UNIT I

STEADY STRESSES AND VARIABLE STRESSES IN MACHINE MEMBERS

CHAPTER 2

Stress

When some external system of forces or loads act on a body, the internal forces (equal and opposite) are set up at various sections of the body, which resist the external forces. This internal force per unit area at any section of the body is known as unit stress or simply a stress. It is denoted by a Greek letter sigma (σ). Mathematically,

$$\text{Stress, } \sigma = P/A$$

Where, P = Force or load acting on a body, and

A = Cross-sectional area of the body.

In S.I. units, the stress is usually expressed in Pascal (Pa) such that 1 Pa = 1 N/m^2. In actual practice, we use bigger units of stress i.e. mega Pascal (MPa) and giga Pascal (GPa), such that

$$1 \text{ MPa} = 1 \times 10^6 \text{ N/m}^2 = 1 \text{ N/mm}^2$$

and

$$1 \text{ GPa} = 1 \times 10^9 \text{ N/m}^2 = 1 \text{ kN/mm}^2$$

Strain

When a system of forces or loads act on a body, it undergoes some deformation. This deformation per unit length is known as unit strain or simply a strain. It is denoted by a Greek letter epsilon (ε). Mathematically,

$$\text{Strain, } \varepsilon = \frac{\delta l}{l} \text{ or}$$

$$\delta l = \varepsilon . l$$

where

δl = Change in length of the body, and l = Original length of the body.

Tensile Stress and Strain

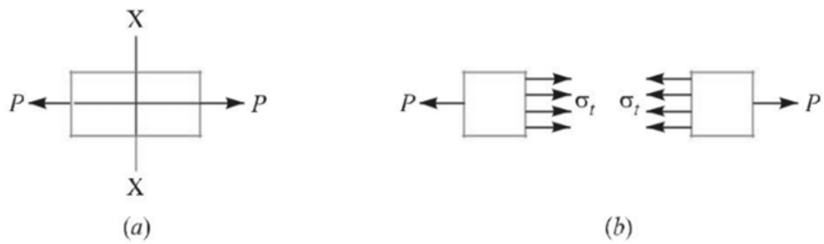

Fig. 2.1 Tensile stress and strain

[Source: "A Textbook of Machine Design" by R.S. Khurmi & J.K. Gupta, Page: 88]

When a body is subjected to two equal and opposite axial pulls P (also called tensile load) as shown in Fig. 2.1 (a), then the stress induced at any section of the body is known as tensile stress as shown in Fig. 2.1 (b). A little consideration will show that due to the tensile load, there will be a decrease in cross-sectional area and an increase in length of the body. The ratio of the increase in length to the original length is known as tensile strain.

Let P = Axial tensile force acting on the body,

A= Cross-sectional area of the body,

l = Original length, and

δl = Increase in length.

∴ Tensile stress, $\sigma_t = P/A$

and

tensile strain, $\varepsilon_t = \delta l / l$

Compressive Stress and Strain

When a body is subjected to two equal and opposite axial pushes P (also called compressive load) as shown in Fig. 2.2 (a), then the stress induced at any section of the body is known as compressive stress as shown in Fig. 2.2 (b). A little consideration will show that due to the compressive load, there will be an increase in cross-sectional area and a decrease in length of the body. The ratio of the decrease in length to the original length is known as compressive strain.

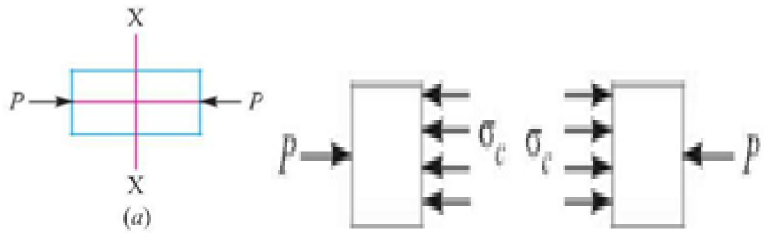

Fig. 2.2 Compressive stress and strain.

