \begin{Bmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{Bmatrix}_k (h_k^2 - h_{k-1}^2)$$

LAMINATE HYGROTHERMAL STRAINS

The changes in moisture concentration and temperature introduce expansional strains in each lamina. The stress-strain relation of an off-axis lamina is then modified as follows

$$\begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_6 \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{16} \\ S_{12} & S_{22} & S_{26} \\ S_{16} & S_{26} & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix} + \begin{Bmatrix} \epsilon_1^e \\ \epsilon_2^e \\ \epsilon_6^e \end{Bmatrix}$$

$$\begin{Bmatrix} \epsilon_1^e \\ \epsilon_2^e \\ \epsilon_6^e \end{Bmatrix} = \begin{Bmatrix} \epsilon_1^H \\ \epsilon_2^H \\ \epsilon_6^H \end{Bmatrix} + \begin{Bmatrix} \epsilon_1^T \\ \epsilon_2^T \\ \epsilon_6^T \end{Bmatrix}$$

$$\begin{Bmatrix} \epsilon_1^H \\ \epsilon_2^H \\ \epsilon_6^H \end{Bmatrix} = \Delta\overline{C} \begin{Bmatrix} \beta_1 \\ \beta_2 \\ \beta_6 \end{Bmatrix} \quad \text{and} \quad \begin{Bmatrix} \epsilon_1^T \\ \epsilon_2^T \\ \epsilon_6^T \end{Bmatrix} = \Delta T \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_6 \end{Bmatrix}$$

where the superscripts e, H, T refer to expansion, moisture and temperature, respectively, ΔC and ΔT are the change in specific moisture concentration and temperature, respectively, and β's and α's are coefficients of moisture expansion and thermal expansion respectively.

Note that the spatial distributions of moisture concentration and temperature are determined from solution of moisture diffusion and heat transfer problems.

Expansional strains transform like mechanical strains i.e.,

$$\{\epsilon'\} = [T_\epsilon]\{\epsilon\}$$

Inversion of Eq. 6.56 yields (see also Eq. 6.26), at any distance z ,

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix}_z = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{12} & Q_{22} & Q_{26} \\ Q_{16} & Q_{26} & Q_{66} \end{bmatrix}_Z \begin{Bmatrix} \epsilon_1 & - & \epsilon_1^e \\ \epsilon_2 & - & \epsilon_2^e \\ \epsilon_6 & - & \epsilon_6^e \end{Bmatrix}_z$$

Thus, for a general laminate

$$\begin{Bmatrix} N_1 \\ N_2 \\ N_6 \\ M_1 \\ M_2 \\ M_6 \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_1^0 \\ \epsilon_2^0 \\ \epsilon_6^0 \\ k_1 \\ k_2 \\ k_6 \end{Bmatrix} + \begin{Bmatrix} N_1^e \\ N_2^e \\ N_6^e \\ M_1^e \\ M_2^e \\ M_6^e \end{Bmatrix}$$

where the expansional force resultants are

$$\begin{Bmatrix} N_1^e \\ N_2^e \\ N_6^e \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{12} & Q_{22} & Q_{26} \\ Q_{16} & Q_{26} & Q_{66} \end{bmatrix}_z \begin{Bmatrix} \in_1^e \\ \in_2^e \\ \in_6^e \end{Bmatrix}_z dz$$

and the expansional moments are

$$\begin{Bmatrix} M_1^e \\ M_2^e \\ M_6^e \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{12} & Q_{22} & Q_{26} \\ Q_{16} & Q_{26} & Q_{66} \end{bmatrix}_z \begin{Bmatrix} \in_1^e \\ \in_2^e \\ \in_6^e \end{Bmatrix}_z z\, dz$$

These expansional force resultants and moments may considerably influence the deformation behaviour of a laminate.

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