CAPACITY IN NON-FADING CHANNELS

Here we derive the capacity equation for MIMO systems in nonfading channels, often known as

“Foschini’s equation”.

Consider the capacity equation for “normal” (single-antenna) Additive White Gaussian Noise (AWGN) channels. As Shannon showed, the information-theoretic (ergodic) capacity of such a channel is

$$C_{\text{shannon}} = \log_2\left(1 + \gamma \cdot |H|^2\right)$$

where γ is the SNR at the RX, and H is the normalized transfer function from the TX to the RX (the transfer function is just a scalar number).

The key statement of this equation is that capacity increases only logarithmically with the SNR, so that boosting the transmit power is a highly ineffective way of increasing capacity.

Consider now the MIMO case, where the channel is represented by matrix .

Let us consider a singular value decomposition of the channel:.

$$\mathbf{H} = \mathbf{W}\boldsymbol{\Sigma}\mathbf{U}^{\dagger}$$

where Σ is a diagonal matrix containing singular values, and W and $U^{\dagger}$ are unitary matrices composed of the left and right singular vectors, respectively.

The received signal is given as

$$\begin{aligned}\mathbf{r} &= \mathbf{H}\mathbf{s} + \mathbf{n} \\ &= \mathbf{W}\boldsymbol{\Sigma}\mathbf{U}^{\dagger}\mathbf{s} + \mathbf{n}\end{aligned}$$

- Then, multiplication of the transmit data vector by matrix U and the received signal vector by $W^{\dagger}$ diagonalizes the channel

$$\mathbf{W}^{\dagger}\mathbf{r} = \mathbf{W}^{\dagger}\mathbf{W}\boldsymbol{\Sigma}\mathbf{U}^{\dagger}\mathbf{U}\tilde{\mathbf{s}} + \mathbf{W}^{\dagger}\mathbf{n}$$

$$\tilde{\mathbf{r}} = \boldsymbol{\Sigma}\tilde{\mathbf{s}} + \tilde{\mathbf{n}}$$

Note that – because U and W are unitary matrices – n has the same statistical properties as n

– i.e., it is independent identically distributed (IID) white Gaussian noise.

The matrix Σ is a diagonal matrix with RH nonzero entries.

σk, where RH is the rank of H (and thus defined as the number of nonzero singular values), and σk is the kth singular value of H.

- The capacity of channel H is thus given by the sum of the capacities of the eigen modes (or antennas) of the channel:

$$C = \sum_{k=1}^{R_H} \log_2 \left[1 + \frac{P_k}{\sigma_n^2} \sigma_k^2 \right]$$

Where

- Pk is the power allocated to the k th eigenmode.
- We assume that ΣPk = P is independent of the number of antennas.
- The distribution of power among the different eigenmodes (or antennas) depends on the amount of CSIT.
- Assume that the RX has perfect CSI. Therefore the capacity increases linearly with min (Nt,Nr).