[Source: "A Textbook of Machine Design" by R.S. Khurmi & J.K. Gupta, Page: 89]

Let P = Axial compressive force acting on the body,

A= Cross-sectional area of the body,

l = Original length, and

δl= Decrease in length.

∴ Compressive stress, $\sigma_c = P/A$

Compressive strain, $\varepsilon_t = \delta l / l$

<u>Young's Modulus or Modulus of Elasticity</u>

Hooke's law states that when a material is loaded within elastic limit, the stress is directly proportional to strain, i.e.

$$\sigma \propto \varepsilon \quad \text{or } \sigma = E.\varepsilon$$

$$\therefore E = \frac{\sigma}{\varepsilon} = \frac{P\,l}{Al}$$

where E is a constant of proportionality known as Young's modulus or modulus of elasticity. In S.I. units, it is usually expressed in GPa i.e. GN/m^2 or kN/mm^2. It may be noted that Hooke's law holds good for tension as well as compression.

The following table shows the values of modulus of elasticity or Young's modulus (E) for the materials commonly used in engineering practice

Table 2.1. Values of E for the commonly used engineering materials.

Material	Modulus of elasticity (E) in GPa i.e. GN/m^2 or kN/mm^2
Steel and Nickel	200 to 220
Wrought iron	190 to 200
Cast iron	100 to 160
Copper	90 to 110
Brass	80 to 90
Aluminium	60 to 80
Timber	10

[Source: "A Textbook of Machine Design" by R.S. Khurmi & J.K. Gupta, Page: 90]

Problem 2.1

A coil chain of a crane required to carry a maximum load of 50 kN, is shown in Fig. 2.3

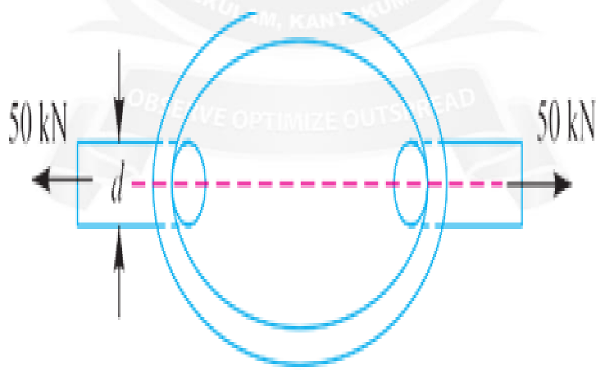

FIG 2.3

[Source: "A Textbook of Machine Design" by R.S. Khurmi & J.K. Gupta, Page: 90]

Find the diameter of the link stock, if the permissible tensile stress in the link material is not to exceed 75 MPa.

Given Data:

$P = 50 \text{ kN} = 50 \times 10^3 \text{ N}$

$\sigma_t = 75 \text{ MPa} = 75 \text{ N/mm}^2$

Let d = Diameter of the link stock in mm.

∴ Area, $A = \frac{\pi}{4} \times d_2 = 0.7854\ d^2$

We know that the maximum load (P),

$50 \times 103 = \sigma t.$

$A = 75 \times 0.7854\ d^2$

$= 58.9\ d_2$

$\therefore d^2 = \frac{50 \times 10^3}{58.9}$

= 850 or d

d = 29.13 say 30 mm

Problem2.2

A cast iron link, as shown in Fig. 2.4, is required to transmit a steady tensile load of 45 kN. Find the tensile stress induced in the link material at sections A-A and B-B

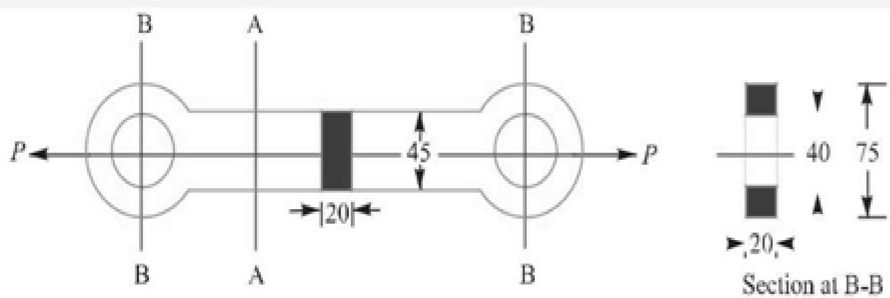

Fig 2. 4 All dimensions in mm

[Source: "A Textbook of Machine Design" by R.S. Khurmi & J.K. Gupta, Page: 90]

Given Data:

$P = 45\ kN = 45 \times 10^3\ N$

Tensile stress induced at section A-A

We know that the cross-sectional area of link at section A-A,

$A_1 = 45 \times 20 = 900\ mm^2$

∴ Tensile stress induced at section A-A,

$\sigma_{t1} = \frac{P}{A_1} = \frac{45\times10^3}{900} = 50$ N/mm^2

$\sigma_{t1} = 50$ MPa

Tensile stress induced at section B-B

We know that the cross-sectional area of link at section B-B,

$A_2 = 20\ (75 - 40) = 700$ mm^2

∴ Tensile stress induced at section B-B,

$\sigma_{t2} = \frac{P}{A_2} = \frac{45\times10^3}{700} = 64.3$ N/mm^2

$\sigma_{t2} = 64.3$ MPa

Problem 2.3

A hydraulic press exerts a total load of 3.5 MN. This load is carried by two steel rods, supporting the upper head of the press. If the safe stress is 85 MPa and E = 210 kN/mm^2, find: 1. diameter of the rods, and 2. extension in each rod in a length of 2.5 m.

Given Data:

$P = 3.5$ MN $= 3.5 \times 10^6$ N

$\sigma_t = 85$ MPa $= 85$ N/mm^2

$E = 210$ kN/mm^2 $= 210 \times 10^3$ N/mm^2

$l = 2.5$ m $= 2.5 \times 10^3$ mm

1. Diameter of the rods

Let d = Diameter of the rods in mm.

∴ Area, $A = \frac{\pi}{4} \times d^2$

$= 0.7854\ d^2$

Since the load P is carried by two rods, therefore load carried by each rod,

$P_1 = \frac{P}{2} = \frac{3.5\times10^6}{2}$

$P_1 = 1.75 \times 10^6$ N

We know that load carried by each rod (P1),

$1.75 \times 10^6 = \sigma_t \times A = 85 \times 0.7854\ d^2$

$= 66.76\ d^2$

$\therefore d^2 = 1.75 \times 106/66.76$

$= 26213$ or

$d = 162$ mm.

2. Extension in each rod

Let δl = Extension in each rod.

We know that Young's modulus (E),

$$210 \times 10^3 = \frac{P \times l}{A \times \delta l}$$

$$= \frac{\sigma_t \times l}{\delta l}$$

$$= \frac{85 \times 2.5 \times 10^3}{\delta l}$$

$$= \frac{212.5}{\delta l} \quad \ldots\ldots\ldots (\frac{P}{A_1} = \sigma_t)$$

$\therefore \delta l = 212.5 \times 103 / (210 \times 10^3)$

$\delta l = 1.012$ mm

Problem 2.4

The piston rod of a steam engine is 50 mm in diameter and 600 mm long. The diameter of the piston is 400 mm and the maximum steam pressure is 0.9 N/mm^2. Find the compression of the piston rod if the Young's modulus for the material of the piston rod is 210 kN/mm^2.

Given Data:

d = 50 mm

l = 600 mm

D = 400 mm

p = 0.9 N/mm^2

$E = 210\ kN/mm^2 = 210 \times 10^3\ N/mm^2$

Let δl = Compression of the piston rod.

We know that cross-sectional area of piston,

$$= \frac{\pi}{4} \times D^2$$

$$= \frac{\pi}{4} \times (400)^2$$

$$= 125680\ mm^2$$

∴ Maximum load acting on the piston due to steam,

P = Cross-sectional area of piston × Steam pressure

$= 125\,680 \times 0.9 = 113110\ N$

We also know that cross-sectional area of piston rod,

$$A = \frac{\pi}{4} \times d^2$$

$$= \frac{\pi}{4} \times (50)^2$$

$$A = 1964\ mm^2$$

and Young's modulus (E),

$$210 \times 10^3 = \frac{P \times l}{A \times \delta l}$$

$$= \frac{113110 \times 600}{1964 \times \delta l}$$

$$= \frac{34555}{\delta l}$$

$$\therefore \delta l = 34\,555 / (210 \times 10^3)$$

$$\delta l = 0.165\ mm$$

Shear Stress and Strain

When a body is subjected to two equal and opposite forces acting tangentially across the resisting section, as a result of which the body tends to shear off the section, then the stress induced is called shear stress.

The corresponding strain is known as shear strain and it is measured by the angular deformation accompanying the shear stress. The shear stress and shear strain are denoted by the Greek letters tau (τ) and phi (φ) respectively. Mathematically,

$$\text{Shear stress, } \tau = \frac{\text{Tangetial Stress}}{\text{Resisting Area}}$$

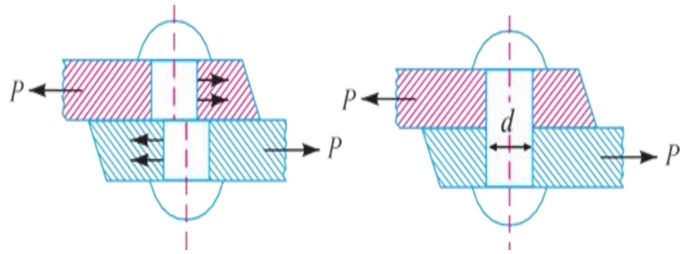

Fig 2.5 Single shearing of a riveted joint.

[Source: "A Textbook of Machine Design" by R.S. Khurmi & J.K. Gupta, Page: 93]

Consider a body consisting of two plates connected by a rivet as shown in Fig. 2.5 (a). In this case, the tangential force P tends to shear off the rivet at one cross-section as shown in Fig. 2.5 (b). It may be noted that when the tangential force is resisted by one cross-section of the rivet (or when shearing takes place at one cross-section of the rivet), then the rivets are said to be in single shear. In such a case, the area resisting the shear off the rivet,

$$A = \frac{\pi}{4} \times d^2$$

and shear stress on the rivet cross-section,

$$\tau = P/A$$

$$= \frac{P}{\frac{\pi}{4} \times d^2}$$

$$\tau = \frac{4P}{\pi \times d^2}$$

Now let us consider two plates connected by the two cover plates as shown in Fig. 2.5 (a). In this case, the tangential force P tends to shear off the rivet at two cross-sections as shown in Fig 2.5 (b). It may be noted that when the tangential force is resisted by two cross-sections of the rivet (or when the shearing takes place at two cross-sections of the rivet), then the rivets are said to be in double shear. In such a case, the area resisting the shear off the rivet,

$$A = 2 \times \frac{\pi}{4} \times d^2 \text{ ....(for double shear)}$$

and shear stress on the rivet cross-section,

$$\tau = P/A$$

$$\tau = \frac{P}{2 \times \frac{\pi}{4} \times d^2}$$

$$\tau = \frac{2P}{\pi \times d^2}$$

Notes: 1. All lap joints and single cover butt joints are in single shear, while the butt joints with double cover plates are in double shear.

2. In case of shear, the area involved is parallel to the external force applied.

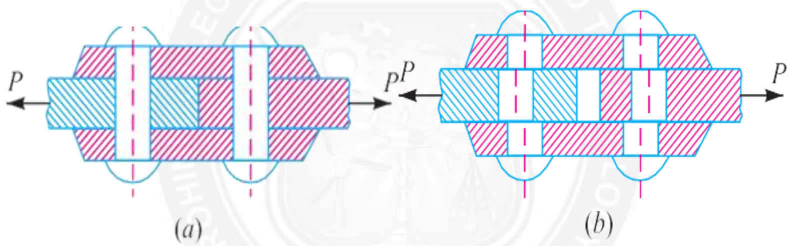

Fig 2.6 Double shearing of a riveted joint.

[Source: "A Textbook of Machine Design" by R.S. Khurmi & J.K. Gupta, Page: 94]

3. When the holes are to be punched or drilled in the metal plates, then the tools used to perform the operations must overcome the ultimate shearing resistance of the material to be cut. If a hole of diameter 'd' is to be punched in a metal plate of thickness 't', then the area to be sheared,

$$A = \pi\, d \times t$$

and the maximum shear resistance of the tool or the force required to punch a hole,

$$P = A \times \tau_u = \pi\, d \times t \times \tau_u$$

where τ_u = Ultimate shear strength of the material of the plate.

Shear Modulus or Modulus of Rigidity

It has been found experimentally that within the elastic limit, the shear stress is directly proportional to shear strain. Mathematically

$\tau \propto \varphi$ or $\tau = C. \varphi$ or $\tau / \varphi = C$

where τ = Shear stress,

φ = Shear strain, and

C = Constant of proportionality, known as shear modulus or modulus of rigidity. It is also denoted by N or G.

The following table shows the values of modulus of rigidity (C) for the materials in everyday use:

Table 2.2 Values of C for the commonly used materials.

Material	Modulus of rigidity (C) in GPa i.e. GN/m^2 or kN/mm^2
Steel	80 to100
Wrought iron	80 to 90
Cast iron	40 to 50
Copper	30 to 50
Brass	30 to 50
Timber	10

[Source: "A Textbook of Machine Design" by R.S. Khurmi & J.K. Gupta, Page: 94]

Problem 2.5

Calculate the force required to punch a circular blank of 60 mm diameter in a plate of 5 mm thick. The ultimate shear stress of the plate is 350 N/mm^2.

Given Data:

$d = 60$ mm

$t = 5$ mm

$\tau_u = 350\ N/mm^2$

We know that area under shear,

$A = \pi\ d \times \tau$

$= \pi \times 60 \times 5$

$= 942.6\ mm^2$

and force required to punch a hole,

$P = A \times \tau_u$

$= 942.6 \times 350$

$= 329910$ N

P = 329.91 kN

Problem 2.6

A pull of 80 kN is transmitted from a bar X to the bar Y through a pin as shown in Fig. 2.7. If the maximum permissible tensile stress in the bars is 100 N/mm2 and the permissible shear stress in the pin is 80 N/mm2, find the diameter of bars and of the pin.

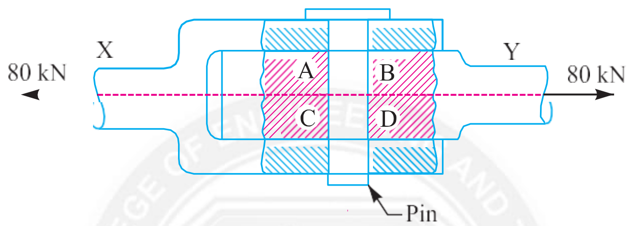

Fig 2.7

[Source: "A Textbook of Machine Design" by R.S. Khurmi & J.K. Gupta, Page: 95]

Given Data:

$P = 80 \text{ kN} = 80 \times 10^3$ N

$\sigma_t = 100$ N/mm^2

$\tau = 80$ N/mm2

Diameter of the bars

Let D_b = Diameter of the bars in mm.

$\therefore$Area, $A_b = \frac{\pi}{4} \times (D_b)^2$

$= 0.7854\ (D_b)^2$

We know that permissible tensile stress in the bar (σ_t),

$$100 = \frac{P}{A_b} = \frac{80\times10^3}{0.7854(D_b{}^2)}$$

$\therefore$ (Db)^2 = 101846 / 100 = 1018.46

or Db = 32 mm.

Diameter of the pin

Let D_p = Diameter of the pin in mm.

Since the tensile load P tends to shear off the pin at two sections i.e. at AB and CD, therefore the pin is in double shear.

$$\therefore \text{Resisting area, } A_p = 2 \times \frac{\pi}{4} (D_p)^2$$

$$= 1.571\ (D_p)^2$$

We know that permissible shear stress in the pin (τ),

$$100 = \frac{P}{A_P} = \frac{80 \times 10^3}{1.571 (D_P{}^2)}$$

$$(D_p)^2 = 50.9 \times 103/80$$

$$= 636.5 \text{ or}$$

$$D_p = 25.2 \text{ mm}$$